National
Qualifications
2023

X847/76/11

Mathematics
Paper 1 (Non-Calculator)

Marking Instructions
THURSDAY, 4 MAY

Strictly Confidential
These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ $\quad\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$ $\quad\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking Information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :

- If the candidate has written in any unexpected areas of their answer booklet
- If the script is legible and that it does not require to be re-scanned
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate
- If the candidate has continued an answer to a question at the back or in a different location in the booklet
- The presence of any non-script related objects.

No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).

Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.

Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.

Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking annotations such as tick/cross or the 

annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.

Link tool: The Link tool  allows you to link pages/additional objects to a particular question item on a response.

In "Full Response View":

- Check which question the candidate's answer relates to
- Click on the question in the marks display panel
- On the left hand side, select the Link Page check box beneath the thumbnail for the page
- Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed **do not** belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking **OR** If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations		
Annotation	**Annotation Name**	**Instructions on use of annotation**
	Correct Point	DO NOT USE THIS ANNOTATION
	Correct Point	A tick should be placed on the script at the point where a mark is awarded (or at the end of that line of working).
	Incorrect Point	DO NOT USE THIS ANNOTATION
	Cross 1	A cross is used to indicate where a mark has not been awarded.
	Highlight	This is used to highlight or underline an error.
	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
	Omission	An omission symbol should be used to show that something is missing, such as part of a solution or a crucial step in the working.
	Tick 1	A tick 1 should be used to indicate 'correct' working where a mark is awarded as a result of follow through from an error.
	Tick 2	A tick 2 should be used to indicate correct working which is irrelevant or insufficient to award any marks. This should also be used for working which is not of equivalent difficulty.
	Horizontal wavy line	A horizontal wavy line should be used to indicate a minor error which is not being penalised, e.g. bad form (bad form only becomes bad form if subsequent working is correct).

Marking instructions for each question

Question	Generic Scheme	Illustrative Scheme	Max Mark
1.	Given that $y = x^{\frac{5}{3}} - \frac{10}{x^4}$, where $x \neq 0$, find $\frac{dy}{dx}$.		
	$\bullet^1$ express second term in differentiable form	$\bullet^1$ $\ldots - 10x^{-4}$ stated or implied by $\bullet^3$	**3**
	$\bullet^2$ differentiate one term	$\bullet^2$ $\frac{5}{3}x^{\frac{2}{3}}\ldots$ or $\ldots + 40x^{-5}$	
	$\bullet^3$ complete differentiation	$\bullet^3$ $\frac{5}{3}x^{\frac{2}{3}} + 40x^{-5}$	

Notes:

1. Where candidates "differentiate over two lines" see Candidates A and B.
2. $\bullet^3$ is only available for differentiating a term with a negative index.
3. Where candidates attempt to integrate throughout, only $\bullet^1$ is available.

Commonly Observed Responses:

Candidate A - differentiating over two lines

$y = x^{\frac{5}{3}} - \frac{10}{x^4}$

$y = \frac{5}{3}x^{\frac{2}{3}} - 10x^{-4}$ $\bullet^1$ ✓

$y = \frac{5}{3}x^{\frac{2}{3}} + 40x^{-5}$ $\bullet^2$ ✓ $\bullet^3$ ✗

Candidate B - differentiating over two lines

$y = x^{\frac{5}{3}} - \frac{10}{x^4}$

$y = \frac{5}{3}x^{\frac{2}{3}} - 10x^{-4}$ $\bullet^1$ ✓

$y = \frac{5}{3}x^{\frac{2}{3}} + 40x^{-3}$ $\bullet^2$ ✓ $\bullet^3$ ✗

Candidate C

$\vdots$

$\frac{5}{3}x^{\frac{2}{3}} + 40x^{-5} + c$ $\bullet^3$ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			P and Q are the points $(-2, 6)$ and $(10, 0)$. Find the equation of the perpendicular bisector of PQ.		
			$\bullet^1$ find midpoint of PQ	$\bullet^1$ $(4,3)$	**4**
			$\bullet^2$ calculate gradient of PQ	$\bullet^2$ $-\frac{1}{2}$ or $-\frac{6}{12}$	
			$\bullet^3$ state perpendicular gradient	$\bullet^3$ 2 stated or implied by $\bullet^4$	
			$\bullet^4$ determine equation of perpendicular bisector	$\bullet^4$ $y = 2x - 5$	

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient of the perpendicular bisector must appear in fully simplified form at $\bullet^3$ or $\bullet^4$ stage for $\bullet^4$ to be awarded.
3. At $\bullet^4$, accept $2x - y = 5$, $y - 2x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3.	Solve $\log_5 x - \log_5 3 = 2$.		
	Method 1 •[1] apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ •[2] write in exponential form •[3] process for x	**Method 1** •[1] $\log_5 \frac{x}{3} \ldots$ stated or implied by •[2] •[2] $\frac{x}{3} = 5^2$ •[3] 75	**3**
	Method 2 •[1] apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ •[2] apply $m\log_5 x = \log_5 x^m$ •[3] process for x	**Method 2** •[1] $\log_5 \frac{x}{3} \ldots$ •[2] $\ldots = \log_5 5^2$ •[3] 75	**3**

Notes:

1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidates A and B for exceptions.
2. Where candidates do not use exponentials at •[2], •[3] is not available – see Candidate C.

Commonly Observed Responses:

Candidate A - incorrect exponential

$\log_5 \frac{x}{3} = 2$ •[1] ✓

$\frac{x}{3} = 2^5$ •[2] ✗

$x = 96$ •[3] ✓[1]

Candidate B

$\log_5 3x = 2$ •[1] ✗

$3x = 5^2$ •[2] ✓[1]

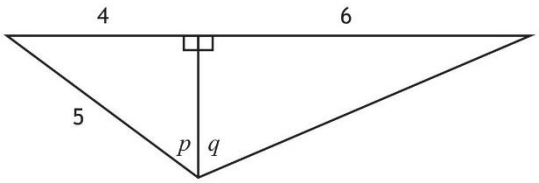
$x = \frac{25}{3}$ •[3] ✓[1]

Candidate C - no use of exponentials

$\log_5 \frac{x}{3} = 2$ •[1] ✓

$\frac{x}{3} = 10$ •[2] ✗

$x = 30$ •[3] ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		The diagram shows two right-angled triangles with angles p and q as marked.  (a) Determine the value of: (i) $\cos p$ (ii) $\cos q$.		
			$\bullet^1$ find $\cos p$	$\bullet^1$ $\frac{3}{5}$	**1**
			$\bullet^2$ find $\cos q$	$\bullet^2$ $\frac{3}{\sqrt{45}}\left(=\frac{1}{\sqrt{5}}\right)$	**1**
Notes:					
1. Accept $\frac{3}{3\sqrt{5}}$ for $\bullet^2$.					
Commonly Observed Responses:					

Question	Generic Scheme	Illustrative Scheme	Max Mark
(b)	(b) Hence determine the value of $\cos(p+q)$.		
	$\bullet^3$ select appropriate formula and express in terms of p and q $\bullet^4$ substitute into addition formula $\bullet^5$ evaluate $\cos(p+q)$	$\bullet^3$ $\cos p\cos q-\sin p\sin q$ stated or implied by $\bullet^4$ $\bullet^4$ $\frac{3}{5}\times\frac{3}{\sqrt{45}}-\frac{4}{5}\times\frac{6}{\sqrt{45}}$ $\bullet^5$ $-\frac{3}{\sqrt{45}}\left(=-\frac{1}{\sqrt{5}}\right)$	**3**

Notes:

2. Award $\bullet^3$ for candidates who write $\cos\left(\frac{3}{5}\right)\times\cos\left(\frac{3}{\sqrt{45}}\right)-\sin\left(\frac{4}{5}\right)\times\sin\left(\frac{6}{\sqrt{45}}\right)$. $\bullet^4$ and $\bullet^5$ are unavailable.
3. For any attempt to use $\cos(p+q)=\cos p\pm\cos q$, $\bullet^4$ and $\bullet^5$ are unavailable.
4. $\bullet^5$ is only available if either the surd part or the non-surd part of the fraction is simplified as far as possible. Accept $-\frac{3}{3\sqrt{5}}$, $-\frac{\sqrt{45}}{15}$, $-\frac{15}{15\sqrt{5}}$ or answers obtained on follow through which do not require simplification. Do not accept $-\frac{15}{5\sqrt{45}}$.
5. $\bullet^5$ is only available for an answer expressed as a single fraction.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.			The equation $2x^2+(3p-2)x+p=0$ has equal roots. Determine the possible values of p.		
			$\bullet^1$ use the discriminant $\bullet^2$ apply condition and express in standard quadratic form $\bullet^3$ process for p	$\bullet^1$ $(3p-2)^2-4\times2\times p$ $\bullet^2$ $9p^2-20p+4=0$ $\bullet^3$ $\frac{2}{9}, 2$	**3**

Notes:

1. Where candidates state an incorrect condition, $\bullet^2$ is not available. However, $\bullet^3$ is available for finding the roots of the quadratic - see Candidate B.
2. Where x appears in any expression, no further marks are available.

Commonly Observed Responses:

Candidate A

(For equal roots) $b^2-4ac=0$

$(3p-2)^2-4\times2\times p$ $\bullet^1$ ✓

$9p^2-20p+4$ $\bullet^2$ ✓

$p=\frac{2}{9}, 2$ $\bullet^3$ ✓

Candidate B

(For equal roots) $b^2-4ac>0$ $\bullet^2$ ✗

$(3p-2)^2-4\times2\times p$ $\bullet^1$ ✓

$9p^2-20p+4=0$

$p=\frac{2}{9}, 2$ $\bullet^3$ $\checkmark_1$

⋮

Question	Generic Scheme	Illustrative Scheme	Max Mark
6.	Find $\int\left(2x^5-6\sqrt{x}\right)dx,\ x\geq 0.$		
	$\bullet^1$ express second term in integrable form	$\bullet^1\ \ldots-6x^{\frac{1}{2}}$	**4**
	$\bullet^2$ integrate one term	$\bullet^2\ \frac{2}{6}x^6\ldots$ or $\ldots-\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$	
	$\bullet^3$ integrate other term	$\bullet^3\ \ldots-\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ or $\frac{2}{6}x^6\ldots$	
	$\bullet^4$ complete integration	$\bullet^4\ \frac{1}{3}x^6-4x^{\frac{3}{2}}+c$	

Notes:

1. The mark for integrating the final term is only available if candidates integrate a term with a fractional index.
2. Do not penalise the appearance of an integral sign and/or dx throughout.
3. Do not penalise the omission of '$+c$' at $\bullet^2$ or $\bullet^3$.
4. All coefficients must be simplified at $\bullet^4$ stage for $\bullet^4$ to be awarded.
5. Accept $\frac{x^6-12x^{\frac{3}{2}}}{3}+c$ for $\bullet^4$ but do not accept $\frac{2x^6-24x^{\frac{3}{2}}}{6}+c$.
6. $\bullet^2$, $\bullet^3$ and $\bullet^4$ are not available within an invalid strategy.

Commonly Observed Responses:

Candidate A

$\int\left(2x^5-6x^{\frac{1}{2}}\right)dx$ $\bullet^1$ ✓

$=\frac{2x^6}{6}-\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}+c$ $\bullet^2$ ✓ $\bullet^3$ ✓

$=\frac{2x^6}{6}-4x^{\frac{3}{2}}+c$

$=\frac{1}{3}x^6-4\sqrt{x}+c$ $\bullet^4$ ×

$\bullet^4$ cannot be awarded over two lines of working

Candidate B - integrating over two lines

$\frac{2x^6}{6}-6x^{\frac{1}{2}}$ $\bullet^1$ ✓

$=\frac{2x^6}{6}-\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}+c$ $\bullet^2$ ✓ $\bullet^3$ ×

$=\frac{1}{3}x^6-4x^{\frac{3}{2}}+c$ $\bullet^4$ ✓1

Candidate C - insufficient evidence

$\int 2x^5-6x^{\frac{1}{2}}dx$ $\bullet^1$ ✓

$\frac{1}{3}x^6-9x^{\frac{3}{2}}+c$ $\bullet^2$ ✓ $\bullet^3$ ^ $\bullet^4$ ×

Candidate D

⋮ $\bullet^1$ ✓

$=\frac{1}{3}x^6-4x^{\frac{3}{2}}$ $\bullet^2$ ✓ $\bullet^3$ ✓

$=\frac{1}{3}x^6-4\sqrt{x^3}+c$ $\bullet^4$ ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		(a) Evaluate $\log_2 5+\log_2 \frac{1}{40}$.		
			$\bullet^1$ use laws of logs $\bullet^2$ evaluate log	$\bullet^1$ $\log_2 \frac{5}{40}$ $\bullet^2$ -3	**2**
Notes:					
1. Do not penalise the omission of the base of the logarithm at $\bullet^1$. 2. Correct answer with no working, award 0/2.					
Commonly Observed Responses:					
Candidate A - introducing a variable $\log_2\left(5\times\frac{1}{40}\right)$ $\bullet^1$ ✓ $\log_2 \frac{1}{8}$ $2^x=\frac{1}{8}$ $x=-3$ $\bullet^2$ ✓					

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		(b) Given that $a\in\mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a.		
			$\bullet^3$ state range	$\bullet^3$ $0<a<1$	**1**
Notes:					
3. At $\bullet^3$ accept "$a>0$ and $a<1$." or "$a>0$, $a<1$".					
Commonly Observed Responses:					

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Question	Generic Scheme	Illustrative Scheme	Max Mark
8.	A function, f, is defined on $\mathbb{R}$, the set of real numbers, by $f(x)=x^3+3x^2-9x+5$. Find the coordinates of the stationary points of f and determine their nature.		
	$\bullet^1$ start to differentiate	$\bullet^1$ $3x^2\ldots$ or $\ldots+6x\ldots$ or $\ldots-9$	**6**
	$\bullet^2$ complete differentiation and equate to 0	$\bullet^2$ $3x^2+6x-9=0$	
	$\bullet^3$ solve for x	$\bullet^3$ -3 and 1	
	$\bullet^4$ process for y	$\bullet^4$ 32 and 0	
	$\bullet^5$ construct nature table(s)	$\bullet^5$ $\bullet^5$ $\bullet^6$ x | … | -3 | … | 1 | … $f'(x)$ | + | 0 | − | 0 | + shape | ↗ | — | ↘ | — | ↗	
	$\bullet^6$ interpret and state conclusions	$\bullet^6$ max at $(-3,32)$; min at $(1,0)$	

Notes:

1. For a numerical approach award 0/6.
2. $\bullet^2$ is only available if '$=0$' appears at the $\bullet^2$ stage or in working leading to $\bullet^3$, however see Candidates A and B.
3. Candidates who equate their derivative to 0, may use division by 3 as a strategy - see candidates B, C and D.
4. $\bullet^3$ is available to candidates who factorise **their** derivative from $\bullet^2$ as long as it is of equivalent difficulty.
5. $\bullet^3$ and $\bullet^4$ may be awarded vertically.
6. $\bullet^5$ is not available where any errors are made in calculating values of $f'(x)$.
7. $\bullet^5$ and $\bullet^6$ may be awarded vertically.
8. $\bullet^6$ is still available in cases where a candidate's table of signs does not lead legitimately to a maximum/minimum shape.
9. Candidates may use the second derivative - see Candidates E and F.
10. Accept "max when $x=-3$" **and** "min when $x=1$" for $\bullet^6$.

Commonly Observed Responses:

Candidate A

Stationary points when $f'(x)=0$

$f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓

$f'(x)=3(x+3)(x-1)$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate B

Stationary points when $f'(x)=0$

$f'(x)=3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ✓

⋮

$f'(x)=(x+3)(x-1)$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate C - division by 3

$3x^2+6x-9=0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$x^2+2x-3=0$

$x=-3,\ 1$ $\bullet^3$ ✓

Candidate D - derivative never equated to 0

$3x^2+6x-9$ $\bullet^1$ ✓ $\bullet^2$ ^

$x^2+2x-3=0$

$x=-3,\ 1$ $\bullet^3$ ✓1

Candidate E - second derivative			**Candidate F - second derivative**		
$f''(x)=6x+6$			$f''(x)=6x+6$		
$\bullet^5$ ✓	$\bullet^6$ ✓		$\bullet^5$ ✓	$\bullet^6$ ✓	
$f''(-3)<0$	$f''(1)>0$	$\bullet^5$ ✓	$f''(-3)=-12$,	$f''(1)=12$	$\bullet^5$ ✓
so max at $(-3,32)$	so min at $(1,0)$	$\bullet^6$ ✓	$-12<0$	$12>0$	
			so max at $(-3,32)$	so min at $(1,0)$	$\bullet^6$ ✓

For the table of signs for a derivative, accept:

x	-3^-	-3	-3^+
$f'(x)$	+	0	−
Slope or shape			

AND

x	1^-	1	1^+
$f'(x)$	−	0	+
Slope or shape			

x	$\to$	-3	$\to$
$f'(x)$	+	0	−
Slope or shape			

Arrows are taken to mean 'in the neighbourhood of'

AND

x	$\to$	1	$\to$
$f'(x)$	−	0	+
Slope or shape			

Arrows are taken to mean 'in the neighbourhood of'

x	a	-3	b
$f'(x)$	+	0	−
Slope or shape			

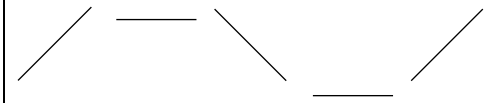
Where $a<-3$ and $-3<b<1$

AND

x	c	1	d
$f'(x)$	−	0	+
Slope or shape			

Where $-3<c<1$ and $d>1$

For the table of signs for a derivative, accept:

x	$\to$	-3	$\to$	1	$\to$
$f'(x)$	+	0	−	0	+
Slope or shape					

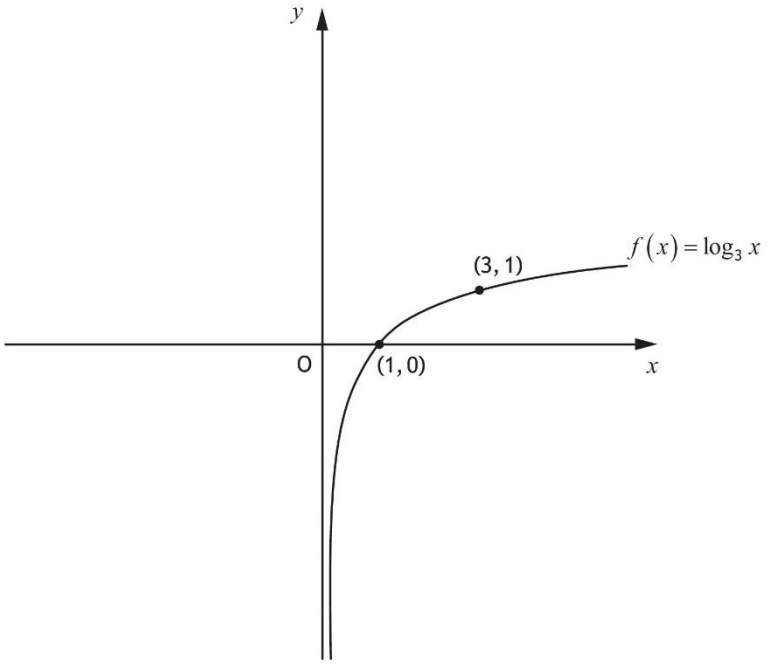
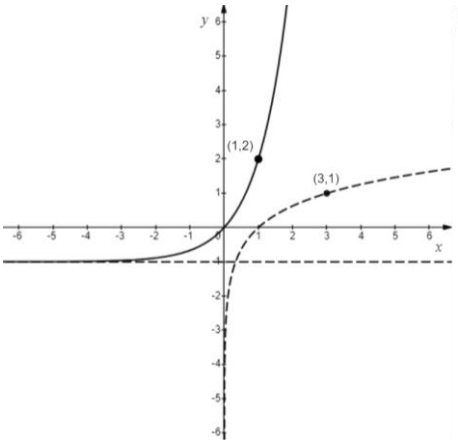
Since the function is continuous $-3\to 1$ is acceptable

x	a	-3	b	1	c
$f'(x)$	+	0	−	0	+
Slope or shape					

Since the function is continuous $-3<b<1$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $f'(x)$ is an acceptable alternative to writing '+' or '−' signs.
- Acceptable variations of $f'(x)$ are: f', $\frac{df}{dx}$, $\frac{dy}{dx}$, $3x^2+6x-9$ and $3(x+3)(x-1)$

  but NOT x^2+2x-3 or $(x+3)(x-1)$.

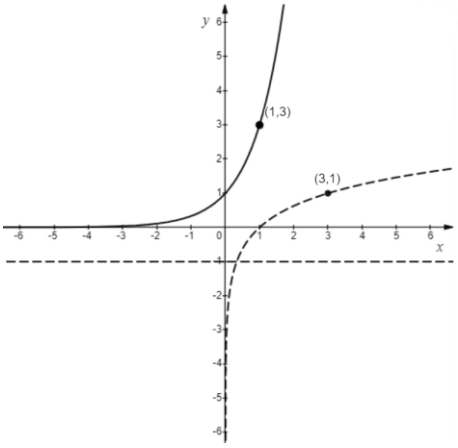
Question	Generic Scheme	Illustrative Scheme	Max Mark
9.	The diagram shows the graph of the function $f(x) = \log_3 x$, where $x > 0$.  The inverse function, f^{-1}, exists. On the diagram in your answer booklet, sketch the graph of $y = f^{-1}(x) - 1$.		
	•[1] reflect the graph in $y = x$ •[2] translate the graph vertically by "-1" units following a reflection in $y = x$ identifiable from graph •[3] sketch the required function	•[1] a concave up curve above the x-axis for $x > 0$ •[2] curve passing through (0,0) and (1,2) •[3] curve approaches the line $y = -1$ from above as $x \to -\infty$  	**3**

Notes:

1. For $\bullet^1$ accept any graph of a function which is concave up within the first quadrant.
2. $\bullet^1$ is only available where the candidate has attempted to reflect the given curve in the line $y = x$.
3. $\bullet^3$ is only available where the curve passes through (0,0) and (1,2).
4. The line $y = -1$ does not need to be shown.
5. For a rotation, award 0/3 – for example see Candidate D.

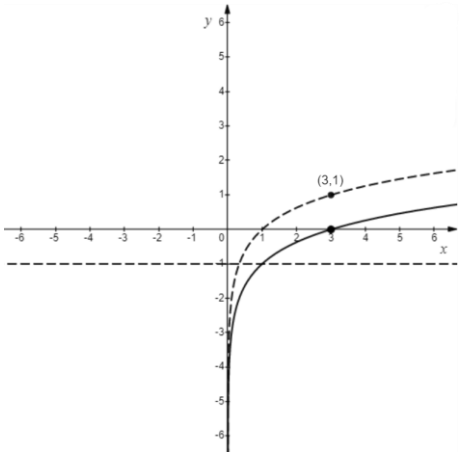
Commonly Observed Responses:

Candidate A - reflection only



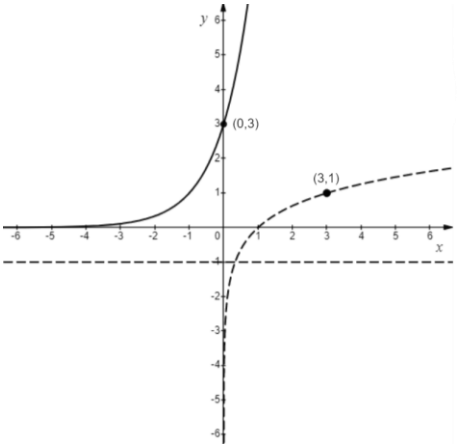
$\bullet^1$ ✓ $\bullet^2$ × $\bullet^3$ ×

Candidate B - translation only



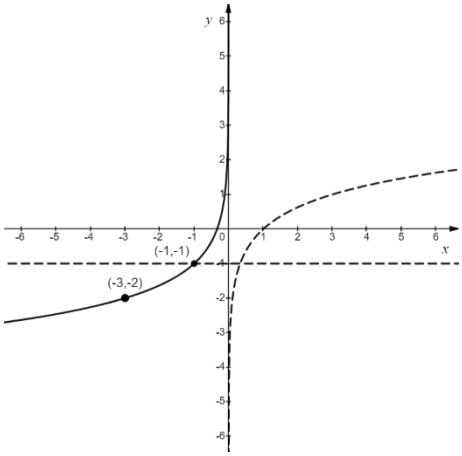
$\bullet^1$ × $\bullet^2$ × $\bullet^3$ ×

Candidate C - incorrect order of transformations



$\bullet^2$ × $\bullet^1$ ✓$_1$ $\bullet^3$ ×

Candidate D - rotation



$\bullet^1$ × $\bullet^2$ × $\bullet^3$ ×

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Question	Generic Scheme	Illustrative Scheme	Max Mark
10. (a)	(a) Show that $(x+5)$ is a factor of $x^4+3x^3-7x^2+9x-30$.		
	$\bullet^1$ use -5 in synthetic division or evaluation of quartic	$\bullet^1$ $-5 \mid 1 \quad 3 \quad -7 \quad 9 \quad -30$ $\quad\quad\quad 1 \quad\quad\quad\quad\quad \square$ or $(-5)^4+3\times(-5)^3-7\times(-5)^2$ $+9\times(-5)-30$	**2**
	$\bullet^2$ complete division/evaluation and interpret result	$\bullet^2$ $-5 \mid 1 \quad 3 \quad -7 \quad 9 \quad -30$ $\quad\quad\quad\quad -5 \quad 10 \quad -15 \quad 30$ $\quad\quad\quad 1 \quad -2 \quad 3 \quad -6 \quad \boxed{0}$ Remainder $=0 \therefore (x+5)$ is a factor or $f(-5)=0 \therefore (x+5)$ is a factor	

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(-5)=0$ so $(x+5)$ is a factor.'
   - 'since remainder = 0, it is a factor.'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the '0' or boxing the '0' without comment.
   - '$x=-5$ is a factor', '… is a root.'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

	x^3			
x	x^4	$-2x^3$		
5	$5x^3$			

$\bullet^1$ ✓

	x^3	$-2x^2$	$+3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

with no remainder

$\therefore (x+5)$ is a factor $\bullet^2$ ✓

Candidate B - grid method

	x^3			
x	x^4	$-2x^3$		
5	$5x^3$			

$\bullet^1$ ✓

	x^3	$-2x^2$	$+3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

$\therefore (x+5)(x^3-2x^2+3x-6)=x^4+3x^3-7x^2+9x-30$

$\therefore (x+5)$ is a factor $\bullet^2$ ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(b)		(a) Show that $(x+5)$ is a factor of $x^4+3x^3-7x^2+9x-30$. (b) Hence, or otherwise, solve $x^4+3x^3-7x^2+9x-30=0,\ x\in\mathbb{R}$.		
			$\bullet^3$ identify cubic and attempt to factorise	$\bullet^3$ eg $1 \mid 1\ \ -2\ \ 3\ \ -6$ $\quad\quad\ \ 1\ \ -1$ $\quad\quad 1\ \ -1\ \ \square$ or $2 \mid 1\ \ -2\ \ 3\ \ -6$ $\quad\quad\ \ 2\ \ 0$ $\quad\quad 1\ \ 0\ \ \square$	**5**
			$\bullet^4$ find second factor	$\bullet^4$ $2 \mid 1\ \ -2\ \ 3\ \ -6$ $\quad\quad\ \ 2\ \ 0\ \ 6$ $\quad\quad 1\ \ 0\ \ 3\ \ 0$ leading to $(x-2)$ or $x=2$	
			$\bullet^5$ identify quadratic	$\bullet^5$ x^2+3	
			$\bullet^6$ interpret lack of solutions of quadratic	$\bullet^6$ $b^2-4ac=-12<0$ $\therefore$ no (further real) solutions OR $x^2=-3$ or ~~$x^2=-3$~~ $\therefore$ no (further real) solutions	
			$\bullet^7$ state solutions	$\bullet^7$ $x=-5,\ x=2$	

Notes:

4. Candidates who arrive at $(x+5)(x-2)(x^2+3)$ by using algebraic long division or by inspection gain $\bullet^3$, $\bullet^4$ and $\bullet^5$.
5. Evidence for $\bullet^6$ may appear in the quadratic formula.
6. At $\bullet^6$ accept interpretations such as "no further roots", "no solutions" and "cannot factorise further" with justification.
7. At $\bullet^6$ accept $x=\sqrt{-3}$ **leading to** "not possible" and "not real".
8. Where there is no reference to b^2-4ac accept '$-12<0$ so no real roots' **with the remaining roots stated** for $\bullet^6$ - see candidates E and F.
9. Do not accept any of the following for $\bullet^6$:
   - $(x+5)(x-2)(x^2+3)$ no further roots / cannot factorise further.
   - $(x+5)(x-2)(\ldots\ \ldots)(\ldots\ \ldots)$ no further roots / cannot factorise further.
10. Where the quadratic factor obtained at $\bullet^5$ can be factorised, $\bullet^6$ and $\bullet^7$ are not available.
11. $\bullet^7$ is only available where $\bullet^6$ has been awarded.

Commonly Observed Responses:

Candidate C

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac=0-12<0$ $\bullet^6$ ✓

so no solutions

$x=-5,\ x=2$ $\bullet^7$ ✓

Candidate D

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac<0$ $\bullet^6$ ^

so no solutions

$x=-5,\ x=2$ $\bullet^7$ ✓2

Candidate E

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$-12<0$ $\bullet^6$ ✓

so no solutions

$x=-5,\ x=2$ $\bullet^7$ ✓

Candidate F

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$-12<0$

so no solutions $\bullet^6$ ^ $\bullet^7$ ^

Candidate G - grid method

(a)

	x^3	$-2x^2$	$3x$	-6
x	x^4	$-2x^3$	$+3x^2$	$-6x$
5	$5x^3$	$-10x^2$	$+15x$	-30

(b)

	x^2	…	…
x	x^3	…	…
…	…	…	…

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	x^2	$0x$	3
x	x^3	0	$3x$
-2	$-2x^2$	0	-6

$\bullet^4$ ✓

$(x+5)(x-2)(x^2+3)$ $\bullet^5$ ✓

$b^2-4ac=-12<0$

$\therefore$ no more solutions $\bullet^6$ ✓

$x=-5,\ x=2$ $\bullet^7$ ✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.	(a)		(a) Evaluate $\int_{\frac{\pi}{2}}^{\pi}(5\sin x-3\cos x)\,dx$.		
			$\bullet^1$ integrate $\bullet^2$ substitute limits $\bullet^3$ evaluate integral	$\bullet^1$ $-5\cos x-3\sin x$ $\bullet^2$ $\left[-5\cos\pi-3\sin\pi\right]$ $-\left[-5\cos\frac{\pi}{2}-3\sin\frac{\pi}{2}\right]$ $\bullet^3$ 8	3

Notes:

1. Where candidates make no attempt to integrate or use another invalid approach award 0/3 - see Candidate A. However see also Candidates B to F.
2. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign.
3. Candidates who change the limits to degrees before integrating cannot gain $\bullet^1$. However, $\bullet^2$ and $\bullet^3$ are still available.
4. $\bullet^3$ is only available where candidates have considered both limits within a trigonometric function.
5. The minimum acceptable response for $\bullet^2$ is $5-(-3)$.

Commonly Observed Responses:

Candidate A - introducing a power

Eg $5\sin x^2-3\cos x^2$ $\bullet^1$ × $\bullet^2$ × $\bullet^3$ ×

⋮

Candidate B - differentiating in full

$5\cos x+3\sin x$ $\bullet^1$ ×

$(5\cos\pi+3\sin\pi)-\left(5\cos\frac{\pi}{2}+3\sin\frac{\pi}{2}\right)$ $\bullet^2$ ✓2

-8 $\bullet^3$ ✓1

Candidate C - integrating one term

$5\cos x-3\sin x$ $\bullet^1$ ×

$(5\cos\pi-3\sin\pi)-\left(5\cos\frac{\pi}{2}-3\sin\frac{\pi}{2}\right)$ $\bullet^2$ ✓1

-2 $\bullet^3$ ✓1

Candidate D - integrating one term

$-5\cos x+3\sin x$ $\bullet^1$ ×

$(-5\cos\pi+3\sin\pi)-\left(-5\cos\frac{\pi}{2}+3\sin\frac{\pi}{2}\right)$ $\bullet^2$ ✓1

2 $\bullet^3$ ✓1

Candidate E - integrating one term

Eg $5\sin x-3\sin x$ $\bullet^1$ ×

$(5\sin\pi-3\sin\pi)-\left(5\sin\frac{\pi}{2}-3\sin\frac{\pi}{2}\right)$ $\bullet^2$ ✓1

-2 $\bullet^3$ ✓2

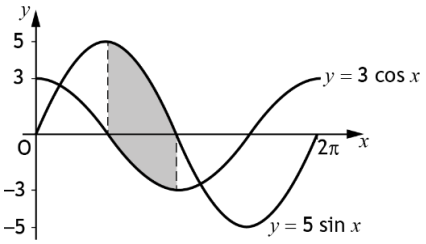
Mark 3 is not of equivalent difficulty - only 2 exact values

Candidate F - obtaining other expressions of the form $a\cos x+b\sin x$

Eg $-\frac{1}{5}\cos x-\frac{1}{3}\sin x$ $\bullet^1$ ×

$\left(-\frac{1}{5}\cos\pi-\frac{1}{3}\sin\pi\right)-\left(-\frac{1}{5}\cos\frac{\pi}{2}-\frac{1}{3}\sin\frac{\pi}{2}\right)$ $\bullet^2$ ✓2

$\frac{8}{15}$ $\bullet^3$ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
(b)	The diagram in your answer booklet shows the graphs with equations $y = 5\sin x$ and $y = 3\cos x$, $0 \le x \le 2\pi$. (b) On the diagram in your answer booklet, shade the area represented by the integral in (a).		
	$\bullet^4$ identify boundaries and shade area	$\bullet^4$ 	1
Notes:			
Commonly Observed Responses:			

Question	Generic Scheme	Illustrative Scheme	Max Mark
12.	Express $-2x^2-12x+7$ in the form $a(x+b)^2+c$.		
	Method 1 $\bullet^1$ identify common factor $\bullet^2$ complete the square $\bullet^3$ process for c and write in required form	**Method 1** $\bullet^1$ $-2(x^2+6x\ldots$ stated or implied by $\bullet^2$ $\bullet^2$ $-2(x+3)^2\ldots$ $\bullet^3$ $-2(x+3)^2+25$	**3**
	Method 2 $\bullet^1$ expand completed square form $\bullet^2$ equate coefficients $\bullet^3$ process for b and c and write in required form	**Method 2** $\bullet^1$ $ax^2+2abx+ab^2+c$ stated or implied by $\bullet^2$ $\bullet^2$ $a=-2,\ 2ab=-12,$ **and** $ab^2+c=7$ $\bullet^3$ $-2(x+3)^2+25$	

Notes:

1. $-2(x+3)^2+25$ with no working gains $\bullet^1$ and $\bullet^2$ only. However, see Candidate E.
2. $\bullet^1$ and $\bullet^3$ are not available in cases where $a>0$. For example, see Candidate F.

Commonly Observed Responses:

Candidate A

$-2(x^2+6)+7$

$-2\left((x+3)^2-9\right)+7$ $\bullet^1$ ✓ $\bullet^2$ ✓

$-2(x+3)^2+25$ $\bullet^3$ ✓

See the exception to marking principle (h)

Candidate B

$ax^2+2abx+ab^2+c$ $\bullet^1$ ✓

$a=-2,\ 2ab=-12,\ ab^2+c=7$ $\bullet^2$ ✓

$b=3,\ c=25$ $\bullet^3$ ^

$\bullet^3$ is lost as answer is not in completed square form

Candidate C

$-2(x^2+12x)+7$ $\bullet^1$ ✗

$-2\left((x+6)^2-36\right)+7$ $\bullet^2$ ✓1

$-2(x+6)^2+79$ $\bullet^3$ ✓1

Candidate D

$-2\left((x+6)^2-36\right)+7$ $\bullet^1$ ✗ $\bullet^2$ ✗

$-2(x+6)^2+79$ $\bullet^3$ ✓1

Candidate E

$-2(x+3)^2+25$ $\bullet^1$ ✓ $\bullet^2$ ✓

Check: $=-2(x^2+6x+9)+25$

$=-2x^2-12x-18+25$

$=-2x^2-12x+7$ $\bullet^3$ ✓

Candidate F

$-2x^2-12x+7$

$=2x^2+12x-7$ $\bullet^1$ ✗

$=2(x^2+6x\ldots$

$=2(x+3)^2\ldots$ $\bullet^2$ ✓1

$=-2(x+3)^2\ldots$ $\bullet^3$ ✗

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Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)		Functions f and g are defined by: • $f(x)=2\sin x$, where $0<x<\frac{\pi}{2}$ • $g(x)=2x$, where $0<x<\frac{\pi}{4}$ (a) (i) Evaluate $f\left(g\left(\frac{\pi}{6}\right)\right)$. (ii) Determine an expression for $f(g(x))$.		
		(i)	$\bullet^1$ state exact value	$\bullet^1$ $\sqrt{3}$	**1**
		(ii)	$\bullet^2$ interpret notation $\bullet^3$ state expression for $f(g(x))$	$\bullet^2$ $f(2x)$ or $2\sin(g(x))$ $\bullet^3$ $2\sin 2x$	**2**

Notes:

1. For $f(g(x))=2\sin 2x$ without working, award both $\bullet^2$ and $\bullet^3$.
2. Working for (a)(ii) may be found in (a)(i).

Commonly Observed Responses:

Candidate A
(a)(ii) $f(g(x))=4\sin x$ $\bullet^2$ ✗ $\bullet^3$ ✓$_1$

Candidate B - Beware of "2 attempts"
$f(g(x))=2\sin x$ $\bullet^2$ ✗ $\bullet^3$ ✗
$f(2x)=2\sin 2x$

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(b)		(b) (i) Given that $f(p)=\frac{1}{3}$, determine the exact value of $\sin p$. (ii) Hence, determine the exact value of $f(g(p))$.		
		(i)	$\bullet^4$ find the value of $\sin p$	$\bullet^4$ $\frac{1}{6}$	**1**
		(ii)	$\bullet^5$ expand $f(g(p))$ using double angle formula $\bullet^6$ find value of $\cos p$ $\bullet^7$ substitute and determine exact value	$\bullet^5$ $2\times 2\sin p\cos p$ or $4\sin p\cos p$ **stated explicitly** $\bullet^6$ $\frac{\sqrt{35}}{6}$ $\bullet^7$ $2\times 2\times\frac{1}{6}\times\frac{\sqrt{35}}{6}$ leading to $\frac{\sqrt{35}}{9}$	**3**

Notes:

6. $\bullet^5$ is not available for expansions which do not involve p. $\bullet^6$ and $\bullet^7$ are still available.
   However, accept $\sin^{-1}\left(\frac{1}{6}\right)$ in place of p - see Candidate C.
7. $\bullet^7$ is only available as a consequence of substituting into a valid formula from $\bullet^5$.
8. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Candidate C

$f(g(p))=4\sin\left(\sin^{-1}\left(\frac{1}{6}\right)\right)\cos\left(\sin^{-1}\left(\frac{1}{6}\right)\right)$ $\bullet^5$ ✓

$4\times\frac{1}{6}\times\frac{\sqrt{35}}{6}$ $\bullet^6$ ✓

$\frac{\sqrt{35}}{9}$ $\bullet^7$ ✓

[END OF MARKING INSTRUCTIONS]