2024 Paper 1

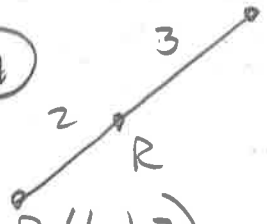
(1) $m = \tan\theta$
$= \tan 30°$
$= \frac{1}{\sqrt{3}}$
$\therefore y - 4 = \frac{1}{\sqrt{3}}(x - 0)$
$y - 4 = \frac{1}{\sqrt{3}}x$
$y = \frac{1}{\sqrt{3}}x + 4$

(2) a) $u_2 = \frac{1}{5}u_1 + 12$
$= \frac{1}{5}(20) + 12$
$= 16$

b)(i) $-1 < \frac{1}{5} < 1$

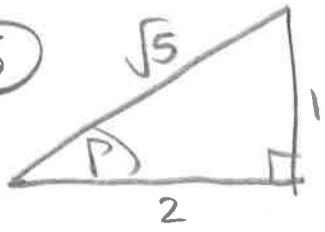
(ii) $L = \frac{12}{1 - \frac{1}{5}}$
$= \frac{12}{\frac{4}{5}}$
$= \frac{60}{4}$
$= 15$

(3) $\frac{dy}{dx} = 7(5x^2+3)^6 \times 10x$
$= 70x(5x^2+3)^6$

(4)


$\underline{r} = \underline{p} + \overrightarrow{PR}$
$= \underline{p} + \frac{2}{5}\overrightarrow{PQ}$
$= \begin{pmatrix}-6\\1\\2\end{pmatrix} + \frac{2}{5}\left[\begin{pmatrix}-1\\11\\-8\end{pmatrix} - \begin{pmatrix}-6\\1\\2\end{pmatrix}\right]$
$= \begin{pmatrix}-6\\1\\2\end{pmatrix} + \frac{2}{5}\begin{pmatrix}5\\10\\-10\end{pmatrix}$
$= \begin{pmatrix}-6\\1\\2\end{pmatrix} + \begin{pmatrix}2\\4\\-4\end{pmatrix}$
$= \begin{pmatrix}-4\\5\\-2\end{pmatrix}$
$\therefore R = (-4, 5, -2)$

(5) $y = 2x^3 - 7$
$2x^3 = y + 7$
$x^3 = \frac{y+7}{2}$
$x = \sqrt[3]{\frac{y+7}{2}}$
$\therefore f^{-1}(x) = \sqrt[3]{\frac{x+7}{2}}$

(6)


a)(i) $\sin 2P$
$= 2\sin P \cos P$
$= 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$
$= \frac{4}{5}$

(ii) $\cos 2P$
$= \cos^2 P - \sin^2 P$
$= \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$
$= \frac{4}{5} - \frac{1}{5}$
$= \frac{3}{5}$

b) $\sin 4p = 2\sin 2p \cos 2p$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \frac{24}{25}$

(7) $x^2 + (2x)^2 - 14x - 8(2x) + 45 = 0$

$x^2 + 4x^2 - 14x - 16x + 45 = 0$

$5x^2 - 30x + 45 = 0$

$5(x^2 - 6x + 9) = 0$

$5(x-3)^2 = 0$

$x = 3$

$y = 2(3)$

$= 6$

$\therefore (3, 6)$

(8) $a = 1$

$b = (m-4)$

$c = (2m-3)$

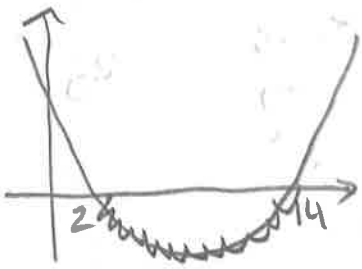
$b^2 - 4ac < 0$

$\therefore (m-4)^2 - 4(1)(2m-3) < 0$

$m^2 - 8m + 16 - 8m + 12 < 0$

$m^2 - 16m + 28 < 0$

$(m-14)(m-2) < 0$



$2 < m < 14$

(9) $\log_a 5 + \log_a 80 - 2\log_a 10$

$= \log_a 400 - \log_a 100$

$= \log_a 4 \quad \therefore k = 4$

(10) a) $f(1) = 2 + 3 - 4 - 3 + 2$

$= 0$

$\therefore (x-1)$ is a factor

$x = 1$ is a root.

b)

$2x^3 + 5x^2 + x - 2$

x	$2x^4$	$+5x^3$	$+x^2$	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	$+2$

Let $g(x) = 2x^3 + 5x^2 + x - 2$

$g(-1) = 2(-1)^3 + 5(-1)^2 + (-1) - 2$

$= -2 + 5 - 1 - 2$

$= 0$

$\therefore (x+1)$ is a factor

$x = -1$ is a root.

$2x^2 + 3x - 2$

x	$2x^3$	$+3x^2$	$-2x$
$+1$	$+2x^2$	$+3x$	-2

$\therefore (x-1)(x+1)(2x^2 + 3x - 2)$

$= (x-1)(x+1)(2x-1)(x+2)$

(11) a) $\cos x + \sqrt{3}\sin x$

$= k\cos\alpha\cos x + k\sin\alpha\sin x$

$\therefore k\sin\alpha = \sqrt{3}$

$k\cos\alpha = 1$

$k^2 = (\sqrt{3})^2 + 1^2$

$= 4$

$k = 2$

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= 60°$

$\therefore 2\cos(x-60)°$

b) Max $= 2$ @ $\cos(x-60) = 1$

$x - 60 = 0$

$x = 60°$

Min $= -2$ @ $\cos(x-60) = -1$

$x - 60 = 180°$

$x = 240°$

y-axis $x = 0$

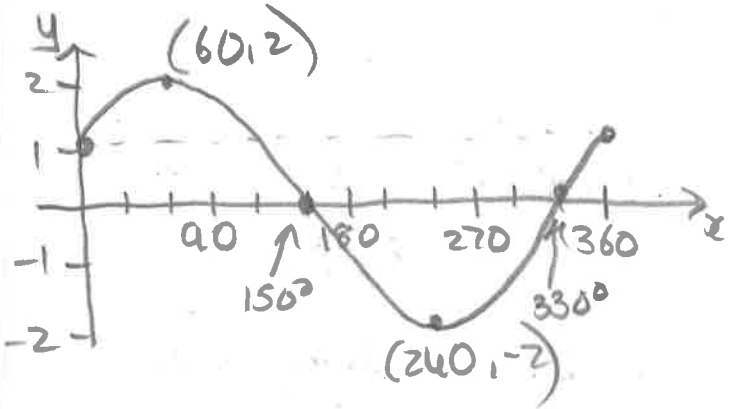
$\therefore 2\cos(0-60)°$

$= 2\cos(-60)°$

$= 2\cos(60)°$

$= 2\left(\frac{1}{2}\right)$

$= 1$



(12) $f(x) = 12x^{1/3}$

$f'(x) = 4x^{-2/3}$

$= \frac{4}{\sqrt[3]{x^2}}$

$f'(a) = 1$

$\therefore \frac{4}{\sqrt[3]{a^2}} = 1$

$4 = \sqrt[3]{a^2}$

$a^2 = 4^3$

$a^2 = 64$ $(a>0)$

$a = -8, 8$

(13) a) Mid $= (5,6)$

$M_{PQ} = \frac{10-2}{4-6}$ $\therefore M_{\perp} = \frac{1}{4}$

$= \frac{8}{-2}$

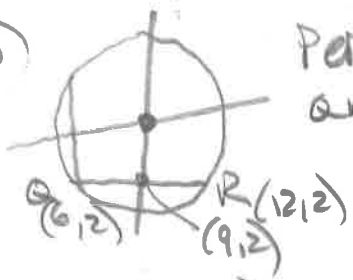
$= -4$

$\therefore y - 6 = \frac{1}{4}(x-5)$

$4y - 24 = x - 5$

$4y = x + 19$

b)



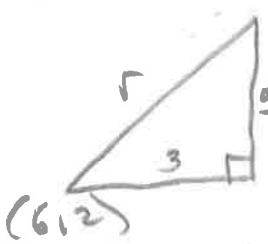
Perp. bisector of QR $\Rightarrow x = 9$

$\therefore 4y = (9) + 19$

$4y = 28$

$y = 7$

$\therefore C = (9,7)$



$r^2 = 5^2 + 3^2$

$= 34$

$\therefore (x-9)^2 + (y-7)^2 = 34$

2024 Paper 2

(1) a) mid = (4, 4)

$M_{BM} = \frac{4+6}{4+1}$

$= \frac{10}{5}$

$= 2$

$\therefore y - 4 = 2(x - 4)$

$y - 4 = 2x - 8$

$y = 2x - 4$

b) $M_{BC} = \frac{0+6}{11+1}$ $\therefore M_L = -2$

$= \frac{6}{12}$

$= \frac{1}{2}$

$y - 0 = -2(x - 11)$

$y = -2x + 22$

c) $y = y$

$2x - 4 = -2x + 22$

$4x = 26$

$x = 6.5$

$y = 2(6.5) - 4$

$= 9$

$\therefore (6.5, 9)$

(2) $y = \frac{8}{2^3}$

$= \frac{8}{8}$ $\therefore (2, 1)$

$= 1$

$y = 8x^{-3}$

$\frac{dy}{dx} = -24x^{-4}$

$= -\frac{24}{x^4}$

$\therefore m = -\frac{24}{2^4}$

$= -\frac{24}{16}$

$= -\frac{3}{2}$

$\therefore y - 1 = -\frac{3}{2}(x - 2)$

$2y - 2 = -3(x - 2)$

$2y - 2 = -3x + 6$

$2y = -3x + 8$

(3) a) $\vec{ED} = \underline{d} - \underline{e}$

$= \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -4 \\ 6 \end{pmatrix}$

$\vec{EF} = \underline{f} - \underline{e}$

$= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

b) (i) $\vec{ED} \cdot \vec{EF} = 2 + (-4) + 18$

$= 16$

(ii) $|\vec{ED}| = \sqrt{1 + 16 + 36}$

$= \sqrt{53}$

$|\vec{EF}| = \sqrt{4 + 1 + 9}$

$= \sqrt{14}$

$\therefore \text{DEF} = \cos^{-1}\left(\frac{16}{\sqrt{53}\sqrt{14}}\right)$

$= 54^\circ$

(4) a) $y = f(x-4)+2$

4 right, 2 up

$\therefore (-1, 3) \rightarrow (3, 5)$

(5) $\int_0^{\frac{\pi}{7}} \sin(5x)\,dx$

$= \left[-\frac{1}{5}\cos(5x)\right]_0^{\frac{\pi}{7}}$

$= \left(-\frac{1}{5}\cos\left(\frac{5\pi}{7}\right)\right) - \left(-\frac{1}{5}\cos(0)\right)$

$= (0.12469\ldots) - (-0.2)$

$= 0.325$

(6) $y = ax^b$

$\log_5 y = \log_5 ax^b$

$\log_5 y = \log_5 x^b + \log_5 a$

$\log_5 y = b\log_5 x + \log_5 a$

$m = \frac{10+2}{4-0}$ $\therefore \log_5 y = 3\log_5 x$

$= \frac{12}{4}$

$= 3$

$\therefore b = 3$ $\log_5 a = -2$

$a = 5^{-2}$

$a = \frac{1}{25}$

(7) Top − (Bottom)

$= 6 + 4x - 2x^2 - (x^3 - 6x^2 + 11x)$

$= 6 + 4x - 2x^2 - x^3 + 6x^2 - 11x$

$= -x^3 + 4x^2 - 7x + 6$

$\therefore \int_0^2 (-x^3 + 4x^2 - 7x + 6)\,dx$

$= \left[\frac{-x^4}{4} + \frac{4x^3}{3} - \frac{7x^2}{2} + 6x\right]_0^2$

$= \left(-\frac{16}{4} + \frac{32}{3} - \frac{28}{2} + 12\right) - (0)$

$= 4\frac{2}{3}u^2$

(8) a) $f(g(x))$
$= f(x+1)$
$= 2(x+1)^2 - 18$

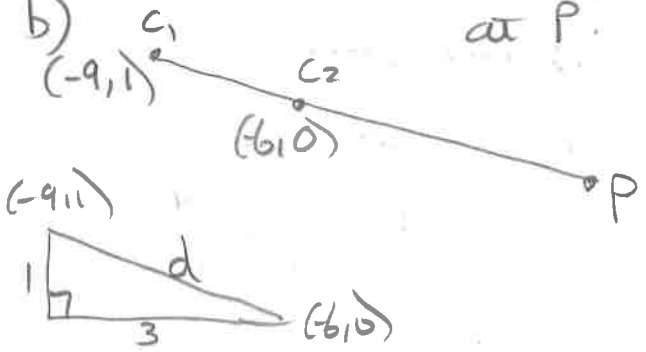
b) $\frac{1}{2(x+1)^2-18}$
$= \frac{1}{2(x^2+2x+1)-18}$
$= \frac{1}{2x^2+4x-16}$
$= \frac{1}{2(x^2+2x-8)}$
$= \frac{1}{2(x+4)(x-2)}$
$\therefore x \neq -4$
$x \neq 2$

(9) a) SP's @ $\frac{dy}{dx} = 0$
$\therefore x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$x = 3 \quad x = -1$
$y = \frac{1}{3}(3)^3 - 3^2 - 3(3) + 1$
$= -8 \quad (3, -8)$
$y = \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) + 1$
$= 2\frac{2}{3} \quad (-1, 2\frac{2}{3})$

b) $x = 6 \quad y = \frac{1}{3}(6)^3 - 6^2 - 3(6) + 1$
$= 19$
$\therefore$ Max $= 19$ @ $x = 6$
Min $= -8$ @ $x = 3$

(10) a) $2g = 18 \quad 2f = -2$
$g = 9 \quad f = -1 \quad c = -8$
Centre $= (-9, 1)$
radius $= \sqrt{9^2 + (-1)^2 - (-8)}$
$= \sqrt{90}$
$= 3\sqrt{10}$

b) Circles touch at P.



$d^2 = 3^2 + 1^2$
$= 10$
$d = \sqrt{10}$
$C_1 \to P = 3\sqrt{10}$
$\therefore C_2 \to P = 2\sqrt{10}$
$\therefore (x+6)^2 + y^2 = 40$

(11) a) $N = 6.8e^{k \times 0}$
$= 6.8$
$\therefore$ 6.8 million vehicles.

b) 2030 $\to t = 10$
$\therefore 12.5 = 6.8e^{10k}$
$e^{10k} = \frac{12.5}{6.8}$

$$10k = \ln\left(\frac{125}{6.8}\right)$$

$$k = \frac{\ln\left(\frac{125}{6.8}\right)}{10}$$

$$= 0.291$$

(12) $2\sin 2x - \sin^2 x = 0$

$4\sin x\cos x - \sin^2 x = 0$

$\sin x\,(4\cos x - \sin x) = 0$

$\sin x = 0$

$x = 0, 180°, \cancel{360°}$ $(0 \le x < 360)$

$4\cos x - \sin x = 0$

$4\cos x = \sin x$

$4 = \frac{\sin x}{\cos x}$

$\tan x = 4$

$x = \tan^{-1}(4)$

$x = 76°, 256°$

$\therefore x = 0, 76°, 180°, 256°$

(13) $y = k(x-3)^2(x+1)(x-5)$

$x = 0$ when $y = -9$

$\therefore -9 = k(-3)^2(1)(-5)$

$-9 = k(9)(-5)$

$-9 = -45k$

$k = \frac{1}{5}$

$\therefore y = \frac{1}{5}(x-3)^2(x+1)(x-5)$