National
Qualifications
2024

X847/76/11

Mathematics
Paper 1 (Non-Calculator)

Marking Instructions

MONDAY, 13 MAY

Strictly Confidential
These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) **Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

$$\begin{array}{lll} & \bullet^5 & \bullet^6 \\ \bullet^5 & x=2 & x=-4 \\ \bullet^6 & y=5 & y=-7 \end{array}$$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$

$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (CORs) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking Information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :

- If the candidate has written in any unexpected areas of their answer booklet
- If the script is legible and that it does not require to be re-scanned
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate
- If the candidate has continued an answer to a question at the back or in a different location in the booklet
- The presence of any non-script related objects.

No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).

Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.

Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.

Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking annotations such as tick/cross
or the SEEN annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.

Link tool: The Link tool 🔗 allows you to link pages/additional objects to a particular question item on a response.

In "Full Response View":

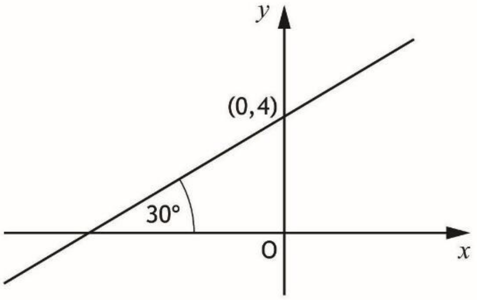
- Check which question the candidate's answer relates to
- Click on the question in the marks display panel
- On the left hand side, select the Link Page check box beneath the thumbnail for the page
- Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed **do not** belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Check if Missing Pages	If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations

Annotation	Annotation Name	Instructions on use of annotation
✓	Tick	A tick should be placed on the script at the point where a mark is awarded (or at the end of that line of working).
✗	Cross	A cross is used to indicate where a mark has not been awarded.
🖍	Highlight	This is used to highlight or underline an error.
SEEN	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
^	Omission	An omission symbol should be used to show that something is missing, such as part of a solution or a crucial step in the working.
✓1	Tick 1	A tick 1 should be used to indicate 'correct' working where a mark is awarded as a result of follow through from an error.
✓2	Tick 2	A tick 2 should be used to indicate correct working which is irrelevant or insufficient to award any marks. This should also be used for working which is not of equivalent difficulty.
〰	Horizontal wavy line	A horizontal wavy line should be used to indicate a minor error which is not being penalised, for example bad form (bad form only becomes bad form if subsequent working is correct).

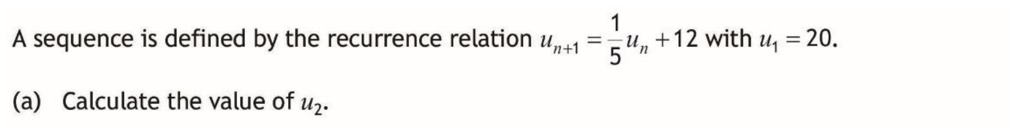
Question	Generic scheme	Illustrative scheme	Max mark
1.	A line passes through the point (0, 4) and makes an angle of 30° with the positive direction of the x-axis as shown in the diagram.  Determine the equation of the line.		
	$\bullet^1$ use $m=\tan\theta$	$\bullet^1$ $m=\tan 30°$	3
	$\bullet^2$ evaluate exact value	$\bullet^2$ $\frac{1}{\sqrt{3}}$	
	$\bullet^3$ determine equation	$\bullet^3$ eg $y=\frac{1}{\sqrt{3}}x+4$ or $\sqrt{3}y-4\sqrt{3}=x$	

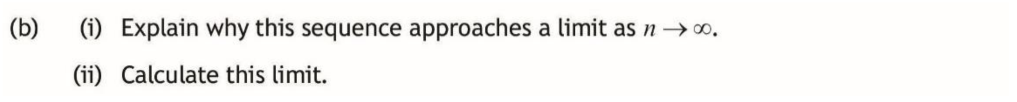
Notes:

1. Do not award $\bullet^1$ for $m=\tan^{-1}30°$. However $\bullet^2$ and $\bullet^3$ are still available.
2. Do not penalise the omission of a degree symbol at $\bullet^1$.
3. Where candidates make no reference to a trigonometric ratio, or use an incorrect trigonometric ratio, $\bullet^1$ and $\bullet^2$ are unavailable. See Candidate A.
4. $\bullet^3$ is only available as a consequence of attempting to use a tan ratio. See Candidate F.
5. $\bullet^3$ is not available for using a gradient of 30.
6. At $\bullet^3$ accept any rearrangement of a candidate's equation where constant terms have been simplified.
7. Accept $y-4=\frac{1}{\sqrt{3}}(x)$ but not $y-4=\frac{1}{\sqrt{3}}(x-0)$ for $\bullet^3$.

Commonly Observed Responses:

Candidate A - no trig ratio $m=\frac{1}{\sqrt{3}}$ $\bullet^1$ ^ $\bullet^2$ ✓$_2$ $y=\frac{1}{\sqrt{3}}x+4$ $\bullet^3$ ✓$_1$	**Candidate B** $m=\tan\theta$ $\bullet^1$ ^ $y=\frac{1}{\sqrt{3}}x+4$ $\bullet^2$ ✓ $\bullet^3$ ✓	**Candidate C** $m=\tan\theta$ $\bullet^1$ ^ $y=\sqrt{3}x+4$ $\bullet^2$ ✗ $\bullet^3$ ✗
Candidate D $m=\tan\theta=30$ $\bullet^1$ ✗ $m=\frac{1}{\sqrt{3}}$ $\bullet^2$ ✓$_1$ $y=\frac{1}{\sqrt{3}}x+4$ $\bullet^3$ ✓$_1$	**Candidate E - no reference to *m*** $\tan 30°=\frac{1}{\sqrt{3}}$ $\bullet^2$ ✓ $y-4=\frac{1}{\sqrt{3}}(x-0)$ $\bullet^1$ ✓ $y=\frac{1}{\sqrt{3}}x+4$ $\bullet^3$ ✓	**Candidate F - not using tan** $m=\sin 30°$ $\bullet^1$ ✗ $m=\frac{1}{2}$ $\bullet^2$ ✓$_2$ $y=\frac{1}{2}x+4$ $\bullet^3$ ✓$_2$

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>2.</td><td>(a)</td><td></td><td colspan="2">
A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{5}u_n + 12$ with $u_1 = 20$.
(a) Calculate the value of u_2.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$\bullet^1$ calculate second term</td><td>$\bullet^1$ 16</td><td>1</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. Candidates who use $u_0 = 20$ and then calculate $u_1 = 16$ gain $\bullet^1$.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="6"></td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		 (b) (i) Explain why this sequence approaches a limit as $n \to \infty$. (ii) Calculate this limit.		
		(i)	•² communicate condition for limit to exist	•² a limit exists as $-1<\frac{1}{5}<1$	1
		(ii)	•³ know how to calculate a limit •⁴ calculate limit	•³ $\frac{12}{1-\frac{1}{5}}$ or $L=\frac{1}{5}L+12$ •⁴ 15	2

Notes:

2. For •² accept:

   any of '$-1<\frac{1}{5}<1$' or '$\left|\frac{1}{5}\right|<1$' or '$0<\frac{1}{5}<1$' with no further comment;

   or statements such as:

   '$\frac{1}{5}$ lies between -1 and 1' or '$\frac{1}{5}$ is a proper fraction'.

3. •² is not available for:

   '$-1\leq\frac{1}{5}\leq 1$' or '$\frac{1}{5}<1$'

   or statements such as:

   'It is between -1 and 1.' or '$\frac{1}{5}$ is a fraction'.

4. Candidates who state $-1<a<1$ can only gain •² if it is explicitly stated that $a=\frac{1}{5}$.
5. Do not accept $L=\frac{b}{1-a}$ with no further working for •³.
6. •³ and •⁴ are not available to candidates who conjecture $L=15$ following the calculation of further terms in the sequence.
7. For $L=15$ with no working award 0/2.
8. •⁴ is only available where •³ has been awarded.

Commonly Observed Responses:

Candidate A	**Candidate B - no explicit reference to *a***
$a=\frac{1}{5}$ $-1<a<1$ so a limit exists •² ✓	$u_{n+1}=au_n+b$ $u_{n+1}=\frac{1}{5}u_n+12$ $-1<a<1$ so a limit exists •² ^

Question			Generic scheme	Illustrative scheme	Max mark
3.			Given that $y=\left(5x^2+3\right)^7$, find $\frac{dy}{dx}$.		
			•^{1} start to differentiate •^{2} complete differentiation	•^{1} $7\left(5x^2+3\right)^6\ldots$ •^{2} $\ldots\times 10x$	2

Notes:

1. •^{1} is awarded for the appearance of $7\left(5x^2+3\right)^6$.
2. For $70x\left(5x^2+3\right)^6$ with no working, award 2/2.
3. Accept $7u^6$ where $u=5x^2+3$ for •^{1}.
4. Do not award •^{2} where the answer includes '$+c$'.

Commonly Observed Responses:

Candidate A - differentiating over two lines $7\left(5x^2+3\right)^6$ •^{1} ✓ $7\left(5x^2+3\right)^6\times 10x$ •^{2} ^	**Candidate B - poor notation** $y=\left(5x^2+3\right)^7$ $y=5x^2+3$ $\frac{dy}{dx}=10x$ $\frac{dy}{dx}=7\left(5x^2+3\right)^6\times 10x$ •^{1} ✓ •^{2} ✓
Candidate C - poor communication $y=\left(5x^2+3\right)^7$ $y=7\left(5x^2+3\right)^6\times 10x$ •^{1} ✓ •^{2} ✓	**Candidate D - insufficient evidence for •^{1}** $70\left(5x^2+3\right)^6$ •^{1} ✗ •^{2} ✗ or $35\left(5x^2+3\right)^6$ •^{1} ✗ •^{2} ✗

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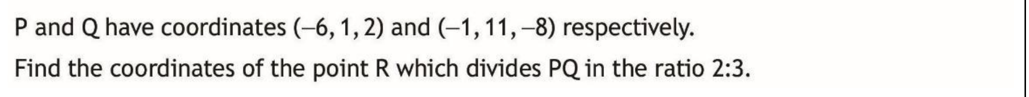
Question			Generic scheme	Illustrative scheme	Max mark
4.			P and Q have coordinates (–6, 1, 2) and (–1, 11, –8) respectively. Find the coordinates of the point R which divides PQ in the ratio 2:3.		
			Method 1 •¹ interpret ratio •² find coordinates of R	**Method 1** •¹ $\begin{pmatrix}2\\4\\-4\end{pmatrix}, \begin{pmatrix}-2\\-4\\4\end{pmatrix}, \begin{pmatrix}3\\6\\-6\end{pmatrix}$ or $\begin{pmatrix}-3\\-6\\6\end{pmatrix}$ •² $(-4,5,-2)$	**2**
			Method 2 •¹ interpret ratio •² find coordinates of R	**Method 2** •¹ eg $\overrightarrow{PR}=\frac{2}{5}\overrightarrow{PQ}$, $\overrightarrow{QR}=\frac{3}{5}\overrightarrow{QP}$ or $\overrightarrow{PR}=\frac{2}{3}\overrightarrow{RQ}$ •² $(-4,5,-2)$	
			Method 3 •¹ use section formula •² find coordinates of R	**Method 3** •¹ $\frac{1}{5}(3\mathbf{p}+2\mathbf{q})$ •² $(-4,5,-2)$	

Notes:

1. For $(-4,5,-2)$ without working award 2/2.
2. For $\begin{pmatrix}-4\\5\\-2\end{pmatrix}$ without working award 1/2.
3. For $(-3,7,-4)$ (ratio of 3:2 with working) award 1/2. See Candidate A.
4. For $\begin{pmatrix}-3\\7\\-4\end{pmatrix}$ without working award 0/2.

Commonly Observed Responses:

Candidate A

$\overrightarrow{PR} = \frac{3}{5}\overrightarrow{PQ}$ $\bullet^1$ ✗

$R = (-3, 7, -4)$ $\bullet^2$ $✓_1$

Candidate B

$\frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{2}{3}$ $\bullet^1$ ✓

$3\overrightarrow{PR} = 2\overrightarrow{RQ}$

$3(\mathbf{r} - \mathbf{p}) = 2(\mathbf{q} - \mathbf{r})$

$5\mathbf{r} = 2\mathbf{q} + 3\mathbf{p}$

Leading to correct answer of

$R = (-4, 5, -2)$ $\bullet^2$ ✓

Candidate C

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 10 \\ -10 \end{pmatrix}$

$R = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\bullet^1$ ✓

$R = \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$

$R = \begin{pmatrix} -4 \\ 5 \\ -2 \end{pmatrix}$

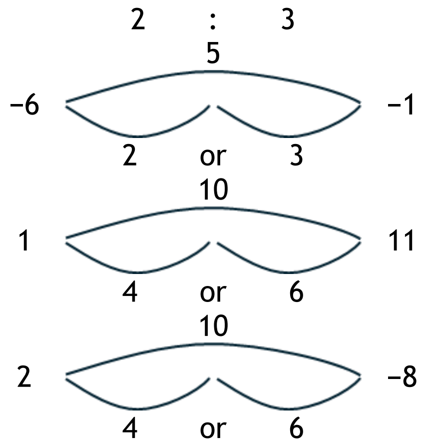
$R(-4, 5, -2)$ $\bullet^2$ ✓

Candidate D

$\overrightarrow{PR} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\bullet^1$ ✓

$R(-8, -3, 6)$ $\bullet^2$ ✗

Candidate E - stepping out using absolute values



$\bullet^1$ ✓

$R(-4, 5, -2)$ $\bullet^2$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.			 A function, h, is defined by $h(x)=2x^3-7$ where $x\in\mathbb{R}$. Find the inverse function, $h^{-1}(x)$.		
			Method 1	**Method 1**	**3**
			•1 equate composite function to x	•1 $h\left(h^{-1}(x)\right)=x$	
			•2 write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$	•2 $2\left(h^{-1}(x)\right)^3-7=x$	
			•3 state inverse function	•3 $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$	
			Method 2	**Method 2**	
			•1 write as $y=h(x)$ and start to rearrange	•1 $y=h(x)\Rightarrow x=h^{-1}(y)$ $y+7=2x^3$	
			•2 express x in terms of y	•2 $x=\sqrt[3]{\frac{y+7}{2}}$	
			•3 state inverse function	•3 $h^{-1}(y)=\sqrt[3]{\frac{y+7}{2}}$ $\Rightarrow h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$	

Notes:

1. In method 1, accept $2\left(h^{-1}(x)\right)^3-7=x$ for •1 and •2.
2. In method 2, accept '$y+7=2x^3$' without reference to $y=h(x)\Rightarrow x=h^{-1}(y)$ at •1.
3. In method 2, accept $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ without reference to $h^{-1}(y)$ at •3.
4. In method 2, beware of candidates with working where each line is not mathematically equivalent. See candidates A and B for example.
5. At •3 stage, accept h^{-1} written in terms of any dummy variable.

   For example $h^{-1}(y)=\sqrt[3]{\frac{y+7}{2}}$.
6. $y=\sqrt[3]{\frac{x+7}{2}}$ does not gain •3.
7. $h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$h(x)=2x^3-7$

$y=2x^3-7$

$x=\sqrt[3]{\frac{y+7}{2}}$ $\bullet^1$ ✓ $\bullet^2$ ✓

$y=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✗

$h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$

Candidate B

$h(x)=2x^3-7$

$y=2x^3-7$

$x=2y^3-7$ $\bullet^1$ ✗

$y=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ $\checkmark_1$

$h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ $\checkmark_1$

Candidate C

$x=2h(x)^3-7$ $\bullet^1$ ✗

$h(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ $\checkmark_1$

$h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ $\checkmark_1$

Candidate D - Method 1

$h\left(h^{-1}(x)\right)=2\left(h^{-1}(x)\right)^3-7$ $\bullet^2$ ✓

$x=2\left(h^{-1}(x)\right)^3-7$ $\bullet^1$ ✓

$h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✓

Candidate E

$x\rightarrow x^3\rightarrow 2x^3\rightarrow 2x^3+3=h(x)$

$\times 2\rightarrow +3$

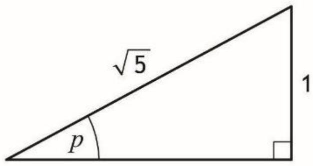
$\therefore -3\rightarrow \div 2$ $\bullet^1$ ✓

$\sqrt[3]{\frac{x+7}{2}}$ $\bullet^2$ ✓

$h^{-1}(x)=\sqrt[3]{\frac{x+7}{2}}$ $\bullet^3$ ✓

Candidate F - BEWARE of incorrect notation

$h'(x)=$ $\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		The right-angled triangle in the diagram is such that $\sin p = \frac{1}{\sqrt{5}}$ and $0 < p < \frac{\pi}{4}$.  (a) Determine the value of: (i) $\sin 2p$ (ii) $\cos 2p$.		
		(i)	$\bullet^1$ find value of $\cos p$ $\bullet^2$ substitute into the formula for $\sin 2p$ $\bullet^3$ simplify answer	$\bullet^1$ $\cos p = \frac{2}{\sqrt{5}}$ stated or implied by $\bullet^2$ $\bullet^2$ $2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}$ $\bullet^3$ $\frac{4}{5}$	**3**
		(ii)	$\bullet^4$ evaluate $\cos 2p$	$\bullet^4$ $\frac{3}{5}$	**1**

Notes:

1. Evidence for $\bullet^1$ may appear in (a)(ii).
2. Where a candidate substitutes an incorrect value for $\cos p$ at $\bullet^2$, $\bullet^2$ may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram or Pythagoras calculation. See Candidates A and B.
3. Where a candidate explicitly states a value for $\cos p$, subsequent working must follow from that value for $\bullet^2$ to be awarded.
4. $\bullet^3$ is only available as a consequence of substituting into a valid formula at $\bullet^2$.
5. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Candidate A – incorrect use of Pythagoras		**Candidate B – no evidence of Pythagoras**	
$\sqrt{\sqrt{5}^2+1^2}=\sqrt{6}$	$\bullet^1$ ✘		$\bullet^1$ ^
$2\times\frac{1}{\sqrt{5}}\times\frac{\sqrt{6}}{\sqrt{5}}$	$\bullet^2$ $\checkmark_1$	$2\times\frac{1}{\sqrt{5}}\times\frac{\sqrt{6}}{\sqrt{5}}$	$\bullet^2$ ✘
$\frac{2\sqrt{6}}{5}$	$\bullet^3$ $\checkmark_1$	$\frac{2\sqrt{6}}{5}$	$\bullet^3$ $\checkmark_1$
Candidate C			
$2\times\sin\frac{1}{\sqrt{5}}\times\cos\frac{2}{\sqrt{5}}$	$\bullet^1$ ✓ $\bullet^2$ ✘		
$\frac{4}{5}$	$\bullet^3$ ✘		

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		 (b) Hence determine the value of $\sin 4p$.		
			$\bullet^5$ evaluate $\sin 4p$	$\bullet^5$ $\frac{24}{25}$	**1**

Notes:

6. $\bullet^5$ is only available for an answer expressed as a single fraction.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.			The line $y=2x$ is a tangent to the circle with equation $x^2+y^2-14x-8y+45=0$. Determine the coordinates of the point of contact.		
			Method 1	**Method 1**	4
			$\bullet^1$ substitute for y	$\bullet^1$ $x^2+(2x)^2-14x-8(2x)+45=0$	
			$\bullet^2$ write in standard quadratic form	$\bullet^2$ $5x^2-30x+45=0$	
			$\bullet^3$ determine x-coordinate	$\bullet^3$ 3	
			$\bullet^4$ determine y-coordinate	$\bullet^4$ 6	
			Method 2	**Method 2**	
			$\bullet^1$ substitute for x	$\bullet^1$ $\left(\frac{y}{2}\right)^2+y^2-14\left(\frac{y}{2}\right)-8y+45=0$	
			$\bullet^2$ write in standard quadratic form	$\bullet^2$ $\frac{5}{4}y^2-15y+45=0$	
			$\bullet^3$ determine y-coordinate	$\bullet^3$ 6	
			$\bullet^4$ determine x-coordinate	$\bullet^4$ 3	
			Method 3	**Method 3**	
			$\bullet^1$ use centre and perpendicular gradient to determine equation of radius through point of contact	$\bullet^1$ $x+2y=15$	
			$\bullet^2$ substitute for y	$\bullet^2$ $x+2(2x)=15$	
			$\bullet^3$ determine x-coordinate	$\bullet^3$ 3	
			$\bullet^4$ determine y-coordinate	$\bullet^4$ 6	

Notes:

1. In Methods 1 and 2, treat an absence of brackets at the $\bullet^1$ stage as bad form only if corrected on the next line of working.
2. In Methods 1 and 2, $\bullet^1$ is only available if the '=0' appears by the $\bullet^2$ stage.
3. In Methods 1 and 2, if a candidate arrives at an equation which is not a quadratic $\bullet^3$ and $\bullet^4$ are unavailable.
4. Where the quadratic obtained at $\bullet^2$ in Methods 1 and 2, does not have repeated roots $\bullet^3$ and $\bullet^4$ are not available.
5. In Method 3 accept $y-4=-\frac{1}{2}(x-7)$, $-\frac{1}{2}=\frac{4-y}{7-x}$ or equivalent for $\bullet^1$.
6. In Method 3 $\bullet^2$, $\bullet^3$ and $\bullet^4$ are unavailable to candidates who find the equation of any other line.
7. For (3,6) without working, award 0/4.
8. For answer of (3,6) verified in both equations, or (3,6) generated by the linear equation and verified in the equation of the circle, award 4/4.

Commonly Observed Responses:	
Candidate A - substitution into the equation of the circle ⋮ $x=3$ $\bullet^3$ ✓ $(3)^2+y^2-14(3)-8y+45=0$ $y^2-8y+12=0$ $(y-2)(y-6)=0$ $y=6$ $\bullet^4$ ✓ no need to explicitly consider $y=2$ **However,** (3,6) and (3,2) $\bullet^4$ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
8.			The equation $x^2+(m-4)x+(2m-3)=0$ has no real roots. Determine the range of values for m. **Justify your answer.**		
			$\bullet^1$ use discriminant	$\bullet^1$ $(m-4)^2-4(1)(2m-3)$	4
			$\bullet^2$ apply condition	$\bullet^2$ $(m-4)^2-4(1)(2m-3)<0$	
			$\bullet^3$ identify roots of quadratic expression	$\bullet^3$ 2, 14	
			$\bullet^4$ state range with justification	$\bullet^4$ $2<m<14$ with eg labelled sketch or table of signs	

Notes:

1. At $\bullet^1$, treat the inconsistent use of brackets: For example $m-4^2-4(1)(2m-3)$ or $(m-4)^2-4\times1\times2m-3$ as bad form only if the candidate deals with the unbracketed terms correctly in the next line of working.
2. Where candidates express a, b and c in terms of m, and then state $b^2-4ac<0$, award $\bullet^2$.
3. If candidates have the condition 'discriminant >0', 'discriminant ≤ 0' or 'discriminant ≥ 0', then $\bullet^2$ is lost but $\bullet^3$ and $\bullet^4$ are available.
4. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
5. If candidates only work with the condition 'discriminant $=0$', then $\bullet^2$ and $\bullet^4$ are unavailable.
6. Accept the appearance of 2 and 14 within inequalities for $\bullet^3$.
7. At $\bullet^4$ accept "$m>2$ and $m<14$" or "$m>2$, $m<14$" together with the required justification.
8. For the appearance of x in any expression of the discriminant, no further marks are available.

Commonly Observed Responses:

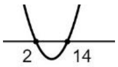
Candidate A - no expressions for *a*, *b* and *c*

No real roots $b^2-4ac<0$

$m^2-16m+28=0$ $\bullet^1$ ✓

$m=2, m=14$ $\bullet^3$ ✓

$2<m<14$



$\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

Candidate B

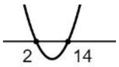
$(m-4)^2-4(1)(2m-3)$ $\bullet^1$ ✓

$m^2-16m+28=0$

$m=2, m=14$ $\bullet^3$ ✓

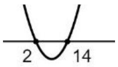
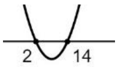
$b^2-4ac<0$

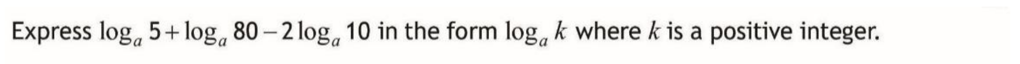
$2<m<14$



$\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

Candidate C $(m-4)^2-4(1)(2m-3)$ $\bullet^1$ ✓ $b^2-4ac=0$ $m^2-16m+28=0$ $m=2,\ m=14$ $\bullet^3$ ✓ $m^2-16m+28<0$ $\bullet^2$ ✓ $2<m<14$  $\bullet^4$ ✓	**Candidate D** $(m-4)^2-4(1)(2m-3)$ $\bullet^1$ ✓ $m^2-16m+28=0$ $\bullet^2$ ✗ $m=2,\ m=14$ $\bullet^3$ ✓ $2<m<14$  $\bullet^4$ ✓$_2$
Candidate E - not solving a quadratic $m-4^2-4(1)(2m-3)<0$ $\bullet^1$ ✗ $\bullet^2$ ✓ $\bullet^3$ ✗ $-7m-4<0$ $m>-\frac{4}{7}$ $\bullet^4$ ✓$_2$	

Question			Generic scheme	Illustrative scheme	Max mark
9.			 Express $\log_a 5+\log_a 80-2\log_a 10$ in the form $\log_a k$ where k is a positive integer.		
			Method 1 •¹ apply $\log_a x+\log_a y=\log_a xy$ •² apply $m\log_a x=\log_a x^m$ •³ apply $\log_a x-\log_a y=\log_a \frac{x}{y}$ and express in required form	**Method 1** •¹ $\log_a(5\times 80)$… stated or implied by •³ •² $\ldots-\log_a 10^2$ stated or implied by •³ •³ $\log_a 4$	3
			Method 2 •¹ apply $m\log_a x=\log_a x^m$ •² apply $\log_a x-\log_a y=\log_a \frac{x}{y}$ •³ apply $\log_a x+\log_a y=\log_a xy$ and express in required form	**Method 2** •¹ $\ldots-\log_a 10^2$ stated or implied by •³ •² $\ldots+\log_a\left(\frac{80}{10^2}\right)$ stated or implied by •³ •³ $\log_a 4$	

Notes:

1. Where an error at the •¹ or •² stage leads to a non-integer value for k, •³ is still available.
2. Each line of working must be equivalent to the line above within a valid strategy. See commonly observed responses.
3. Where candidates apply the laws of logarithms in the incorrect order see Candidates A and B.
4. Where candidates do not consider the '2', a maximum of 1/3 is available. See Candidate C.
5. Do not penalise the omission of the base of the logarithm.
6. Correct answer with no working, award 3/3.
7. Where candidates form an invalid equation, •¹ and •² may only be awarded for working with $\log_a 5+\log_a 80-2\log_a 10$ on one side of the equation; •³ is not available.

Commonly Observed Responses:

Candidate A

$\log_a 5+2\log_a\left(\frac{80}{10}\right)$

$2\log_a\left(\frac{5\times 80}{10}\right)$

$\log_a(40)^2$

$\log_a 1600$

Award 1/3

Candidate B

$\log_a 400-2\log_a 10$

$2\log_a\left(\frac{400}{10}\right)$

$\log_a(40)^2$

$\log_a 1600$

Award 2/3

Candidate C - ignoring the 2 $\log_a 5+\log_a 80-2\log_a 10$ $\log_a 5+\log_a \frac{80}{10}$ $\log_a 40$ **Award 1/3**	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		(a) Show that $(x-1)$ is a factor of $2x^4+3x^3-4x^2-3x+2$.		
			$\bullet^1$ use 1 in synthetic division or in evaluation of quartic	$\bullet^1$ $\begin{array}{r\|rrrrr} 1 & 2 & 3 & -4 & -3 & 2 \\ \hline & 2 & & & & \boxed{} \end{array}$ or $2\times(1)^4+3\times(1)^3-4\times(1)^2$ $-3\times(1)+2$	**2**
			$\bullet^2$ complete division/evaluation and interpret result	$\bullet^2$ $\begin{array}{r\|rrrrr} 1 & 2 & 3 & -4 & -3 & 2 \\ & & 2 & 5 & 1 & -2 \\ \hline & 2 & 5 & 1 & -2 & \boxed{0} \end{array}$ Remainder $=0 \therefore (x-1)$ is a factor or $f(1)=0 \therefore (x-1)$ is a factor	

Notes:

1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded.
2. Accept any of the following for $\bullet^2$:
   - '$f(1)=0$ so $(x-1)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow$, $\Rightarrow$ etc.
3. Do not accept any of the following for $\bullet^2$:
   - double underlining the '0' or boxing the '0' without comment
   - '$x=1$ is a factor', '… is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Candidate A - grid method

$$\begin{array}{c|c|c|c|c|} & 2x^3 & & & \\ \hline x & 2x^4 & 5x^3 & & \\ \hline -1 & -2x^3 & & & \\ \hline \end{array}$$ $\bullet^1$ ✓

$$\begin{array}{c|c|c|c|c|} & 2x^3 & 5x^2 & x & -2 \\ \hline x & 2x^4 & 5x^3 & x^2 & -2x \\ \hline -1 & -2x^3 & -5x^2 & -x & 2 \\ \hline \end{array}$$

'with no remainder'

$\therefore (x-1)$ is a factor $\bullet^2$ ✓

Candidate B - grid method

$$\begin{array}{c|c|c|c|c|} & 2x^3 & & & \\ \hline x & 2x^4 & 5x^3 & & \\ \hline -1 & -2x^3 & & & \\ \hline \end{array}$$ $\bullet^1$ ✓

$$\begin{array}{c|c|c|c|c|} & 2x^3 & 5x^2 & x & -2 \\ \hline x & 2x^4 & 5x^3 & x^2 & -2x \\ \hline -1 & -2x^3 & -5x^2 & -x & 2 \\ \hline \end{array}$$

$\therefore (x-1)(2x^3+5x^2+x-2)=2x^4+3x^3-4x^2-3x+2$

$\therefore (x-1)$ is a factor $\bullet^2$ ✓

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Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		(b) Hence, or otherwise, factorise $2x^4+3x^3-4x^2-3x+2$ fully.		
			$\bullet^3$ identify cubic and attempt to factorise	$\bullet^3$ eg $\begin{array}{r\|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & \\ \hline & 2 & 3 & & \boxed{} \end{array}$ **or** $\begin{array}{r\|rrrr} -2 & 2 & 5 & 1 & -2 \\ & & -4 & -2 & \\ \hline & 2 & 1 & & \boxed{} \end{array}$	**4**
			$\bullet^4$ find second factor	$\bullet^4$ eg $\begin{array}{r\|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & \boxed{0} \end{array}$ leading to $(x+1)$ **or** $\begin{array}{r\|rrrr} -2 & 2 & 5 & 1 & -2 \\ & & -4 & -2 & 2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$ leading to $(x+2)$	
			$\bullet^5$ identify quadratic	$\bullet^5$ $2x^2+3x-2$ **or** $2x^2+x-1$	
			$\bullet^6$ complete factorisation	$\bullet^6$ $(x-1)(x+1)(2x-1)(x+2)$ **stated explicitly**	

Notes:

4. Ignore the appearance of '$=0$'.
5. Candidates who arrive at $(x-1)(x+1)(2x^2+3x-2)$ or $(x-1)(x+2)(2x^2+x-1)$ by using algebraic long division or by inspection, gain $\bullet^3$, $\bullet^4$ and $\bullet^5$.
6. Where a candidate only identifies additional factors from a quartic, only $\bullet^4$ is available.
7. $\bullet^3$ and $\bullet^4$ may be awarded for applications of synthetic division even when previous trials contain errors. $\bullet^5$ and $\bullet^6$ are available.

Commonly Observed Responses:

Candidate C - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	$\cdots$	$\cdots$
x	$2x^3$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	$3x$	-2
x	$2x^3$	$3x^2$	$-2x$
$+1$	$2x^2$	$3x$	-2

$\bullet^4$ ✓

$2x^2+3x-2$ $\bullet^5$ ✓

$(x-1)(x+1)(2x-1)(x+2)$ $\bullet^6$ ✓

Candidate D - grid method

(a)

	$2x^3$	$5x^2$	x	-2
x	$2x^4$	$5x^3$	x^2	$-2x$
-1	$-2x^3$	$-5x^2$	$-x$	2

(b)

	$2x^2$	$\cdots$	$\cdots$
x	$2x^3$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$

$\bullet^3$ ✓

$\bullet^3$ is awarded for evidence of the cubic expression (which may be in the grid from part (a)) **AND** the terms in the diagonal boxes summing to the second and third terms in the cubic respectively.

	$2x^2$	x	-1
x	$2x^3$	x^2	$-x$
$+2$	$4x^2$	$2x$	-2

$\bullet^4$ ✓

$2x^2+x-1$ $\bullet^5$ ✓

$(x-1)(x+2)(x+1)(2x-1)$ $\bullet^6$ ✓

Candidate E

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 5 & 1 & -2 \\ & & 1 & 3 & 2 \\ \hline & 2 & 6 & 4 & \boxed{0} \end{array}$$

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$(2x-1)\left(x^2+3x+2\right)$

$(x-1)(2x-1)(x+1)(x+2)$ $\bullet^6$ ✓

Candidate F

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 5 & 1 & -2 \\ & & 1 & 3 & 2 \\ \hline & 2 & 6 & 4 & \boxed{0} \end{array}$$

$\bullet^3$ ✓ $\bullet^4$ ✓

$\left(x-\frac{1}{2}\right)\left(2x^2+6x+4\right)$ $\bullet^5$ ✓

$\left(x-\frac{1}{2}\right)(2x+2)(x+2)$

$(x-1)\left(x-\frac{1}{2}\right)(2x+2)(x+2)$ $\bullet^6$ ^

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		(a) Express $\cos x° + \sqrt{3}\sin x°$ in the form $k\cos(x-a)°$, where $k>0$ and $0<a<360$.		
			$\bullet^1$ use compound angle formula	$\bullet^1$ $k\cos x°\cos a° + k\sin x°\sin a°$ **stated explicitly**	**4**
			$\bullet^2$ compare coefficients	$\bullet^2$ $k\cos a° = 1, k\sin a° = \sqrt{3}$ **stated explicitly**	
			$\bullet^3$ process for k	$\bullet^3$ $k=2$	
			$\bullet^4$ process for a and express in required form	$\bullet^4$ $2\cos(x-60)°$	

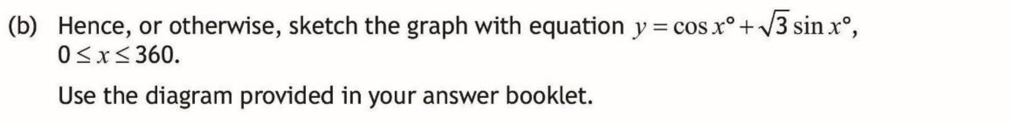
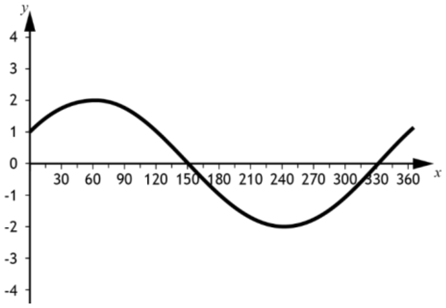
Notes:

1. Accept $k(\cos x°\cos a° + \sin x°\sin a°)$ for $\bullet^1$. Treat $k\cos x°\cos a° + \sin x°\sin a°$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
2. Do not penalise the omission of degree signs.
3. $2\cos x°\cos a° + 2\sin x°\sin a°$ or $2(\cos x°\cos a° + \sin x°\sin a°)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x° = 1, k\sin x° = \sqrt{3}$, however $\bullet^4$ may still be gained- see Candidate E
5. $\bullet^3$ is only available for a single value of $k, k>0$.
6. $\bullet^3$ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without explicitly simplifying at any stage. $\bullet^4$ is still available.
7. $\bullet^4$ is not available for a value of a given in radians.
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\cos(x-a)°$.
9. Evidence for $\bullet^4$ may not appear until part (b).

Commonly Observed Responses:

Candidate A	**Candidate B - inconsistency**	**Candidate C**
$\bullet^1$ ^	$k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓	$\cos x°\cos a° + \sin x°\sin a°$ $\bullet^1$ ✗
$2\cos a° = 1$	$\cos a° = 1$	$\cos a° = 1$
$2\sin a° = \sqrt{3}$ $\bullet^2$ ✓ $\bullet^3$ ✓	$\sin a° = \sqrt{3}$ $\bullet^2$ ✗	$\sin a° = \sqrt{3}$ $\bullet^2$ $\checkmark_2$
		$k=2$ $\bullet^3$ ✓
$\tan a° = \sqrt{3}$ $a=60$	$\tan a° = \sqrt{3}$ $a=60$	$\tan a° = \sqrt{3}$ $a=60$
$2\cos(x-60)°$ $\bullet^4$ ✓	$2\cos(x-60)°$ $\bullet^3$ ✓ $\bullet^4$ ✗	$2\cos(x-60)°$ $\bullet^4$ ✗

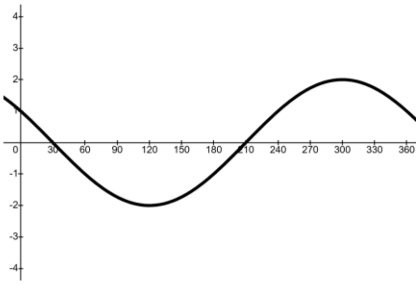
Candidate D - errors at $\bullet^2$	**Candidate E - use of x at $\bullet^2$**	**Candidate F**
$k\cos x^\circ\cos a^\circ+k\sin x^\circ\sin a^\circ$ $\bullet^1$✓	$k\cos x^\circ\cos a^\circ+k\sin x^\circ\sin a^\circ$ $\bullet^1$✓	$k\sin \mathrm{A}\cos \mathrm{B}+k\cos \mathrm{A}\sin \mathrm{B}$ $\bullet^1$ ✗
$k\cos a^\circ=\sqrt{3}$ $k\sin a^\circ=1$ $\bullet^2$ ✗	$k\cos x^\circ=1$ $k\sin x^\circ=\sqrt{3}$ $\bullet^2$ ✗	$k\cos \mathrm{A}=1$ $k\sin \mathrm{A}=\sqrt{3}$ $\bullet^2$ ✗
$\tan a^\circ=\frac{1}{\sqrt{3}}$ $a=30$	$\tan x^\circ=\sqrt{3}$ $x=60$	$\tan \mathrm{A}=\sqrt{3}$
$2\cos(x-30)^\circ$ $\bullet^3$✓ $\bullet^4$✓$_1$	$2\cos(x-60)^\circ$ $\bullet^3$✓ $\bullet^4$✓$_1$	$2\cos(x-60)^\circ$ $\bullet^3$✓ $\bullet^4$✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		 (b) Hence, or otherwise, sketch the graph with equation $y = \cos x° + \sqrt{3}\sin x°$, $0 \le x \le 360$. Use the diagram provided in your answer booklet.		
			$\bullet^5$ exactly two roots identifiable from graph	$\bullet^5$ $(150,0)$ and $(330,0)$	**3**
			$\bullet^6$ coordinates of exactly two turning points identifiable from graph	$\bullet^6$ $(60,2)$ and $(240,-2)$	
			$\bullet^7$ y-intercept and value of y at $x = 360$ identifiable from graph	$\bullet^7$ 1  	

Notes:

10. $\bullet^5$, $\bullet^6$ and $\bullet^7$ are only available for attempting to draw a "cosine" graph with a period of $360°$.
11. Ignore any part of a graph drawn outwith $0 \le x \le 360$.
12. Vertical marking is not applicable to $\bullet^5$ and $\bullet^6$.
13. Candidate's sketch in (b) must be consistent with the equation obtained in (a), see also Candidates G and H.
14. For any incorrect horizontal translation of the graph of the wave function arrived at in part (a) only $\bullet^6$ is available.

Commonly Observed Responses:

Candidate G - incorrect translation	**Candidate H - incorrect equation**
(a) $2\cos(x-60)°$ - correct equation (b) Incorrect translation: Sketch of $2\cos(x+60)°$ only $\bullet^6$ is available	(a) $2\cos(x+60)°$ - incorrect equation (b) Sketch of $2\cos(x+60)°$ all 3 marks available 

Question	Generic scheme	Illustrative scheme	Max mark
12.	The function f is given by $f(x)=12\sqrt[3]{x}$, $x>0$. When $x=a$ the rate of change of f with respect to x is 1. Determine the value of a.		
	$\bullet^1$ write in differentiable form	$\bullet^1$ $12x^{\frac{1}{3}}$ stated or implied by $\bullet^2$	4
	$\bullet^2$ differentiate	$\bullet^2$ $12\times\frac{1}{3}\times x^{-\frac{2}{3}}$	
	$\bullet^3$ solve for $a^{-\frac{2}{3}}$ or $a^{\frac{2}{3}}$	$\bullet^3$ $a^{-\frac{2}{3}}=\frac{1}{4}$ or $a^{\frac{2}{3}}=4$	
	$\bullet^4$ solve for a	$\bullet^4$ $a=8$	

Notes:

1. $\bullet^2$ is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate or make no attempt to differentiate, only $\bullet^1$ is available.
3. Accept $x^{-\frac{2}{3}}=\frac{1}{4}$ or $x^{\frac{2}{3}}=4$ at $\bullet^3$. See Candidates A and B.
4. $\bullet^4$ is only available where the expression at $\bullet^2$ is of the form $kx^{-\frac{m}{n}}$ where $m\neq 1$.
5. Do not penalise the inclusion of -8 at $\bullet^4$.

Commonly Observed Responses:

Candidate A - working in terms of x throughout

…	$\bullet^1$ ✓ $\bullet^2$ ✓
$x^{-\frac{2}{3}}=\frac{1}{4}$	$\bullet^3$ ✓
$x=8$	$\bullet^4$ ✗

Candidate B

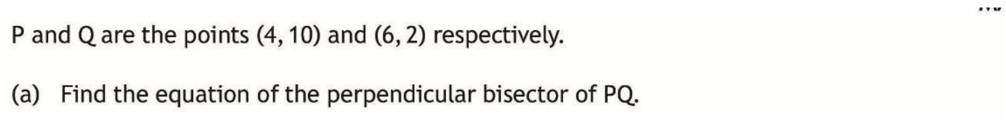
…	$\bullet^1$ ✓ $\bullet^2$ ✓
$x^{-\frac{2}{3}}=\frac{1}{4}$	$\bullet^3$ ✓
$(x=8)$	
$a=8$	$\bullet^4$ ✓

Candidate C

$f(x)=12x^{\frac{3}{2}}$	$\bullet^1$ ✗
$f'(x)=18x^{\frac{1}{2}}$	$\bullet^2$ $✓_1$
$a^{\frac{1}{2}}=\frac{1}{18}$	$\bullet^3$ $✓_1$
$a=\frac{1}{324}$	$\bullet^4$ $✓_2$

Candidate D - partly differentiated

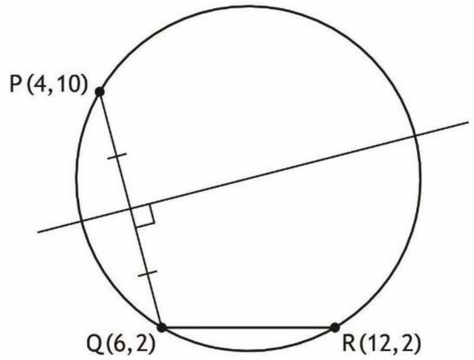
$f(x)=12x^{\frac{1}{3}}$	$\bullet^1$ ✓
$f'(x)=12\times\frac{1}{3}x^{\frac{4}{3}}$	$\bullet^2$ ✗
$1=4a^{\frac{4}{3}}$	
$\frac{1}{4}=a^{\frac{4}{3}}$	$\bullet^3$ $✓_1$
$a=\frac{1}{\sqrt{8}}$	$\bullet^4$ $✓_2$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		 P and Q are the points (4, 10) and (6, 2) respectively. (a) Find the equation of the perpendicular bisector of PQ.		
			$\bullet^1$ find midpoint of PQ	$\bullet^1$ (5,6)	**4**
			$\bullet^2$ find gradient of PQ	$\bullet^2$ -4 or $-\frac{8}{2}$	
			$\bullet^3$ find perpendicular gradient	$\bullet^3$ $\frac{1}{4}$	
			$\bullet^4$ find equation of perpendicular bisector	$\bullet^4$ $4y = x + 19$	

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient and a mid-point.
2. The gradient of the perpendicular bisector must appear in fully simplified form at $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be awarded.
3. At $\bullet^4$ accept $4y - x = 19$, $4y - x - 19 = 0$, or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		The point R has coordinates (12, 2). A circle passes through the points P, Q and R. The chord QR is horizontal.  (b) Find the equation of the circle.		
			$\bullet^5$ identify x-coordinate of centre	$\bullet^5$ 9	4
			$\bullet^6$ find y-coordinate of centre	$\bullet^6$ 7	
			$\bullet^7$ find radius	$\bullet^7$ $\sqrt{34}$	
			$\bullet^8$ state equation of circle	$\bullet^8$ $(x-9)^2+(y-7)^2=34$	

Notes:

4. Do not accept "centre = (9,2)" as evidence of $\bullet^5$.
5. Where candidates use PQ, QR or PR as the diameter of the circle no marks are available.
6. $\bullet^7$ and $\bullet^8$ are only available as a consequence of using the point of intersection of two perpendicular bisectors and a point on the circumference of the circle.
7. Accept $r^2=34$ for $\bullet^7$.
8. $(x-9)^2+(y-7)^2=\left(\sqrt{34}\right)^2$ does not gain $\bullet^8$.

Commonly Observed Responses:

Candidate A - horizontal line through midpoint of PQ	**Candidate B - perpendicular bisector of PR**
Centre = (9,6) $\bullet^5$ ✓ $\bullet^6$ ✗ Radius = 5 $\bullet^7$ ✗ Equation: $(x-9)^2+y^2=25$ $\bullet^8$ ✗	Perpendicular bisector of PR: $y=x-2$ Centre = (9,7) $\bullet^5$ ✓ $\bullet^6$ ✓ ⋮

[END OF MARKING INSTRUCTIONS]