2019 Paper 1

① $\frac{dy}{dx} = 2x^3 - 6x^2$

$\therefore 2x^3 - 6x^2 = 0$

$2x^2(x-3) = 0$

$x = 0 \quad x = 3$

② $b^2 - 4ac = 0$

$(k-5)^2 - 4(1)(1) = 0$

$k^2 - 10k + 25 - 4 = 0$

$k^2 - 10k + 21 = 0$

$(k-7)(k-3) = 0$

$k = 7, \; k = 3$

③ $r_1 = \sqrt{(-3)^2 + (-1)^2 + 26}$

$= \sqrt{36}$

$= 6$

$\therefore r_2 = 6$

$\therefore (x-4)^2 + (y+2)^2 = 36$

④ $6m + c = 9$ ①

$9m + c = 11$ ②

② − ① $3m = 2$

$m = \frac{2}{3}$

$6\left(\frac{2}{3}\right) + c = 9$

$4 + c = 9$

$c = 5$

⑤ a) $\vec{AB} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 8 \\ -9 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix}$

$\vec{AB} = 3\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad \vec{BC} = 4\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\therefore \vec{AB}$ and $\vec{BC}$ are parallel

B is common, ∴ points are collinear.

b) 3 : 4

⑥ $y = (1-3x)^{-5}$

$\frac{dy}{dx} = -5(1-3x)^{-6} \times (-3)$

$= 15(1-3x)^{-6}$

$= \frac{15}{(1-3x)^6}$

⑦ $m_1 = \tan\theta$

$= \tan 30^\circ$

$= \frac{1}{\sqrt{3}}$

$m_2 = -\sqrt{3}$

$\therefore y = -\sqrt{3}x - 4$

⑧ Top − (bottom)

a) $= x^2 + 2x + 3 - (2x^2 + x + 1)$

$= -x^2 + x + 2$

$\therefore \int_{-1}^{2} (-x^2 + x + 2)\,dx$

b) $\left[\frac{-x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{2}$

$= \left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$

$= \left(\frac{-16}{6} + \frac{12}{6} + \frac{24}{6}\right) - \left(\frac{2}{6} + \frac{3}{6} - \frac{12}{6}\right)$

$= \frac{20}{6} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2} u^2$

9 a) (i) $\underline{u} \cdot \underline{v}$

$= p(2p+16) + (-2)\times(-3) + 4\times 6$

$= 2p^2 + 16p + 6 + 24$

$= 2p^2 + 16p + 30$

(ii) $\underline{u} \cdot \underline{v} = 0$

$\therefore 2p^2 + 16p + 30 = 0$

$2(p^2 + 8p + 15) = 0$

$2(p+3)(p+5) = 0$

$p = -3 \quad p = -5$

b) As $-2 \to -3$ and $4 \to 6$, to be parallel

$\underline{v} = \frac{3}{2}\underline{u}$

$\therefore 2p + 16 = \frac{3}{2}p$

$4p + 32 = 3p$

$p = -32$

10 a) $a = 3$

b) $(2,5)$ has gone up 3 $\therefore$ would be $(2,2)$

$(2,-1)$ has flipped to $(2,2)$

$\therefore k = -2$

11 $\left[\frac{1}{3}\sin\left(3x - \frac{\pi}{6}\right)\right]_0^{\frac{\pi}{9}}$

$= \left(\frac{1}{3}\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3}\sin\left(-\frac{\pi}{6}\right)\right)$

$= \left(\frac{1}{3}\sin\left(\frac{\pi}{6}\right)\right) - \left(-\frac{1}{3}\sin\left(\frac{\pi}{6}\right)\right)$

$= \left(\frac{1}{3}\left(\frac{1}{2}\right)\right) - \left(-\frac{1}{3}\left(\frac{1}{2}\right)\right)$

$= \frac{1}{6} - \left(-\frac{1}{6}\right)$

$= \frac{1}{3}$

12 a) $f(5-x) = \frac{1}{\sqrt{5-x}}$

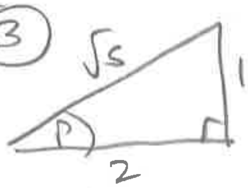
b) undefined where

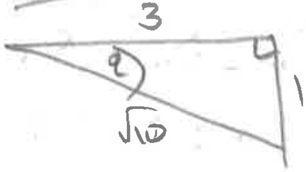
$5 - x \leq 0$

$\therefore -x \leq -5$

$x \geq 5$

13





a) $\cos p = \frac{2}{\sqrt{5}} \qquad \cos q = \frac{3}{\sqrt{10}}$

b) $\sin(p+q)$

$= \sin p \cos q + \cos p \sin q$

$= \left(\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}}\right) + \left(\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}\right)$

$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$

$= \frac{5}{\sqrt{50}}$

$= \frac{5}{5\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

14 a) $\log_{10} 4 + 2\log_{10} 5$

$= \log_{10} 4 + \log_{10} 5^2$

$= \log_{10} 100$

$= 2$

b) $\log_2(7x-2) - \log_2 3 = 5$

$\log_2\left(\frac{7x-2}{3}\right) = 5$

$\frac{7x-2}{3} = 2^5$

$\frac{7x-2}{3} = 32$

$7x-2 = 96$

$7x = 98$

$x = 14$

15 $\sin 2x + 6\cos x = 0$

a) $2\sin x\cos x + 6\cos x = 0$

$2\cos x(\sin x + 3) = 0$

$2\cos x = 0$ $\quad$ $\sin x = -3$

$x = 90°, 270°$ $\quad$ NO SOLUTIONS

b) $\sin 4x + 6\cos 2x = 0$

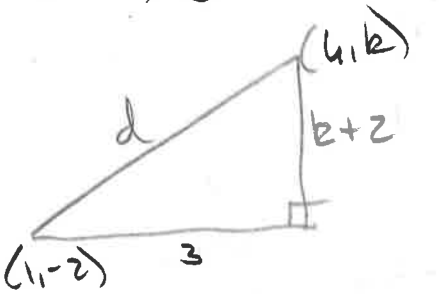
$2\cos 2x(\sin 2x + 3) = 0$

$2\cos 2x = 0$ $\quad$ $\sin 2x = -3$

$2x = 90, 270, 450°, 630°$ $\quad$ NO SOLUTIONS

$x = 45°, 135°, 225°, 315°$

16 a) $C = (1,-2)$



$d^2 = (k+2)^2 + 3^2$

$= k^2 + 4k + 4 + 9$

$= k^2 + 4k + 13$

$\therefore d = \sqrt{k^2+4k+13}$

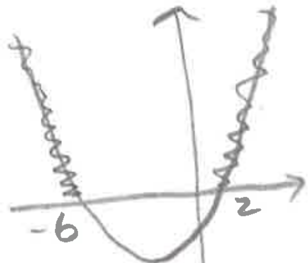
b) Radius $= 5$

$\therefore \sqrt{k^2+4k+13} > 5$

$k^2 + 4k + 13 > 25$

$k^2 + 4k - 12 > 0$

$(k+6)(k-2) > 0$



$k < -6, \ k > 2$

17 a) $(\sin x - \cos x)^2$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x - 2\sin x\cos x$

$= 1 - \sin 2x$

b) $\int (1 - \sin 2x)\,dx$

$= x + \frac{1}{2}\cos 2x + C$

2019 Paper 2

1 a) Mid $= (-4, -3)$

$m = \frac{-8+3}{11+4}$

$= \frac{-5}{15}$

$= -\frac{1}{3}$

$\therefore y+3 = -\frac{1}{3}(x+4)$

$3y+9 = -x-4$

$x+3y+13=0$

b) $m_{BC} = \frac{6+8}{-3-11}$

$= \frac{14}{-14}$

$= -1$

$\therefore m_{alt} = 1$

$\therefore y+12 = 1(x+5)$

$x-y-7=0$

c) $x+3y = -13$ ①

$x - y = 7$ ②

①-② $4y = -20$

$y = -5$

$x+5 = 7$

$x = 2 \quad \therefore (2,-5)$

2 $\int (6x^{1/2} - 4x^{-3} + 5)\,dx$

$= \frac{6x^{3/2}}{\frac{3}{2}} - \frac{4x^{-2}}{-2} + 5x + C$

$= 4x^{3/2} + 2x^{-2} + 5x + C$

3 a) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$

$= -\underline{p} + \underline{r}$

$= \underline{r} - \underline{p}$

b) $\overrightarrow{EF} = \overrightarrow{EA} + \overrightarrow{AB} + \frac{3}{4}\overrightarrow{BC}$

$= -\underline{r} + \underline{p} + \frac{3}{4}\underline{q}$

4 a) $-2.7\% \rightarrow 97.3\%$

$= 0.973$

$\therefore u_{n+1} = 0.973u_n + 30$

b)(i) $-1 < 0.973 < 1$

(ii) $L = 0.973L + 30$

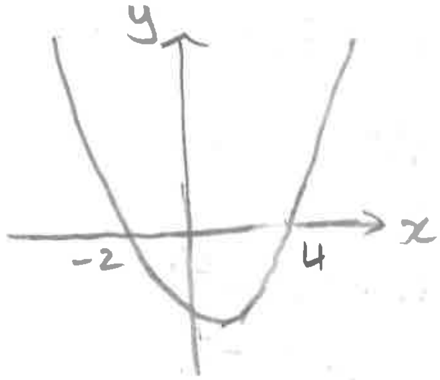
$0.027L = 30$

$L = \frac{30}{0.027}$

$= 1111.111\ldots$

$= 1100$ mice

5



6 $k\cos(x+\alpha)$

$= k(\cos x\cos\alpha - \sin x\sin\alpha)$

$= k\cos\alpha\cos x - k\sin\alpha\sin x$

$2\cos x - 3\sin x$

$\therefore \boxed{\begin{aligned} k\cos\alpha &= 2 \\ k\sin\alpha &= 3 \end{aligned}}$

$k^2 = 2^2+3^2$

$k^2 = 13$

$k = \sqrt{13}$

$\tan\alpha = \frac{3}{2}$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$

$\alpha = 56.3^\circ$

S	A ✓
T	C ✓

$k\sin\alpha = pos$

$k\cos\alpha = pos$

$\therefore \sqrt{13}\cos(x+56.3)^\circ$

b) $\sqrt{13}\cos(x+56.3)^\circ = 3$

$\cos(x+56.3)^\circ = \frac{3}{\sqrt{13}}$

~~S~~	A
~~T~~	C

$x + 56.3^\circ = 33.7^\circ, 326.3^\circ$

$x = -22.6^\circ, 270^\circ$

$\therefore x = 270^\circ, 337.4^\circ$

7 a) $px^2 + 2pqx + pq^2 + r$

$-6x^2 + 24x - 25$

$\therefore p = -6$

$2pq = 24$

$12q = 24$

$q = 2$

$pq^2 + r = -25$

$-6(2)^2 + r = -25$

$-24 + r = -25$

$r = -1$

$\therefore -6(x+2)^2 - 1$

b) $f'(x) = -6x^2 + 24x - 25$

$= -6(x+2)^2 - 1$

$\therefore$ Q1 $y = f'(x)$ has a maximum TP @ $(-2, -1)$

$\therefore$ Maximum of $f'(x) =$

$\therefore f'(x)$ is always < 0

$\therefore f(x)$ is strictly decrea

8 a) $y = \sqrt[3]{x} + 8$

$\sqrt[3]{x} = y - 8$

$x = (y-8)^3$

$\therefore f^{-1}(x) = (x-8)^3$

b) Domain of $f^{-1}(x)$ is the range of $f(x)$

$\therefore f(1) = 1 + 8 = 9$

$f(1000) = 10 + 8 = 18$

$\therefore 9 \le x \le 18$

9 a) 120W

b) $0.85 = e^{-0.0079t}$

$\ln 0.85 = -0.0079t$

$t = \frac{\ln 0.85}{-0.0079}$

$t = 20.6$ years

(10) a) $F(x) = 3x^4 + 10x^3 + x^2 - 8x - 6$

$F(-3) = 3(81) + 10(-27) + 9 + 24 - 6$

$= 243 - 270 + 27$

$= 0$

$\therefore (x+3)$ is a factor

$x = -3$ is a root.

b)

	$3x^3$	x^2	$-2x$	-2
x	$3x^4$	x^3	$-2x^2$	$-2x$
$+3$	$9x^3$	$3x^2$	$-6x$	-6

$\therefore (x+3)(3x^3 + x^2 - 2x - 2)$

$g(x) = 3x^3 + x^2 - 2x - 2$

$g(1) = 3 + 1 - 2 - 2$

$= 0$

$\therefore (x-1)$ is a factor

$x = 1$ is a root.

	$3x^2$	$+4x$	$+2$
x	$3x^3$	$+4x^2$	$+2x$
-1	$-3x^2$	$-4x$	-2

$\therefore (x+3)(x-1)(3x^2 + 4x + 2)$

$b^2 - 4ac = 4^2 - 4(3)(2)$

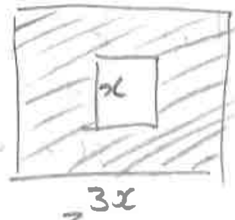
$= 16 - 24$

$= -8$

$\therefore 3x^2 + 4x + 2$ has no real roots

$\therefore (x+3)(x-1)(3x^2 + 4x + 2)$

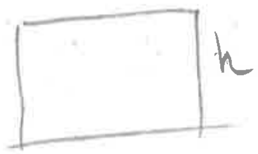
(11) a) Top



$A = 9x^2 - x^2$

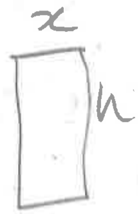
$= 8x^2$

outsides



$A = 3xh$

insides



$A = xh$

$\therefore$ Total $= 2(8x^2) + 4(3xh) + 4(xh)$

$= 16x^2 + 16xh$

Volume $= 9x^2h - x^2h$

$= 8x^2h$

$\therefore 8x^2h = 2000$

$h = \frac{2000}{8x^2}$

$h = \frac{250}{x^2}$

$\therefore A = 16x^2 + 16x\left(\frac{250}{x^2}\right)$

$= 16x^2 + \frac{4000}{x}$

(as required)

b) $A = 16x^2 + 4000x^{-1}$

$\therefore \dfrac{dA}{dx} = 32x - 4000x^{-2}$

$\therefore 32x - \dfrac{4000}{x^2} = 0$

$32x = \dfrac{4000}{x^2}$

$x^3 = \dfrac{4000}{32}$

$x^3 = 125$

$\underline{x = 5}$

x	$\xrightarrow{4}$	5	$\xrightarrow{6}$
$\dfrac{dA}{dx}$	$-$	0	$+$
shape	\	—	/

$\therefore$ Minimum @ $x = 5$

$\therefore A = 16(5)^2 + \dfrac{4000}{5}$

$= \underline{1200\,cm^2}$

12) $\log_4 y = \log_4 ab^x$

$\log_4 y = \log_4 a + \log_4 b^x$

$\log_4 y = (\log_4 b)x + \log_4 a$

$m = \dfrac{8+1}{3-0} = \underline{3}$ $\quad \therefore \log_4 y = 3x - 1$

$\therefore \log_4 b = 3$ $\quad \log_4 a = -1$

$b = 4^3$ $\quad a = 4^{-1}$

$\underline{b = 64}$ $\quad \underline{a = \frac{1}{4}}$

13) $f(x) = \int (3x^2 - 16x + 11)\,dx$

$f(x) = x^3 - 8x^2 + 11x + c$

$\therefore 0 = 7^3 - 8(7)^2 + 11(7) + c$

$0 = 28 + c$

$c = -28$

$\therefore \underline{f(x) = x^3 - 8x^2 + 11x - 28}$

14) $\underline{u}.(\underline{u}+\underline{v}) = 21$

$\underline{u}.\underline{u} + \underline{u}.\underline{v} = 21$

$\underline{u}.\underline{u} = |\underline{u}||\underline{u}| \cos 0$

$= \underline{16}$

$\therefore \underline{u}.\underline{v} = 21 - 16$

$= \underline{5}$

$\therefore \underline{u}.\underline{v} = |\underline{u}||\underline{v}| \cos\theta$

$\therefore 20\cos\theta = 5$

$\cos\theta = \frac{1}{4}$

$\underline{\theta = 75.5^\circ}$

15) a) $m_r = \dfrac{13-12}{5-8} = -\dfrac{1}{3}$

$\therefore m_T = 3$

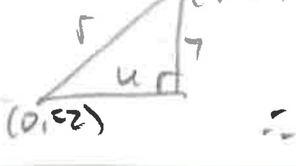
$\therefore y - 13 = 3(x-5)$

$y - 13 = 3x - 15$

$\underline{y = 3x - 2}$

b) $T = (0,-2)$

TC = diameter $\therefore$ centre $= (4,$



$r^2 = 4^2 + 7^2$

$= 65$

$\therefore (x-4)^2 + (y-5)^2 = 6$