

2023 Paper 1

① $y = x^{5/3} - 10x^{-4}$

$$\frac{dy}{dx} = \frac{5}{3}x^{2/3} + 40x^{-5}$$

② Midpoint = (4, 3)

$$MPQ = \frac{6-0}{-2-10}$$

$$= \frac{6}{-12}$$

$$= -\frac{1}{2}$$

$$\therefore MPB = 2$$

$$\therefore y-3 = 2(x-4)$$

$$y-3 = 2x-8$$

$$\underline{\underline{y = 2x - 5}}$$

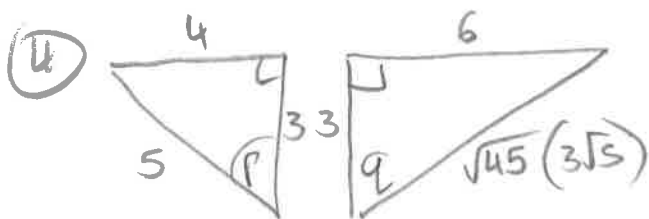
③ $\log_5 x - \log_5 3 = 2$

$$\log_5 \left(\frac{x}{3}\right) = 2$$

$$\frac{x}{3} = 5^2$$

$$\frac{x}{3} = 25$$

$$\underline{\underline{x = 75}}$$



a) $\cos p = \frac{3}{5}$ $\cos q = \frac{3}{3\sqrt{5}}$
 $= \frac{1}{\sqrt{5}}$

b) $\cos(p+q)$

$$= \cos p \cos q - \sin p \sin q$$

$$= \left(\frac{3}{5} \times \frac{1}{\sqrt{5}}\right) - \left(\frac{4}{5} \times \frac{2}{\sqrt{5}}\right)$$

$$= \frac{3}{5\sqrt{5}} - \frac{8}{5\sqrt{5}}$$

$$= -\frac{5}{5\sqrt{5}}$$

$$= -\frac{1}{\sqrt{5}}$$

c) $b^2 - 4ac = 0$

$$(3p-2)^2 - 4(2)(p) = 0$$

$$9p^2 - 12p + 4 - 8p = 0$$

$$9p^2 - 20p + 4 = 0$$

$$(9p-2)(p-2) = 0$$

$$\underline{\underline{p = \frac{2}{9}, p = 2}}$$

⑤ $\int (2x^5 - 6x^{1/2}) dx$

$$= \frac{2x^6}{6} - \frac{6x^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{x^6}{3} - 4x^{3/2} + C$$

⑦ a) $\log_2 5 + \log_2 \frac{1}{40}$

$$= \log_2 \frac{5}{40}$$

$$= \log_2 \frac{1}{8}$$

$$= -3$$

b) $0 < a < 1$

8) $f'(x) = 3x^2 + 6x - 9$

$\therefore 3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

$f(-3) = -27 + 27 + 27 + 5$
 $= 32$

$\therefore (-3, 32)$

$f(1) = 1 + 3 - 9 + 5$
 $= 0$

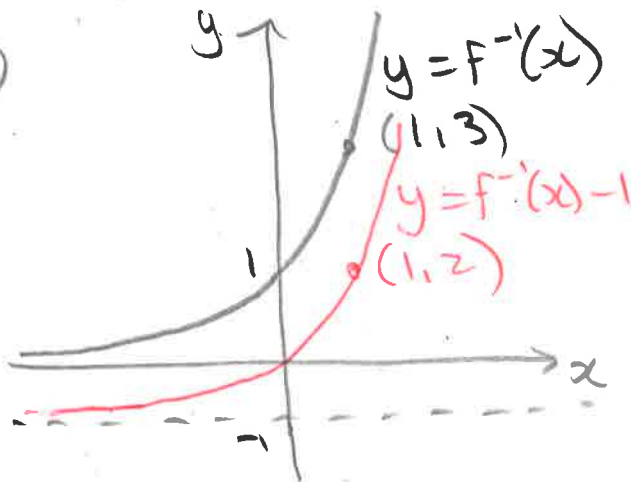
$\therefore (1, 0)$

x	$\xrightarrow{-4}$	-3	$\xrightarrow{-2}$	0	$\xrightarrow{1}$	$\xrightarrow{2}$
$f'(x)$	+	0	-	-	0	+
shape	/	-	\	\	-	/

\therefore Max TP @ $(-3, 32)$

Min TP @ $(1, 0)$

9)



10) a) $f(x) = x^4 + 3x^3 - 7x^2 + 9x$

$f(-5) = 625 + 3(-125) - 7(25) + 9(-5)$
 $= 625 - 375 - 175 - 45 - 30$
 $= 0$

$\therefore (x+5)$ is a factor
 $x = -5$ is a root

b) $x^3 - 2x^2 + 3x - 6$

x	x^4	$-2x^3$	$+3x^2$	$-6x$
$+5$	$+5x^3$	$-10x^2$	$+15x$	-30

$\therefore (x+5)(x^3 - 2x^2 + 3x - 6) = 0$

$g(x) = x^3 - 2x^2 + 3x - 6$

$g(1) = 1 - 2 + 3 - 6 \neq 0$

$g(-1) = -1 - 2 - 3 - 6 \neq 0$

$g(2) = 8 - 8 + 6 - 6$
 $= 0$

$\therefore (x-2)$ is a factor

$x = 2$ is a root

$x^2 \quad 0 \quad +3$

x	x^3	$+0$	$+3x$
-2	$-2x^2$	0	-6

$\therefore (x+5)(x-2)(x^2+3) = 0$

$x = -5 \quad x = 2$

$$\textcircled{11} \text{ a) } \int_{\frac{\pi}{2}}^{\pi} (5\sin x - 3\cos x) dx$$

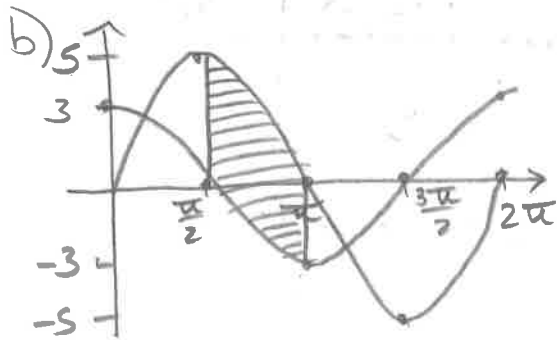
$$= \left[-5\cos x - 3\sin x \right]_{\frac{\pi}{2}}^{\pi}$$

$$= (-5\cos \pi - 3\sin \pi) - (-5\cos \frac{\pi}{2} - 3\sin \frac{\pi}{2})$$

$$= (-5(-1) - 3(0)) - (-5(0) - 3(1))$$

$$= 5 - (-3)$$

$$= \underline{\underline{8}}$$



$$\textcircled{12} \quad ax^2 + 2abx + ab^2 + c$$

$$-2x^2 - 12x + 7$$

$$\therefore \underline{a = -2}$$

$$2ab = -12$$

$$-4b = -12$$

$$\underline{b = 3}$$

$$ab^2 + c = 7$$

$$-2(3)^2 + c = 7$$

$$-18 + c = 7$$

$$\underline{c = 25}$$

$$\therefore \underline{\underline{-2(x+3)^2 + 25}}$$

$$\textcircled{13} \text{ a) (i) } g\left(\frac{\pi}{6}\right) = 2\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3}$$

$$\therefore f\left(\frac{\pi}{3}\right) = 2\sin\left(\frac{\pi}{3}\right)$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \underline{\underline{\sqrt{3}}}$$

$$\text{(ii) } f(2x) = 2\sin 2x$$

$$\text{b) } 2\sin p = \frac{1}{3}$$

$$\underline{\underline{\sin p = \frac{1}{6}}}$$

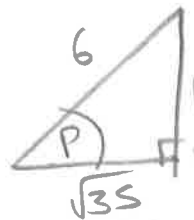
$$\text{(ii) } f(g(p)) = 2\sin 2p$$

$$= 2\sin p \cos p$$

$$= 2\left(\frac{1}{6}\right)\left(\frac{\sqrt{35}}{6}\right)$$

$$= \frac{2\sqrt{35}}{36}$$

$$= \underline{\underline{\frac{\sqrt{35}}{18}}}$$



Paper 2

$$\begin{aligned} \textcircled{1} \text{ a) } M_{QR} &= \frac{8-3}{-2-13} \\ &= \frac{5}{-15} \\ &= -\frac{1}{3} \end{aligned}$$

$$\therefore \text{Mark} = 3$$

$$y+1 = 3(x-5)$$

$$y+1 = 3x-15$$

$$y = 3x-16$$

$$\begin{aligned} \text{b) } M_{PR} &= \frac{3+1}{13-5} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

$$m = \tan \theta$$

$$\begin{aligned} \therefore \theta &= \tan^{-1}\left(\frac{1}{2}\right) \\ &= \underline{\underline{26.6^\circ}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad y &= 2(1)^5 - 3(1) \\ &= 2-3 \\ &= -1 \quad \therefore (1, -1) \end{aligned}$$

$$\frac{dy}{dx} = 10x^4 - 3$$

$$\begin{aligned} \therefore m &= 10(1)^4 - 3 \\ &= 10-3 \end{aligned}$$

$$= \underline{\underline{7}}$$

$$\therefore y+1 = 7(x-1)$$

$$y+1 = 7x-7$$

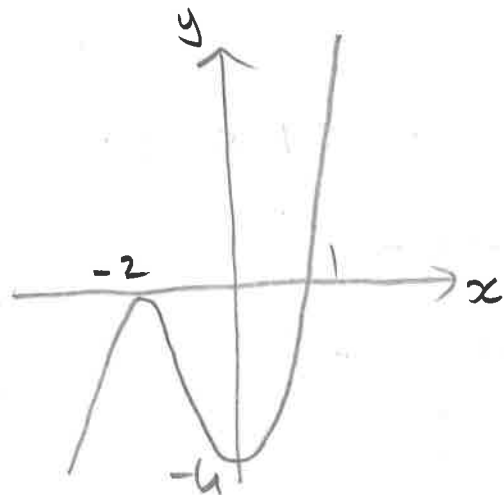
$$y = 7x-8$$

$$\begin{aligned} \textcircled{3} \quad \int 7 \cos\left(4x + \frac{\pi}{3}\right) dx \\ = \underline{\underline{\frac{7}{4} \sin\left(4x + \frac{\pi}{3}\right) + C}} \end{aligned}$$

$$\textcircled{4} \quad y = 2f(-x)$$

stretch $\times 2$ y flip over y -axis

	$2f(x)$	$2f(-x)$
$(-1, 0)$	$(-1, 0)$	$(1, 0)$
$(0, -2)$	$(0, -4)$	$(0, -4)$
$(2, 0)$	$(2, 0)$	$(-2, 0)$



$$\begin{aligned} \textcircled{5} \quad F'(x) &= 4(3-2x)^3 \times (-2) \\ &= \underline{\underline{-8(3-2x)^3}} \end{aligned}$$

$$\begin{aligned} F'(4) &= -8(3-2(4))^3 \\ &= -8(-5)^3 \\ &= -8(-125) \\ &= \underline{\underline{1000}} \end{aligned}$$

$$(6) y = \frac{2}{x} + 3$$

$$y - 3 = \frac{2}{x}$$

$$x(y - 3) = 2$$

$$x = \frac{2}{y - 3}$$

$$\therefore f^{-1}(x) = \frac{2}{x - 3}$$

$$(7) \sin x + 2 = 3 \cos 2x$$

$$\sin x + 2 = 3(1 - 2\sin^2 x)$$

$$\sin x + 2 = 3 - 6\sin^2 x$$

$$6\sin^2 x + \sin x - 1 = 0$$

$$(3\sin x - 1)(2\sin x + 1) = 0$$

$$\sin x = \frac{1}{3} \quad \sin x = -\frac{1}{2}$$

$$x = 19.5^\circ, 160.5^\circ \quad x = 210^\circ, 330^\circ$$

$$\therefore x = 19.5^\circ, 160.5^\circ, 210^\circ, 330^\circ$$

(8) Top - (bottom)

$$= x^3 - 2x^2 - 6x + 1 - (x - 5)$$

$$= x^3 - 2x^2 - 5x + 6$$

$$\therefore \int_{-2}^1 (x^3 - 2x^2 - 5x + 6) dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_{-2}^1$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right) - \left(\frac{16}{4} + \frac{16}{3} - \frac{20}{2} - 12 \right)$$

$$= \frac{63}{4} \quad \therefore \text{Area} = \frac{63}{4} \text{ u}^2$$

$$(9) a) k \sin(x + \alpha)$$

$$= k (\sin x \cos \alpha + \cos x \sin \alpha)$$

$$= k \cos \alpha \sin x + k \sin \alpha \cos x$$

$$= -3 \sin x + 7 \cos x$$

$$\therefore \begin{cases} k \sin \alpha = 7 \\ k \cos \alpha = -3 \end{cases}$$

$$k^2 = 7^2 + (-3)^2$$

$$= 49 + 9$$

$$k = \sqrt{58}$$

$$\tan \alpha = -\frac{7}{3}$$

$$\alpha = \tan^{-1}\left(\frac{7}{3}\right)$$

$$= 66.8^\circ$$

S	A
T	C

$$k \sin \alpha = \text{POS} \quad \therefore \text{Q2}$$

$$k \cos \alpha = \text{NEG}$$

$$\therefore \alpha = 180 - 66.8$$

$$= 113.2^\circ$$

$$\therefore \sqrt{58} \sin(x + 113.2^\circ)$$

$$b) (i) 14 \cos x - 6 \sin x$$

$$= 2(7 \cos x - 3 \sin x)$$

$$= 2\sqrt{58} \sin(x + 113.2^\circ)$$

$$\text{Max} = \underline{\underline{2\sqrt{58}}}$$

$$(ii) 2\sqrt{58} \sin(x + 113.2^\circ) = 2\sqrt{58}$$

$$\sin(x + 113.2^\circ) = 1$$

$$x + 113.2 = 90$$

$$x = -23.2$$

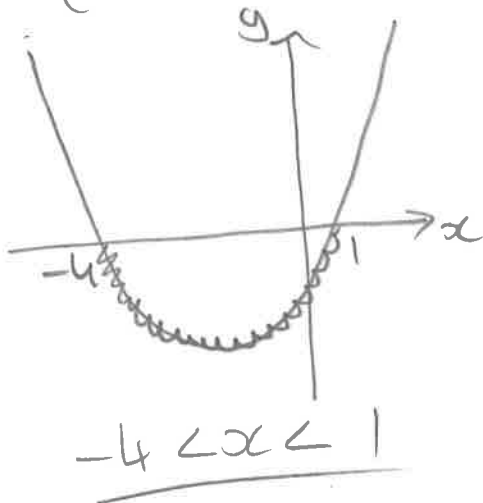
$$x = \underline{\underline{336.8^\circ}}$$

⑩ Decreasing for $f'(x) < 0$

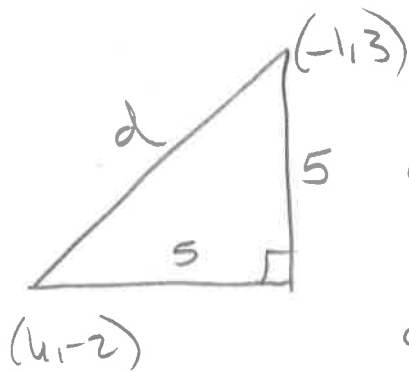
$$\therefore 6x^2 + 18x - 24 < 0$$

$$6(x^2 + 3x - 4) < 0$$

$$6(x+4)(x-1) < 0$$



⑪ a) Centres $C_1 = (4, -2)$
 $C_2 = (-1, 3)$



$$d^2 = 5^2 + 5^2$$

$$= 50$$

$$d = \sqrt{50}$$

$$= \underline{\underline{5\sqrt{2}}}$$

b) Radius $C_1 = \sqrt{37}$

$$C_2 = \sqrt{1^2 + (-3)^2 - (-7)}$$

$$= \sqrt{1+9+7}$$

$$= \underline{\underline{\sqrt{17}}}$$

$$r_1 + r_2 = \sqrt{37} + \sqrt{17}$$

$$= 10.21$$

$$d = \sqrt{50} = 7.07$$

$$r_1 + r_2 > d$$

\therefore circles intersect in two places.

⑫ $y = \int (8x^3 + 3) dx$
 $= \frac{8x^4}{4} + 3x + C$
 $= 2x^4 + 3x + C$

$$\therefore 3 = 2(-1)^4 + 3(-1) + C$$

$$3 = 2 - 3 + C$$

$$3 = -1 + C$$

$$C = 4$$

$$\therefore y = \underline{\underline{2x^4 + 3x + 4}}$$

⑬ a) $C_{30} = 11e^{-0.0053 \times 30}$
 $= \underline{\underline{9.38 \text{ mg/L}}}$

b) $0.66 = 11e^{-0.0053t}$

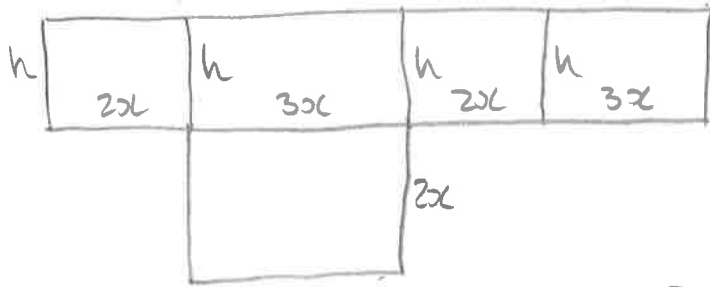
$$\frac{0.66}{11} = e^{-0.0053t}$$

$$\ln\left(\frac{0.66}{11}\right) = -0.0053t$$

$$t = \frac{\ln\left(\frac{0.66}{11}\right)}{-0.0053}$$

$$= \underline{\underline{530.8 \text{ mins}}}$$

(14) a) (i)



$$A = 2(2hx) + 2(3hx) + 6x^2$$

$$= 6x^2 + 10hx$$

(ii) $6x^2 + 10hx = 7200$
 $10hx = 7200 - 6x^2$
 $h = \frac{7200 - 6x^2}{10x}$

$$V = L B H$$

$$= (3x)(2x)(h)$$

$$= 6x^2 h$$

$$\therefore V = 6x^2 \left(\frac{7200 - 6x^2}{10x} \right)$$

$$= 6x^2 \left(\frac{7200}{10x} - \frac{6x^2}{10x} \right)$$

$$= 6x^2 \left(\frac{720}{x} - \frac{3x}{5} \right)$$

$$= \frac{4320x^2}{x} - \frac{18x^3}{5}$$

$$= 4320x - \frac{18}{5}x^3$$

(as required)

b) $\frac{dV}{dx} = 4320 - \frac{54}{5}x^2$
 $\therefore 4320 - \frac{54}{5}x^2 = 0$
 $\frac{54x^2}{5} = 4320$

$$54x^2 = 21600$$

$$x^2 = 400$$

$$x^2 - 400 = 0$$

$$(x+20)(x-20) = 0$$

$$x = \cancel{20} \quad \underline{\underline{x = 20}}$$

x	$\xrightarrow{14}$	20	$\xrightarrow{21}$	
$\frac{dV}{dx}$		$+$	0	$-$
shape		\swarrow	\searrow	\swarrow

\therefore Max @ $x = 20$

(15) $x + 3y = 17$
 $3y = -x + 17$
 $y = -\frac{1}{3}x + \frac{17}{3}$
 $\therefore M_T = -\frac{1}{3} \rightarrow M_R = 3$
 $y - 5 = 3(x - 2)$
 $y = 3x - 1$

on y-axis, $x = 0$
 $y = 3(0) - 1$
 $= -1$

\therefore centre = $(0, -1)$

