2022 Paper 1

① $5x + 2y = 7$

$2y = -5x + 7$

$y = -\frac{5}{2}x + \frac{7}{2}$

$m_1 = -\frac{5}{2} \quad \therefore m_2 = \frac{2}{5}$

$y - 6 = \frac{2}{5}(x+1)$

$5y - 30 = 2(x+1)$

$5y - 30 = 2x + 2$

$\underline{5y = 2x + 32}$

② $2\log_3 6 - \log_3 4$

$= \log_3 36 - \log_3 4$

$= \log_3 9$

$= \underline{\underline{2}}$

③ $y = 4 + \frac{1}{3}x$

$\frac{1}{3}x = y - 4$

$x = 3y - 12$

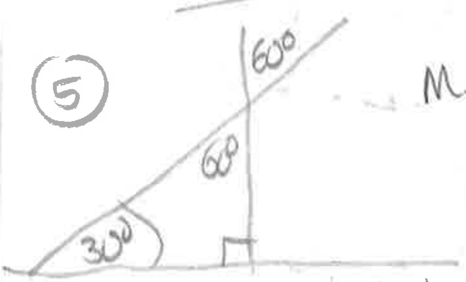
$\therefore \underline{h^{-1}(x) = 3x - 12}$

④ $y = \sqrt{x^3} - 2x^{-1}$

$= x^{3/2} - 2x^{-1}$

$\underline{\frac{dy}{dx} = \frac{3}{2}x^{1/2} + 2x^{-2}}$

⑤



$m = \tan 30°$

$= \frac{1}{\sqrt{3}}$

$y - 0 = \frac{1}{\sqrt{3}}(x+2)$

$\underline{\sqrt{3}y = x + 2}$

⑥ $\left[\frac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2} \times (-3)}\right]_{-5}^{2}$

$= \left[-\frac{2}{3}\sqrt{10-3x}\right]_{-5}^{2}$

$= \left(-\frac{2}{3}\sqrt{4}\right) - \left(-\frac{2}{3}\sqrt{25}\right)$

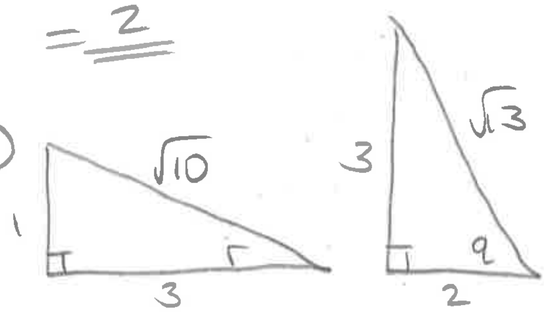
$= \left(-\frac{2}{3}(2)\right) - \left(-\frac{2}{3}(5)\right)$

$= -\frac{4}{3} - \left(-\frac{10}{3}\right)$

$= \frac{6}{3}$

$= \underline{\underline{2}}$

⑦



a) $\sin r = \frac{1}{\sqrt{10}} \qquad \sin q = \frac{3}{\sqrt{13}}$

b) $\sin(q - r)$

$= \sin q \cos r - \cos q \sin r$

$= \left(\frac{3}{\sqrt{13}} \times \frac{3}{\sqrt{10}}\right) - \left(\frac{2}{\sqrt{13}} \times \frac{1}{\sqrt{10}}\right)$

$= \frac{9}{\sqrt{130}} - \frac{2}{\sqrt{130}}$

$= \underline{\underline{\frac{7}{\sqrt{130}}}}$

⑧ $\log_6 x + \log_6(x+5) = 2$

$\log_6(x^2 + 5x) = 2$

$x^2 + 5x = 6^2$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$\cancel{x = 9} \quad \underline{x = 4}$

(9) $\cos 2x = 5\cos x - 3$

$2\cos^2 x - 1 - 5\cos x + 3 = 0$

$2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

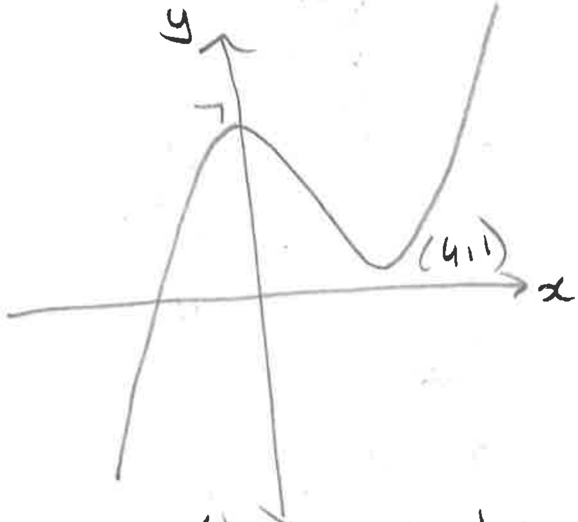
$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = 2$

NO SOLUTIONS

$x = 60°, 300°$

(10) a) $y = 2f(x) + 1$

stretch $\times 2$ y $\qquad$ slide up 1 y



b) $y = f(\frac{1}{2}x)$ stretches $x \times 2$

$\therefore (8,0), (0,3)$

(11) $px^2 + 2pqx + pq^2 + r$

$2x^2 + 12x + 23$

$\therefore p = 2$

$2pq = 12$

$4q = 12$

$q = 3$

$pq^2 + r = 23$

$2(3)^2 + r = 23$

$18 + r = 23$

$r = 5$

$\therefore 2(x+3)^2 + 5$

(12) $f'(x) = 4\cos(3x - \frac{\pi}{3}) \times 3$

$= 12\cos(3x - \frac{\pi}{3})$

$f'(\frac{\pi}{6}) = 12\cos(3(\frac{\pi}{6}) - \frac{\pi}{3})$

$= 12\cos(\frac{\pi}{2} - \frac{\pi}{3})$

$= 12\cos(\frac{\pi}{6})$

$= 12(\frac{\sqrt{3}}{2})$

$= 6\sqrt{3}$

(13) a) $f(-2) = -8 - 2(4) - 20(-2)$

$= -8 - 8 + 40 - 24$

$= 0$

$\therefore (x+2)$ is a factor

$x = -2$ is a root.

(ii)

	x^2	$-4x$	-12
x	x^3	$-4x^2$	$-12x$
$+2$	$+2x^2$	$-8x$	-24

$\therefore x^3 - 2x^2 - 20x - 24 = 0$

$(x+2)(x^2 - 4x - 12) = 0$

$(x+2)(x+2)(x-6) = 0$

$x = -2$ $\qquad$ $x = 6$

b) SP on x-axis $= (-2,0)$

For $(1,0)$, move 3 right

$\therefore k = 3$

(14)

a) (i) $(7,-5)$, $r=10$

$x=-2$, $y=7$

$\therefore (x-7)^2+(y+5)^2$

$=(-2-7)^2+(7+5)^2$

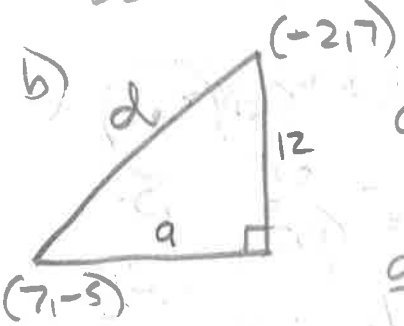
$=(-9)^2+12^2$

$=81+144$

$=225.$

$225>100$ $\therefore$ outside.

b)



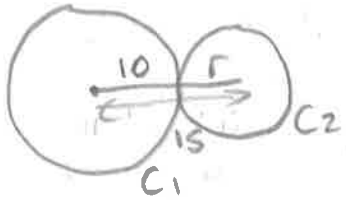
$d^2=9^2+12^2$

$=225$

$\underline{d=15}$

$\therefore$ centres 15 units apart.

Option 1



$\underline{r=5}$

Option 2

$\underline{r=25}$

Paper 2

(1) a) $m_{AB}=\frac{-1+4}{-1-2}$

$=\frac{3}{-3}$

$=\underline{\underline{-1}}$

$\therefore m_{alt}=1$

$y-3=1(x-7)$

$\underline{\underline{y=x-4}}$

b) $mid_{AC}=(3,1)$

$m_{med}=\frac{1+4}{3-2}$

$=\underline{\underline{5}}$

$y-1=5(x-3)$

$y-1=5x-15$

$\underline{y=5x-14}$

c) $y=y$

$5x-14=x-4$

$4x=10$

$x=\frac{5}{2}$

$y=\frac{5}{2}-4$

$=\frac{5}{2}-\frac{8}{2}$

$=-\frac{3}{2}$

$\therefore \underline{\underline{\left(\frac{5}{2},-\frac{3}{2}\right)}}$

② $b^2 - 4ac > 0$

$(-8)^2 - 4(2)(4-p) > 0$

$64 - 8(4-p) > 0$

$64 - 32 + 8p > 0$

$8p + 32 > 0$

$8p > -32$

$\underline{p > -4}$

③ a) $k\sin(x+\alpha)$

$= k(\sin x\cos\alpha + \cos x\sin\alpha)$

$= k\cos\alpha\sin x + k\sin\alpha\cos x$

$4\sin x + 5\cos x$

$\therefore \boxed{\begin{aligned} k\sin\alpha &= 5 \\ k\cos\alpha &= 4 \end{aligned}}$

$k^2 = 5^2 + 4^2$

$= 41$

$k = \sqrt{41}$

$\tan\alpha = \frac{5}{4}$

$\alpha = \tan^{-1}\left(\frac{5}{4}\right)$

$= 51^\circ$

S	A ✓
T	C

$k\sin\alpha = \text{pos}$

$k\cos\alpha = \text{pos}$ $\therefore$ Q1

$\frac{51\pi}{180} = 0.896$

$\therefore \sqrt{41}\sin(x + 0.896)$

b) $\sqrt{41}\sin(x+51) = 5.5$

$\sin(x+51) = \frac{5.5}{\sqrt{41}}$

$x + 51 = \sin^{-1}\left(\frac{5.5}{\sqrt{41}}\right)$

$x + 51 = 59^\circ, 121^\circ$

$x = 8^\circ, 70^\circ$

$x = \frac{8\pi}{180}, \frac{70\pi}{180}$

$x = 0.14, 1.22$

④ a) $\int_{-1}^{2}(x^3 - 5x^2 + 2x + 8)\,dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_{-1}^{2}$

$= \left(\frac{16}{4} - \frac{40}{3} + 4 + 16\right) - \left(\frac{1}{4} + \frac{5}{3} + 1 -\right.$

$= \frac{63}{4}u^2$

b) $\int_{2}^{4}(x^3 - 5x^2 + 2x + 8)\,dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_{2}^{4}$

$= \left(\frac{256}{4} - \frac{320}{3} + 16 + 32\right)$

$-\left(\frac{16}{4} - \frac{40}{3} + 4 + 1\right.$

$= -\frac{16}{3}$

$\therefore$ Total $= \frac{63}{4} + \frac{16}{3}$

$= \frac{253}{12}u^2$

⑤ a) $f(3x+5)$

$= (3x+5)^2 - 2$

$= 9x^2 + 30x + 25 - 2$

$= 9x^2 + 30x + 23$

$g(x^2 - 2)$

$= 3(x^2 - 2) + 5$

$= 3x^2 - 6 + 5$

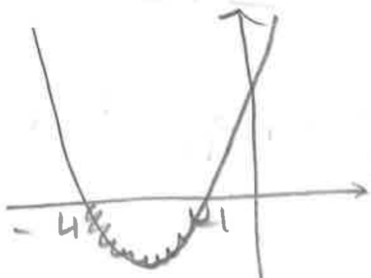
$= 3x^2 - 1$

b) $9x^2+30x+23 < 3x^2-1$

$6x^2+30x+24<0$

$6(x^2+5x+4)<0$

$6(x+4)(x+1)<0$



$-4<x<-1$

6) $F(x)=\int(1-3x^{-2})\,dx$

$= x - \frac{3x^{-1}}{-1}+c$

$= x+3x^{-1}+c$

$= x+\frac{3}{x}+c$

$\therefore 6 = 3+\frac{3}{3}+c$

$6=4+c$

$c=2$

$y=x+\frac{3}{x}+2$

7) $m=\frac{3+1}{0-2}$ $\quad \log_5 y = -2\log_5 x+3$

$=-2$

$\log_5 y = \log_5 kx^n$

$\log_5 y=\log_5 k+\log_5 x^n$

$\log_5 y = n\log_5 x+\log_5 k$

$\therefore n=-2$ $\quad \log_5 k=3$

$k=5^3$

$k=125$

8) Pond: Length $= x-3$

breadth $= y-2$

$\therefore$ Area $=(x-3)(y-2)$

$= xy-2x-3y+6$

Total area $= xy$

$\therefore xy=150$

$y=\frac{150}{x}$

$\therefore A(x) = x\left(\frac{150}{x}\right)-2x-3\left(\frac{150}{x}\right)+6$

$=150-2x-\frac{450}{x}+6$

$=156-2x-\frac{450}{x}$

(as required)

b) $A(x)=156-2x-450x^{-1}$

$A'(x)=-2+450x^{-2}$

$\therefore -2+\frac{450}{x^2}=0$

$\frac{450}{x^2}=2$

$x^2=225$

$x=15$

x	$\xrightarrow{14}$	15	$\xrightarrow{16}$
$A'(x)$	+	0	−
shape	/	—	\

$\therefore$ Max @ $x=15$

$\therefore A(15)=156-2(15)-\frac{450}{15}$

$=96\text{m}^2$

9 a)

$x^2 + (3x+7)^2 - 4x - 6(3x+7) - 7 = 0$

$x^2 + 9x^2 + 42x + 49 - 4x - 18x - 42 - 7 = 0$

$10x^2 + 20x = 0$

$10x(x+2) = 0$

$x = 0 \qquad x = -2$

$y = 7 \qquad y = 1$

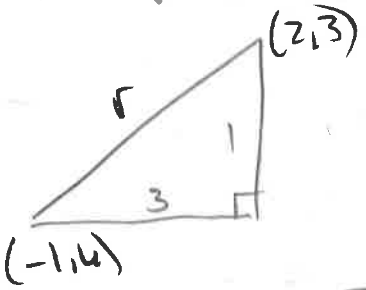
$\therefore P = (-2, 1)$

$Q = (0, 7)$

b) $x^2 + y^2 - 4x - 6y - 7 = 0$

centre $= (2, 3)$

Midpoint $= (-1, 4)$



$r^2 = 3^2 + 1^2$

$= 10$

$\therefore (x-2)^2 + (y-3)^2 = 10$

10 a) $P = 4.99087(42.5 - 24.55)^{1.81}$

$= 929$ points

b) $850 = 0.188807(600 - 210)^k$

$850 = 0.188807(390)^k$

$390^k = \dfrac{850}{0.188807}$

$\ln(390^k) = \ln\left(\dfrac{850}{0.188807}\right)$

$k \ln 390 = \ln\left(\dfrac{850}{0.188807}\right)$

$k = \dfrac{\ln\left(\dfrac{850}{0.188807}\right)}{\ln 390}$

$k = 1.41$