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X847/76/12

Mathematics
Paper 2

Amended Marking Instructions

FRIDAY, 6 MAY

Strictly Confidential
These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

Version 2
30/05/22

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ •6 $y = 5$ and $y = -7$

Vertical: •5 $x = 2$ and $y = 5$ •6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

$\left(x^3+2x^2+3x+2\right)(2x+1)$ written as

$\left(x^3+2x^2+3x+2\right)\times 2x+1$

$=2x^4+5x^3+8x^2+7x+2$

gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :

- If the candidate has written in any unexpected areas of their answer booklet
- If the script is legible and that it does not require to be re-scanned
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate
- If the candidate has continued an answer to a question at the back or in a different location in the booklet
- The presence of any non-script related objects.

No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).

Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.

Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.

Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking annotations such as tick/cross
or the 

annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.

Link tool: The Link tool  allows you to link pages/additional objects to a particular question item on a response.

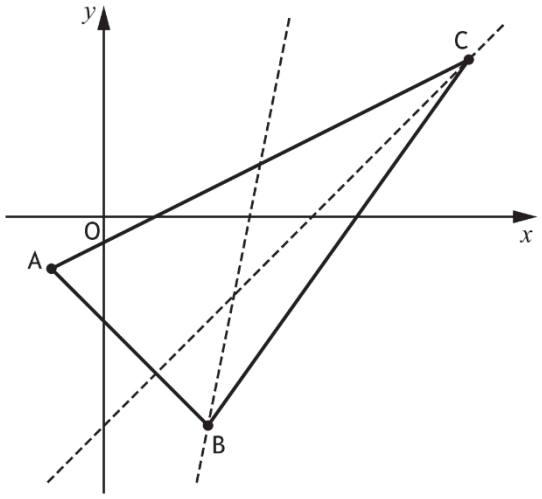
In "Full Response View":

- Check which question the candidate's answer relates to
- Click on the question in the marks display panel
- On the left hand side, select the Link Page check box beneath the thumbnail for the page
- Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed **do not** belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking **OR** If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations		
Annotation	**Annotation Name**	**Instructions on use of annotation**
	Tick	A tick should be placed on the script at the point where a mark is awarded (or at the end of that line of working).
	Cross	A cross is used to indicate where a mark has not been awarded.
	Highlight	This is used to highlight or underline an error.
	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
	Omission	An omission symbol should be used to show that something is missing, such as part of a solution or a crucial step in the working.
	Tick 1	A tick 1 should be used to indicate 'correct' working where a mark is awarded as a result of follow through from an error.
	Tick 2	A tick 2 should be used to indicate correct working which is irrelevant or insufficient to award any marks. This should also be used for working which is not of equivalent difficulty.
	Horizontal wavy line	A horizontal wavy line should be used to indicate a minor error which is not being penalised, e.g. bad form (bad form only becomes bad form if subsequent working is correct).

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		Triangle ABC has vertices A(−1, −1), B(2, −4) and C(7, 3).  (a) Find the equation of the altitude through C.		
			•[1] determine gradient of AB •[2] determine gradient of altitude •[3] find equation	•[1] -1 •[2] 1 •[3] $y = x - 4$	3

Notes:

1. •[3] is only available to candidates who find and use a perpendicular gradient.
2. At •[3], accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A – BEWARE

Correct gradient from incorrect substitution

$m_{AB} = \dfrac{2-(-1)}{-4-(-1)} = -1$ •[1] ✗

$m_{\perp} = 1$ •[2] ✓ 1

$y = x - 4$ •[3] ✓ 1

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		(b) Find the equation of the median through B.		
			•4 determine midpoint of AC •5 determine gradient of median •6 find equation	•4 $(3,1)$ •5 5 •6 $y=5x-14$	3

Notes:

3. •5 is only available to candidates who use a midpoint to find a gradient.
4. •6 is only available as a consequence of using a 'midpoint' of AC and the point B.
5. At •6, accept any arrangement of a candidate's equation where constant terms have been simplified.
6. •6 is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A – Perpendicular bisector of AC

Midpoint$_{AC}$ $(3,1)$ •1 ✓

$m_{AC}=\frac{1}{2}\Rightarrow m_{\perp}=-2$ •2 ✗

$y+2x=7$ •3 ✓ 2

For other perpendicular bisectors award 0/3

Candidate B – Altitude through B

$m_{AC}=\frac{1}{2}$ •1 ^

$m_{\perp}=-2$ •2 ✗

$y+2x=0$ •3 ✓ 2

Candidate C – Median through A

Midpoint$_{BC}$ $\left(\frac{9}{2},-\frac{1}{2}\right)$ •1 ✗

$m_{AM}=\frac{1}{11}$ •2 ✓ 1

$11y=x-10$ •3 ✓ 2

Candidate D – Median through C

Midpoint$_{AB}$ $\left(\frac{1}{2},-\frac{5}{2}\right)$ •1 ✗

$m_{CM}=\frac{11}{13}$ •2 ✓ 1

$13y=11x-38$ •3 ✓ 2

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		(c) Determine the coordinates of the point of intersection of the altitude through C and the median through B.		
			•7 determine x-coordinate •8 determine y-coordinate	•7 2.5 •8 -1.5	2

Notes:

7. For $\left(\frac{10}{4},-\frac{6}{4}\right)$ award 1/2 (do not penalise repeated lack of simplification – *general marking principle* (*l*)).

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
2.	The equation $2x^2-8x+(4-p)=0$ has two real and distinct roots. Determine the range of values for p.		
	$\bullet^1$ use discriminant $\bullet^2$ apply condition and simplify $\bullet^3$ state range	$\bullet^1$ $(-8)^2-4(2)(4-p)$ $\bullet^2$ $32+8p>0$ or $8p>-32$ $\bullet^3$ $p>-4$	**3**

Notes:

1. At $\bullet^1$, treat the inconsistent use of brackets eg $(-8)^2-4\times2\times4-p$ or $-8^2-4(2)(4-p)$ as bad form only if the candidate deals with the unbracketed terms correctly in the next line of working.
2. If candidates have the condition 'discriminant $=0$', then $\bullet^2$ and $\bullet^3$ are unavailable. However, see Candidate E.
3. If candidates have the condition 'discriminant <0', 'discriminant ≤ 0' or 'discriminant ≥ 0' then $\bullet^2$ is lost but $\bullet^3$ is available.

Commonly Observed Responses:

Candidate A - bad form

$(-8)^2-4\times2\times4-p>0$

$32+8p>0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$p>-4$ $\bullet^3$ ✓

Candidate B - no coefficient of p

$(-8)^2-4\times2\times4-p>0$

$32-p>0$ $\bullet^1$ ✗ $\bullet^2$ ✓ 2

$p<32$ $\bullet^3$ ✓ 2

Candidate C - bad form

$-8^2-4\times2\times(4-p)>0$

$32+8p>0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$p>-4$ $\bullet^3$ ✓

Candidate D - not bad form

$-8^2-4\times2\times(4-p)>0$

$-96+8p>0$ $\bullet^1$ ✗ $\bullet^2$ ✓ 2

$p>12$ $\bullet^3$ ✓ 1

Candidate E - condition stated initially

Real and distinct roots $b^2-4ac>0$

$(-8)^2-4(2)(4-p)=0$ $\bullet^1$ ✓

$32+8p=0$

$p=-4$

so $p>-4$ $\bullet^2$ ✓ $\bullet^3$ ✓

Candidate F

$8^2-4(2)(4-p)>0$ $\bullet^1$ ✗

$32+8p>0$ $\bullet^2$ ✓ 1

$p>-4$ $\bullet^3$ ✓ 1

However, $64-4(2)(4-p)>0$ as the first line of working may be awarded $\bullet^1$

Question			Generic scheme	Illustrative scheme	Max mark
3.			(a) Express $4\sin x+5\cos x$ in the form $k\sin(x+a)$ where $k>0$ and $0<a<2\pi$.		
	(a)		$\bullet^1$ use compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process for k $\bullet^4$ process for a and express in required form	$\bullet^1$ $k\sin x\cos a+k\cos x\sin a$ stated explicitly $\bullet^2$ $k\cos a=4$ and $k\sin a=5$ stated explicitly $\bullet^3$ $k=\sqrt{41}$ $\bullet^4$ $\sqrt{41}\sin(x+0.896\ldots)$	4

Notes:

1. Accept $k(\sin x\cos a+\cos x\sin a)$ at $\bullet^1$.
2. Treat $k\sin x\cos a+\cos x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain k.
3. $\sqrt{41}\sin x\cos a+\sqrt{41}\cos x\sin a$ or $\sqrt{41}(\sin x\cos a+\cos x\sin a)$ are acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $k\cos x=4$ and $k\sin x=5$, however $\bullet^4$ may still be gained. See Candidate E.
5. $\bullet^3$ is only available for a single value of k, $k>0$.
6. $\bullet^4$ is not available for a value of a given in degrees.
7. Accept values of a which round to 0.9.
8. Candidates may use any form of the wave function for $\bullet^1$, $\bullet^2$ and $\bullet^3$. However, $\bullet^4$ is only available if the wave is interpreted in the form $k\sin(x+a)$.
9. Evidence for $\bullet^4$ may not appear until part (b) and must appear by the $\bullet^5$ stage.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ^

$\sqrt{41}\cos a=4$

$\sqrt{41}\sin a=5$ $\bullet^2$ ✓ $\bullet^3$ ✓

$\tan a=\frac{5}{4}$

$a=0.896\ldots$

$\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^4$ ✓

Candidate B

$k\sin x\cos a+k\cos x\sin a$ $\bullet^1$ ✓

$\cos a=4$

$\sin a=5$ $\bullet^2$ ✗

$\tan a=\frac{5}{4}$

$a=0.896\ldots$

Not consistent with equations at $\bullet^2$.

$\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^3$ ✓ $\bullet^4$ ✗

Candidate C

$\sin x\cos a+\cos x\sin a$ $\bullet^1$ ✗

$\cos a=4$

$\sin a=5$ $\bullet^2$ ✓ 2

$k=\sqrt{41}$ $\bullet^3$ ✓

$\tan a=\frac{5}{4}$

$a=0.896\ldots$

$\sqrt{41}\sin(x+0.896\ldots)$ $\bullet^4$ ✗

Candidate D - errors at •2	Candidate E - use of x at •2	Candidate F
$k\sin x\cos a + k\cos x\sin a$ •1 ✓	$k\sin x\cos a + k\cos x\sin a$ •1 ✓	$k\sin A\cos B + k\cos A\sin B$ •1 ✗
$k\cos a = 5$ $k\sin a = 4$ •2 ✗	$k\cos x = 4$ $k\sin x = 5$ •2 ✗	$k\cos A = 4$ $k\sin A = 5$ •2 ✗
$\tan a = \frac{4}{5}$ $a = 0.674\ldots$	$\tan x = \frac{5}{4}$ $x = 0.896\ldots$	$\tan A = \frac{5}{4}$ $A = 0.896\ldots$
$\sqrt{41}\sin(x+0.674\ldots)$ •3 ✓ •4 ✓ 1	$\sqrt{41}\sin(x+0.896\ldots)$ •3 ✓ •4 ✓ 1	$\sqrt{41}\sin(x+0.896\ldots)$ •3 ✓ •4 ✓ 1

Question			Generic scheme	Illustrative scheme	Max mark
			(b) Hence solve $4\sin x + 5\cos x = 5.5$ for $0 \le x < 2\pi$.		
	(b)		•5 link to (a) •6 solve for $(x+a)$ •7 solve for x	•5 $\sqrt{41}\sin(x+0.896\ldots) = 5.5$ •6 •7 •6 1.033…, 2.108… •7 0.137…, 1.212…	3

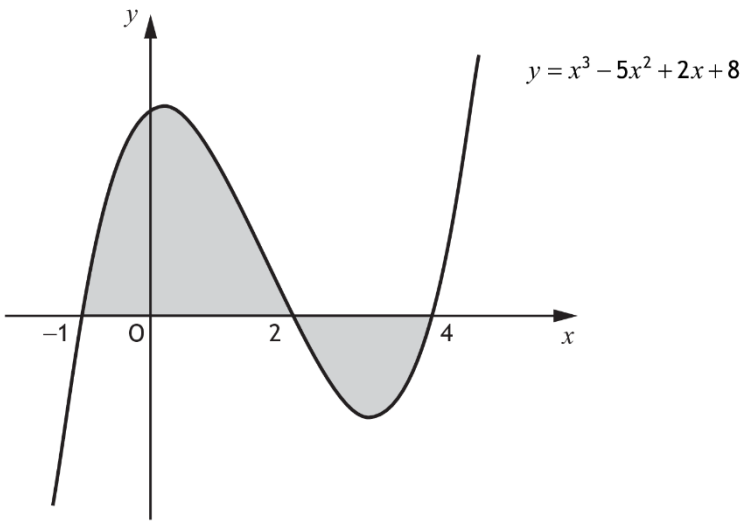
Notes:

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. •7 is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At •7 accept values of x which round to 0.1 or 1.2

Commonly Observed Responses:

Candidate G - converting to radians	Candidate H - working in degrees and truncation
⋮ •1 ✓ •2 ✓ •3 ✓	⋮ •1 ✓ •2 ✓ •3 ✓
$\sqrt{41}\sin(x+51.3\ldots)$ •4 ✗	$\sqrt{41}\sin(x+51.3)$ •4 ✗
$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ •5 ✓ 1	$\sqrt{41}\sin(x+51.3) = 5.5$ •5 ✓ 1
$x+51.3\ldots = 59.1\ldots, 120.8\ldots$ $x = 7.8\ldots, 69.4\ldots$ •6 ✓ 1	$x+51.3 = 59.1, 120.9$ $x = 7.8, 69.6$ •6 ✓ 1 •7 ^
$x = \frac{7.9\pi}{180}, \frac{69.5\pi}{180}$ •7 ✓ 1	

Candidate I - working in degrees	Candidate J - working in degrees
⋮ •1 ✓ •2 ✓ •3 ✓	⋮ •1 ✓ •2 ✓ •3 ✓
$\sqrt{41}\sin(x+51.3\ldots)$ •4 ✗	$\sqrt{41}\sin(x+51.3\ldots)$ •4 ✗
$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ •5 ✓ 1	$\sqrt{41}\sin(x+51.3\ldots) = 5.5$ •5 ✓ 1
$x+51.3\ldots = 59.1\ldots$ $x = 7.8\ldots$ •6 ^ •7 ^	$x+51.3\ldots = 59.1\ldots, 120.8\ldots$ •6 ^ •7 ^

Question			Generic scheme	Illustrative scheme	Max mark
4.			The graph shown has equation $y = x^3 - 5x^2 + 2x + 8$. The total shaded area is bounded by the curve and the x-axis.  (a) Calculate the shaded area above the x-axis.		
	(a)		$\bullet^1$ state appropriate integral	$\bullet^1 \int_{-1}^{2} \left(x^3 - 5x^2 + 2x + 8\right) dx$	**4**
			$\bullet^2$ integrate	$\bullet^2 \frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2x^2}{2} + 8x$	
			$\bullet^3$ substitute limits	$\bullet^3 \left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + (2)^2 + 8(2)\right)$ $-\left(\frac{1}{4}(-1)^4 - \frac{5}{3}(-1)^3 + (-1)^2 + 8(-1)\right)$	
			$\bullet^4$ evaluate area	$\bullet^4$ $\frac{63}{4}$ or 15.75	

Notes:

1. Limits and 'dx' must appear at the $\bullet^1$ stage for $\bullet^1$ to be awarded.
2. Where a candidate differentiates one or more terms at $\bullet^2$, then $\bullet^3$ and $\bullet^4$ are not available.
3. Candidates who substitute limits without integrating, do not gain $\bullet^3$ or $\bullet^4$.
4. Do not penalise the inclusion of '$+c$'.
5. Do not penalise the continued appearance of the integral sign after $\bullet^1$.
6. $\bullet^4$ is not available where solutions include statements such as $-\frac{63}{4} = \frac{63}{4}$. See Candidate C.

Commonly Observed Responses:	
Candidate A $\int_{-1}^{2}\left(x^3-5x^2+2x+8\right)$ $\bullet^1$ ✗ $=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ $\bullet^2$ ✓ $\bullet^3$ ^ $=\frac{63}{4}$ $\bullet^4$ ✓ 1	**Candidate B - evidence of substitution using a calculator** $\int\left(x^3-5x^2+2x+8\right)dx$ $\bullet^1$ ✗ $=\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x$ $\bullet^2$ ✓ $=\frac{32}{3}-\left(-\frac{61}{12}\right)$ $\bullet^3$ ✓ $=\frac{63}{4}$ $\bullet^4$ ✓
Candidate C - communication for $\bullet^4$ $\int_{2}^{-1}\left(x^3-5x^2+2x+8\right)dx$ $\bullet^1$ ✓ ... $\bullet^2$ ✓ $\bullet^3$ ✓ $=-\frac{63}{4}$, hence area is $\frac{63}{4}$. $\bullet^4$ ✓ However $-\frac{63}{4}=\frac{63}{4}$ square units does not gain $\bullet^4$	

Question			Generic scheme	Illustrative scheme	Max mark
			(b) Hence calculate the total shaded area.		
	(b)		**Method 1** $\bullet^5$ state appropriate integral $\bullet^6$ evaluate integral $\bullet^7$ interpret result and evaluate total area	**Method 1** $\bullet^5$ $\int_2^4 \left(x^3-5x^2+2x+8\right)dx$ $\bullet^6$ $-\frac{16}{3}$ $\bullet^7$ $\frac{253}{12}$ or 21.083...	**3**
			Method 2 $\bullet^5$ state appropriate integral $\bullet^6$ substitute limits $\bullet^7$ evaluate total area	**Method 2** $\bullet^5$ $\int_2^4 \left(0-\left(x^3-5x^2+2x+8\right)\right)dx$ $\bullet^6$ $-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)\right)$ $\bullet^7$ $\frac{253}{12}$ or 21.083...	

Notes:

7. For candidates who only consider $\int_{-1}^{4} \ldots dx$ or any other invalid integral, award 0/3.
8. In part (b), at $\bullet^5$ do not penalise the omission of 'dx'.
9. In Method 1, $\bullet^5$ may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_2^4$ or $\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)-\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)$.
10. In Method 2, $\bullet^5$ may be awarded for $\left[\frac{1}{4}x^4-\frac{5}{3}x^3+\frac{2x^2}{2}+8x\right]_4^2$ or $\bullet^5$ and $\bullet^6$ may be awarded for $\left(\frac{1}{4}(2)^4-\frac{5}{3}(2)^3+(2)^2+8(2)\right)-\left(\frac{1}{4}(4)^4-\frac{5}{3}(4)^3+(4)^2+8(4)\right)$.
11. $\bullet^7$ is not available to candidates where solutions include statements such as $-\frac{16}{3}=\frac{16}{3}$ square units. See Candidate D.
12. In Method 1, where a candidate's integral leads to a positive value, $\bullet^7$ is not available.
13. Where a candidate has differentiated in both parts of the question see Candidate E.

Commonly Observed Responses:

Candidate D - communication for $\bullet^7$

$\int_2^4 \left(x^3-5x^2+2x+8\right)dx = -\frac{16}{3}$ $\bullet^5$ ✓ $\bullet^6$ ✓

$\frac{63}{4}+\frac{16}{3}=\frac{253}{12}$ $\bullet^7$ ✓

However, $\bullet^7$ is not available where statements such as "$-\frac{16}{3}=\frac{16}{3}$ square units" or "ignore negative" appear.

Candidate E - differentiation in (a) and (b)

(a) $\int_{-1}^{2} \left(x^3-5x^2+2x+8\right)dx$ $\bullet^1$ ✓

$=3x^2-10x+2$ $\bullet^2$ ✗

$=\left(3(2)^2-10(2)+2\right)-\left(3(-1)^2-10(-1)+2\right)$ $\bullet^3$ ✗

$=-21$

Area $=21$ $\bullet^4$ ✗

(b) $\left(3(4)^2-10(4)+2\right)-\left(3(2)^2-10(2)+2\right)=16$ $\bullet^5$ ✓ $\bullet^6$ ✓ 1

Total Area $=5$ $\bullet^7$ ✓ 2 see note 12

Question			Generic scheme	Illustrative scheme	Max mark
5.			Functions f and g are given by $f(x)=x^2-2$ and $g(x)=3x+5$, $x\in\mathbb{R}$. (a) Find expressions for: (i) $f(g(x))$ and (ii) $g(f(x))$.		
	(a)	(i)	$\bullet^1$ interpret notation $\bullet^2$ state expression for $f(g(x))$	$\bullet^1$ $f(3x+5)$ or $(g(x))^2-2$ $\bullet^2$ $(3x+5)^2-2$	**2**
		(ii)	$\bullet^3$ state expression for $g(f(x))$	$\bullet^3$ $3(x^2-2)+5$	**1**

Notes:

1. For $f(g(x))=(3x+5)^2-2$ without working, award both $\bullet^1$ and $\bullet^2$.

Commonly Observed Responses:

Candidate A

(a)(i) $f(g(x))=3(x^2-2)+5$ $\bullet^1$ ✗ $\bullet^2$ ✓ 1

(a)(ii) $g(f(x))=(3x+5)^2-2$ $\bullet^3$ ✓ 1

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		(b) Determine the range of values of x for which $f\left(g\left(x\right)\right) < g\left(f\left(x\right)\right)$.		
			$\bullet^4$ interpret information and expand $\bullet^5$ express inequality in standard quadratic form $\bullet^6$ determine zeros of quadratic equation $\bullet^7$ state range with justification	$\bullet^4$ $9x^2+30x+25-2<3x^2-6+5$ $\bullet^5$ $6x^2+30x+24<0$ $\bullet^6$ $-4,\ -1$ $\bullet^7$ $-4<x<-1$ with eg sketch or table of signs	4

Notes:

2. Candidates who do not work with an inequation from the outset lose $\bullet^4$, $\bullet^5$ and $\bullet^7$. However, $\bullet^6$ is still available. See Candidate D.
3. Accept the appearance of $-4,-1$ within inequalities for $\bullet^6$.
4. At $\bullet^7$ accept "$x>-4$ and $x<-1$" or "$x>-4$, $x<-1$" together with the required justification.

Commonly Observed Responses:

Candidate B

$9x^2+30x+25-2<3x^2-6+5$	$\bullet^4$ ✓
$6x^2+30x+24<0$	$\bullet^5$ ✓
$6x^2+30x+24=0$	
$x=-1,\ x=-4$	$\bullet^6$ ✓
$-4<x<-1$ with sketch	$\bullet^7$ ✓

Candidate C

$9x^2+30x+25-2<3x^2-6+5$	$\bullet^4$ ✓
$6x^2+30x+24=0$	$\bullet^5$ ✗
$x=-1,\ x=-4$	$\bullet^6$ ✓
$-4<x<-1$ with sketch	$\bullet^7$ ✗

Candidate D

$9x^2+30x+25-2=3x^2-6+5$	$\bullet^4$ ✗
$6x^2+30x+24=0$	$\bullet^5$ ✗
$x=-1,\ x=-4$	$\bullet^6$ ✓
For $f\left(g\left(x\right)\right)<g\left(f\left(x\right)\right)$	
$-4<x<-1$ with sketch	$\bullet^7$ ✗

Question	Generic scheme	Illustrative scheme	Max mark
6.	A curve with equation $y = f(x)$ is such that $\frac{dy}{dx} = 1 - \frac{3}{x^2}$, where $x > 0$. The curve passes through the point $(3, 6)$. Express y in terms of x.		
	$\bullet^1$ write in integrable form $\bullet^2$ integrate one term $\bullet^3$ complete integration $\bullet^4$ interpret information given and substitute for x and y $\bullet^5$ state expression for y	$\bullet^1$ $1 - 3x^{-2}$ $\bullet^2$ x or $\cdots - \frac{3x^{-1}}{-1}$ $\bullet^3$ $\cdots - \frac{3x^{-1}}{-1} + c$ or $x \ldots + c$ $\bullet^4$ $6 = 3 + 3(3)^{-1} + c$ $\bullet^5$ $y = x + 3x^{-1} + 2$	**5**

Notes:

1. For candidates who make no attempt to integrate only $\bullet^1$ is available.
2. For candidates who omit $+ c$ only $\bullet^1$ and $\bullet^2$ are available.
3. For candidates who differentiate either term, $\bullet^3$, $\bullet^4$, and $\bullet^5$ are not available.

Commonly Observed Responses:

Candidate A - incomplete substitution

$y = x + 3x^{-1} + c$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

$y = 3 + 3(3)^{-1} + c$

$c = -4$ $\bullet^4$ ^

$y = x + 3x^{-1} - 4$ $\bullet^5$ ✓ 1

Candidate B - partial integration

$y = 1 + 3x^{-1} + c$ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✗

$6 = 1 + 3(3)^{-1} + c$ $\bullet^4$ ✓ 1

$c = 4$

$y = x + 3x^{-1} + 4$ $\bullet^5$ ✓ 1

Candidate C - inconsistent working

$\frac{dy}{dx} = 1 - \frac{3}{x^2}$

$x - 3x^{-2}$ $\bullet^1$ ✗

$y = x - \frac{3x^{-1}}{-1} + c$ $\bullet^2$ ✓ 1 $\bullet^3$ ✓ 1

Candidate D - inconsistent working

$\frac{dy}{dx} = 1 - \frac{3}{x^2}$

$x - 3x^{-2}$ $\bullet^1$ ✗

$y = \frac{x^2}{2} - \frac{3x^{-1}}{-1} + c$ $\bullet^2$ ✓ 1 $\bullet^3$ ✓ 1

Candidate E
integration not complete at $\bullet^3$ stage

$\frac{dy}{dx} = 1 - 3x^{-2}$ $\bullet^1$ ✓

$y = x - \frac{3x^{-1}}{-1}$ $\bullet^2$ ✓ $\bullet^3$ ✗

$y = x + 3x^{-1} + c$

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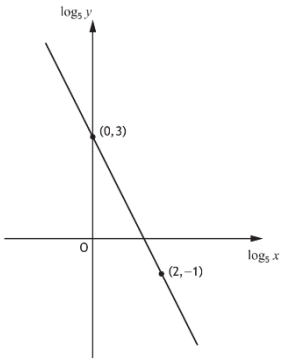
Question	Generic scheme	Illustrative scheme	Max mark
7.	Two variables, x and y, are connected by the equation $y = kx^n$. The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown. Find the values of k and n.		
	Method 1	**Method 1**	**5**
	•[1] state equation of line	•[1] $\log_5 y = -2\log_5 x + 3$	
	•[2] introduce logs	•[2] $\log_5 y = -2\log_5 x + 3\log_5 5$	
	•[3] use laws of logs	•[3] $\log_5 y = \log_5 x^{-2} + \log_5 5^3$	
	•[4] use laws of logs	•[4] $\log_5 y = \log_5 5^3 x^{-2}$	
	•[5] state k and n	•[5] $k = 125,\ n = -2$	
	Method 2	**Method 2**	
	•[1] state equation of line	•[1] $\log_5 y = -2\log_5 x + 3$	
	•[2] use laws of logs	•[2] $\log_5 y = \log_5 x^{-2} + 3$	
	•[3] use laws of logs	•[3] $\log_5 \dfrac{y}{x^{-2}} = 3$	
	•[4] use laws of logs	•[4] $\dfrac{y}{x^{-2}} = 5^3$	
	•[5] state k and n	•[5] $k = 125, n = -2$	
	Method 3	**Method 3** **The equations at •[1], •[2], and •[3] must be stated explicitly.**	
	•[1] introduce logs to $y = kx^n$	•[1] $\log_5 y = \log_5 kx^n$	
	•[2] use laws of logs	•[2] $\log_5 y = n\log_5 x + \log_5 k$	
	•[3] interpret intercept	•[3] $\log_5 k = 3$	
	•[4] use laws of logs	•[4] $k = 125$	
	•[5] interpret gradient	•[5] $n = -2$	

		Method 4 $\bullet^1$ interpret point on log graph $\bullet^2$ convert from log to exponential form $\bullet^3$ interpret point and convert $\bullet^4$ substitute into $y = kx^n$ and evaluate k $\bullet^5$ substitute other point into $y = kx^n$ and evaluate n	**Method 4** $\bullet^1$ $\log_5 x = 0$ and $\log_5 y = 3$ $\bullet^2$ $x = 1, y = 5^3$ $\bullet^3$ $\log_5 x = 2$ **and** $\log_5 y = -1$ $x = 5^2$ and $y = 5^{-1}$ $\bullet^4$ $5^3 = k(1)^n \Rightarrow k = 125$ $\bullet^5$ $5^{-1} = 5^3 \times 5^{2n}$ $\Rightarrow 3 + 2n = -1$ $\Rightarrow n = -2$	

Notes:

1. In any method, marks may only be awarded within a valid strategy using $y = kx^n$.
2. Markers must identify the method which best matches the candidates approach; markers must not mix and match between methods.
3. Penalise the omission of base 5 at most once in any method.
4. In Method 4, candidates may use $(2, -1)$ for $\bullet^1$ and $\bullet^2$ and (0,3) for $\bullet^3$.
5. Do not accept $k = 5^3$.
6. In Method 3, do not accept $m = -2$ or gradient $= -2$ for $\bullet^5$.
7. Accept $y = 125x^{-2}$ for $\bullet^5$.

Commonly Observed Responses:

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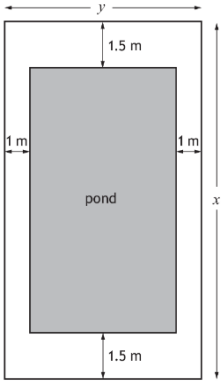
Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		A rectangular plot consists of a rectangular pond surrounded by a path. The length and breadth of the plot are x metres and y metres respectively. The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown. The total area of the **pond and path together** is 150 square metres. (a) Show that the area of the pond, A square metres, is given by $A(x)=156-2x-\frac{450}{x}$.  		
			•[1] determine expression for area of pond •[2] obtain expression for y •[3] demonstrate result	•[1] $(x-3)(y-2)$ stated or implied by •[3] •[2] $y=\frac{150}{x}$ •[3] $A(x)=(x-3)\left(\frac{150}{x}-2\right)$ eg $A(x)=\frac{150x}{x}-\frac{450}{x}-2x+6$ $A(x)=156-2x-\frac{450}{x}$	**3**

Notes:

1. Accept any legitimate variations for the area of the pond in •[1], eg $A=150-2(x-3)-2(y)(1.5)$.
2. Do not penalise the omission of brackets at •[1]. See Candidate A.
3. The substitution for y at •[3] must be clearly shown for •[3] to be available.

Commonly Observed Responses:

Candidate A

$A(x)=x-3\times y-2$ •[1] ✓

$A(x)=x-3\times\frac{150}{x}-2$ •[2] ✓

$A(x)=156-2x-\frac{450}{x}$ •[3] ^

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$A(x) = 156 - 2x - \frac{450}{x}$. (b) Determine the maximum area of the pond.		
			$\bullet^4$ express A in differentiable form $\bullet^5$ differentiate $\bullet^6$ equate expression for derivative to 0 $\bullet^7$ solve for x $\bullet^8$ verify nature of stationary point $\bullet^9$ determine maximum area	$\bullet^4$ $156 - 2x - 450x^{-1}$ stated or implied by $\bullet^5$ $\bullet^5$ $-2 + 450x^{-2}$ $\bullet^6$ $-2 + 450x^{-2} = 0$ $\bullet^7$ $x = 15$ $\bullet^8$ table of signs for derivative $\therefore$ maximum or $A''(x) = -900x^{-3}$ and $A''(15) < 0$ $\therefore$ maximum $\bullet^9$ $A = 96\left(\text{m}^2\right)$	6

Notes:

4. For a numerical approach award 0/6.
5. $\bullet^6$ can be awarded for $450x^{-2} = 2$.
6. For candidates who integrate any term at the $\bullet^5$ stage, only $\bullet^6$ is available on follow through for setting their 'derivative' to 0.
7. $\bullet^7$, $\bullet^8$, and $\bullet^9$ are only available for working with a derivative which contains an index ≤ -2.
8. $\sqrt{\frac{450}{2}}$ must be simplified at $\bullet^7$ or $\bullet^8$ for $\bullet^7$ to be awarded.
9. Ignore the appearance of -15 at mark $\bullet^7$.
10. $\bullet^8$ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 15.
11. $\bullet^9$ is still available in cases where a candidate's table of signs does not lead legitimately to a maximum at $\bullet^8$.
12. $\bullet^8$ and $\bullet^9$ are not available to candidates who state that the maximum exists at a negative value of x.

For the table of signs for a derivative, accept:

x	15^-	15	15^+
$A'(x)$	+	0	−
Slope or shape			

x	$\rightarrow$	15	$\rightarrow$
$A'(x)$	+	0	−
Slope or shape			

Arrow are taken to mean 'in the neighbourhood of'

x	a	15	b
$A'(x)$	+	0	−
Slope or shape			

Where $0 < a < 15$ and $b > 15$

For the table of signs for a derivative, **do not** accept:

x	$\rightarrow$	-15	$\rightarrow$	15	$\rightarrow$
$A'(x)$	−	0	+	0	−
Slope or shape					

Since the function is discontinuous
$-15 \rightarrow 15$ is not acceptable

x	a	-15	b	15	c
$A'(x)$	−	0	+	0	−
Slope or shape					

Since the function is discontinuous
$-15 < b < 15$ is not acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $A'(x)$ is an acceptable alternative to writing '+' or '−' signs.
- Acceptable variations of $A'(x)$ are: A', $a'(x)$, $\frac{dA}{dx}$, and $-2+450x^{-2}$.

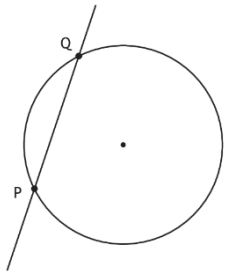
Commonly Observed Responses:

Candidate B - differentiating over multiple lines

	•4 ^
$A'(x) = -2-450x^{-1}$	
$A'(x) = -2+450x^{-2}$	•5 ✘
$-2+450x^{-2} = 0$	•6 ✓ 1

Candidate C - differentiating over multiple lines

$A(x) = 156-2x-450x^{-1}$	•4 ✓
$A'(x) = -2-450x^{-1}$	
$A'(x) = -2+450x^{-2}$	•5 ✘
$-2+450x^{-2} = 0$	•6 ✓ 1

Question			Generic scheme	Illustrative scheme	Max mark
9.			The line $y=3x+7$ intersects the circle $x^2+y^2-4x-6y-7=0$ at the points P and Q. (a) Find the coordinates of P and Q. 		
	(a)		$\bullet^1$ substitute for y in equation of circle $\bullet^2$ arrange in standard quadratic form $\bullet^3$ factorise $\bullet^4$ state x coordinates $\bullet^5$ state corresponding y coordinates	$\bullet^1$ $x^2+(3x+7)^2-4x-6(3x+7)-7=0$ $\bullet^2$ $10x^2+20x=0$ $\bullet^3$ $10x(x+2)=0$ $\bullet^4$ 0 $\quad$ -2 $\bullet^5$ 7 $\quad$ 1	**5**

Notes:

1. $\bullet^1$ is only available if '$=0$' appears by the $\bullet^3$ stage.
2. At $\bullet^3$, the quadratic must lead to two distinct real roots for $\bullet^4$ and $\bullet^5$ to be available.
3. At $\bullet^3$ do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
4. If a candidate arrives at an equation which is not a quadratic at $\bullet^2$ stage, then $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available
5. $\bullet^3$ is available for substituting correctly into the quadratic formula.
6. $\bullet^4$ and $\bullet^5$ may be marked either horizontally or vertically.
7. Ignore incorrect labelling of P and Q.

Commonly Observed Responses:

Candidate A - substituting for y

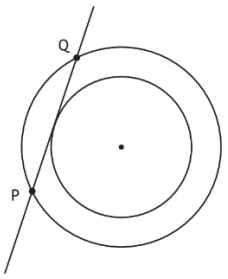
$\left(\frac{y-7}{3}\right)^2+y^2-4\left(\frac{y-7}{3}\right)-6y-7=0$ $\bullet^1$ ✓

$\frac{10y^2-80y+70}{9}=0$ $\bullet^2$ ✓

$10(y-1)(y-7)=0$ $\bullet^3$ ✓

$y=1$ or $y=7$ $\bullet^4$ ✓

$x=-2$ or $x=0$ $\bullet^5$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
			PQ is a tangent to a second, smaller circle. This circle is concentric with the first. (b) Determine the equation of the smaller circle. 		
	(b)		•6 state centre of circle •7 calculate midpoint of PQ •8 calculate radius of small circle •9 state equation of small circle	•6 $(2, 3)$ •7 $(-1, 4)$ •8 $\sqrt{10}$ •9 $(x-2)^2+(y-3)^2=10$	4

Notes:

8. Evidence for •6 may appear in part (a).
9. Where a candidate uses coordinates for P and Q without supporting working, •7 is not available, however •8 and •9 may be awarded.
10. Where candidates find the equation of the larger circle •8 and •9 are not available.

Commonly Observed Responses:

Candidate B - using substitution

Equation of smaller circle of form

$(x-2)^2+(y-3)^2=r^2$ •6 ✓

Midpoint PQ $(-1, 4)$ •7 ✓

$(-1-2)^2+(4-3)^2=r^2$

$r^2=10$ •8 ✓

$(x-2)^2+(y-3)^2=10$ •9 ✓

Candidate C - using tangency

Equation of smaller circle of form

$(x-2)^2+(y-3)^2=r^2$ •6 ✓

Since $y=3x+7$ is tangent to smaller circle

$10x^2+20x+20-r^2=0$ has equal roots

$\Rightarrow 20^2-4(10)(20-r^2)=0$ •7 ✓

$\Rightarrow r^2=10$ •8 ✓

$(x-2)^2+(y-3)^2=10$ •9 ✓

Candidate D - using P or Q to mid-point as radius

⋮

$r=\sqrt{(-2+1)^2+(1-4)^2}=\sqrt{10}$ •8 ✗

or

$r=\sqrt{(0+1)^2+(7-4)^2}=\sqrt{10}$ •8 ✗

$(x-2)^2+(y-3)^2=10$ •9 ✓ 2

Question			Generic scheme	Illustrative scheme	Max mark
10.			The heptathlon is an athletics contest made up of seven events. Athletes score points for each event. In the 200 metres event, the points are calculated using the formula $$P = 4.99087\left(42.5 - T\right)^{1.81}$$ where P is the number of points awarded, and T is the athlete's time, in seconds. (a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event.		
	(a)		•[1] evaluate P for $t = 24.55$	•[1] 929	**1**
Notes:					
1. Accept any answer which rounds 929.0368007… to at least 2 significant figures.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
			In the long jump event, the points are calculated using the formula $P = 0.188807(D-210)^k$ where P is the number of points awarded, D is the distance jumped, in centimetres, and k is a constant. (b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k.		
	(b)		$\bullet^2$ substitute for P and D $\bullet^3$ arrange equation in the form $a = b^k$ $\bullet^4$ write in logarithmic form $\bullet^5$ solve for k	$\bullet^2$ $850 = 0.188807(600-210)^k$ $\bullet^3$ $\frac{850}{0.188807} = (600-210)^k$ $\bullet^4$ eg $\ln\left(\frac{850}{0.188807}\right) = \ln(600-210)^k$ or $k = \log_{(600-210)} \frac{850}{0.188807}$ $\bullet^5$ 1.41	4

Notes:

2. $\bullet^3$ may be implied by $\bullet^4$.
3. Any base may be used at $\bullet^4$ stage.
4. Accept 1.4 at $\bullet^5$.
5. The calculation at $\bullet^5$ must follow from the valid use of exponentials and logarithms at $\bullet^3$ and $\bullet^4$. See Candidate A.
6. For candidates who take an iterative approach to arrive at the value $t = 1.41$ award 1/4. However, if, in the iterations P is calculated for $t = 1.405$ and $t = 1.415$ then award 4/4.

Commonly Observed Responses:

Candidate A - invalid use of exponentials

$850 = 0.188807(600-210)^k$ $\bullet^2$ ✓

$850 = 73.63473^k$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗

$\log_{73.63473} 850 = k$

1.56...

Candidate B - transcription error

$850 = 0.18807(600-210)^k$ $\bullet^2$ ✗

$4519.59... = 390^k$ $\bullet^3$ ✓ 1

$\log_{390} 4519.59...$ $\bullet^4$ ✓ 1

1.41... $\bullet^5$ ✓ 1

[END OF MARKING INSTRUCTIONS]