National
Qualifications
2022

X847/76/11

Mathematics
Paper 1 (Non-Calculator)

Amended Marking Instructions

FRIDAY, 6 MAY

Version 2
30/05/22

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

	$x^2 + 5x + 7 = 9x + 4$
This is a transcription error and so the mark is not awarded.	$x - 4x + 3 = 0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x = 1$

The following example is an exception to the above

	$x^2 + 5x + 7 = 9x + 4$
This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x - 4x + 3 = 0$
	$(x-3)(x-1) = 0$
	$x = 1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :

- If the candidate has written in any unexpected areas of their answer booklet
- If the script is legible and that it does not require to be re-scanned
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate
- If the candidate has continued an answer to a question at the back or in a different location in the booklet
- The presence of any non-script related objects.

No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).

Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.

Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.

Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking annotations such as tick/cross
or the SEEN annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.

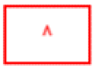
Link tool: The Link tool allows you to link pages/additional objects to a particular question item on a response.

In "Full Response View":

- Check which question the candidate's answer relates to
- Click on the question in the marks display panel
- On the left hand side, select the Link Page check box beneath the thumbnail for the page
- Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed **do not** belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking **OR** If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations		
Annotation	**Annotation Name**	**Instructions on use of annotation**
	Tick	A tick should be placed on the script at the point where a mark is awarded (or at the end of that line of working).
	Cross	A cross is used to indicate where a mark has not been awarded.
	Highlight	This is used to highlight or underline an error.
	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
	Omission	An omission symbol should be used to show that something is missing, such as part of a solution or a crucial step in the working.
	Tick 1	A tick 1 should be used to indicate ‘correct’ working where a mark is awarded as a result of follow through from an error.
	Tick 2	A tick 2 should be used to indicate correct working which is irrelevant or insufficient to award any marks. This should also be used for working which is not of equivalent difficulty.
	Horizontal wavy line	A horizontal wavy line should be used to indicate a minor error which is not being penalised, e.g. bad form (bad form only becomes bad form if subsequent working is correct).

Question			Generic scheme	Illustrative scheme	Max mark
1.			Determine the equation of the line perpendicular to $5x+2y=7$, passing through $(-1,6)$.		
			•¹ state gradient •² state perpendicular gradient •³ find equation of line	•¹ $-\frac{5}{2}$ •² $\frac{2}{5}$ •³ $5y=2x+32$	3

Notes:

1. At •¹, ignore any errors in processing the constant term.
2. At •¹ and •², ignore the appearance of 'x'.
3. •³ is only available as a consequence of using a perpendicular gradient.
4. At •³, accept any arrangement of a candidate's equation where constant terms have been simplified.

Commonly Observed Responses:

Candidate A A perpendicular gradient has been clearly stated $5x+2y=7$ $m_{\perp}=\frac{2}{5}$ •¹ ✓ •² ✓ $5y=2x+32$ •³ ✓	**Candidate B** No communication for perpendicular gradient $5x+2y=7$ $y=-\frac{5}{2}x+\frac{7}{2}$ $m=\frac{2}{5}$ •¹ ^ •² $\boxed{✓1}$ $5y=2x+32$ •³ $\boxed{✓1}$
Candidate C $m=5$ •¹ ✗ $m_{\perp}=-\frac{1}{5}$ •² $\boxed{✓1}$ $x+5y=29$ •³ $\boxed{✓1}$	

Question			Generic scheme	Illustrative scheme	Max mark
2.			Evaluate $2\log_3 6 - \log_3 4$.		
			$\bullet^1$ apply $m\log_n x = \log_n x^m$ $\bullet^2$ apply $\log_n x - \log_n y = \log_n \frac{x}{y}$ $\bullet^3$ evaluate	$\bullet^1$ $\log_3 6^2$ $\bullet^2$ $\log_3 \frac{6^2}{4}$ $\bullet^3$ 2	**3**

Notes:

1. Do not penalise the omission of the base of the logarithm at $\bullet^1$ or $\bullet^2$.
2. Correct answer with no working, award 0/3.

Commonly Observed Responses:

Candidate A - introducing a variable		**Candidate B**	
$\log_3 9$	$\bullet^1$ ✓ $\bullet^2$ ✓	$2\log_3\left(\frac{6}{4}\right)$	$\bullet^2$ ✗
$3^x = 9$		$\log_3\left(\frac{6}{4}\right)^2$	$\bullet^1$ $\boxed{✓ 1}$ $\bullet^3$ ^
$x = 2$	$\bullet^3$ ✓		

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Question			Generic scheme	Illustrative scheme	Max mark
3.			A function, h, is defined by $h(x) = 4 + \frac{1}{3}x$, where $x \in \mathbb{R}$. Find the inverse function, $h^{-1}(x)$.		
			Method 1 •1 equate composite function to x •2 write $h\left(h^{-1}(x)\right)$ in terms of $h^{-1}(x)$ •3 state inverse function	**Method 1** •1 $h\left(h^{-1}(x)\right) = x$ •2 $4 + \frac{1}{3}h^{-1}(x) = x$ •3 $h^{-1}(x) = 3(x-4)$	3
			Method 2 •1 write as $y = h(x)$ and start to rearrange •2 express x in terms of y •3 state inverse function	**Method 2** •1 $y = h(x) \Rightarrow x = h^{-1}(y)$ $y - 4 = \frac{1}{3}x$ or $3y = 12 + x$ •2 $x = 3(y-4)$ •3 $h^{-1}(y) = 3(y-4)$ $\Rightarrow h^{-1}(x) = 3(x-4)$	

Notes:

1. In Method 1, accept $4 + \frac{1}{3}h^{-1}(x) = x$ for •1 and •2.
2. In Method 2, accept '$y - 4 = \frac{1}{3}x$' without reference to $y = h(x) \Rightarrow x = h^{-1}(y)$ at •1.
3. In Method 2, accept $h^{-1}(x) = 3(x-4)$ without reference to $h^{-1}(y)$ at •3.
4. In Method 2, beware of candidates with working where each line is not mathematically equivalent. See Candidates A and B for example.
5. At •3 stage, accept h^{-1} written in terms of any dummy variable eg $h^{-1}(y) = 3(y-4)$.
6. $y = 3(x-4)$ does not gain •3.
7. $h^{-1}(x) = 3(x-4)$ with no working gains 3/3.

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<tr><th colspan="2">Commonly Observed Responses:</th></tr>
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Candidate A

$h(x)=4+\frac{1}{3}x$

$y=4+\frac{1}{3}x$

$x=3(y-4)$ $\bullet^1$ ✓ $\bullet^2$ ✓

$y=3(x-4)$ $\bullet^3$ ✗

$h^{-1}(x)=3(x-4)$

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Candidate B

$h(x)=4+\frac{1}{3}x$

$y=4+\frac{1}{3}x$

$x=4+\frac{1}{3}y$ $\bullet^1$ ✗

$y=3(x-4)$ $\bullet^2$ $\boxed{\checkmark\ 1}$

$h^{-1}(x)=3(x-4)$ $\bullet^3$ $\boxed{\checkmark\ 1}$

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Candidate C - BEWARE

$h'=\ldots$ $\bullet^3$ ✗

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Candidate D

$x\to x\div 3\to x\div 3+4=h(x)$

$\div 3\to +4$

$\therefore -4\to \times 3$ $\bullet^1$ ✓

$3(x-4)$ $\bullet^2$ ✓

$h^{-1}(x)=3(x-4)$ $\bullet^3$ ✓

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Question			Generic scheme	Illustrative scheme	Max mark
4.			Differentiate $y=\sqrt{x^3}-2x^{-1}$, where $x>0$.		
			•1 express first term in differentiable form •2 differentiate first term •3 differentiate second term	•1 $y=x^{\frac{3}{2}}\ldots$ stated or implied by •2 •2 $\frac{3}{2}x^{\frac{1}{2}}\ldots$ •3 $\ldots+2x^{-2}$	3

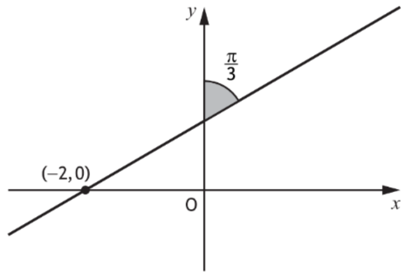
Notes:

1. •2 is only available for differentiating a term with a fractional index.
2. Where candidates attempt to integrate throughout, only •1 is available.

Commonly Observed Responses:

Candidate A - differentiating over two lines

$y=x^{\frac{3}{2}}+2x^{-2}$	•1 ✓
$y=\frac{3}{2}x^{\frac{1}{2}}+2x^{-2}$	•2 ✓ •3 ✗

Question			Generic scheme	Illustrative scheme	Max mark
5.			A line makes an angle of $\frac{\pi}{3}$ radians with the y-axis, and passes through the point $(-2, 0)$ as shown below. Determine the equation of the line.		
			$\bullet^1$ use $m = \tan\theta$ $\bullet^2$ evaluate exact value $\bullet^3$ determine equation	$\bullet^1$ $m = \tan\frac{\pi}{6}$ or $m = \tan 30°$ $\bullet^2$ $\frac{1}{\sqrt{3}}$ $\bullet^3$ eg $y\sqrt{3} = x + 2$ or $y = \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$	3

Notes:

1. Do not award $\bullet^1$ for $m = \tan^{-1}\frac{\pi}{6}$. However $\bullet^2$ and $\bullet^3$ are still available
2. Where candidates make no reference to a trigonometric ratio or use an incorrect trigonometric ratio, $\bullet^1$ and $\bullet^2$ are unavailable.
3. $\bullet^3$ is only available as a consequence of attempting to use a tan ratio. See Candidate F
4. Accept $y = \frac{1}{\sqrt{3}}(x+2)$ for $\bullet^3$, but do not accept $y - 0 = \frac{1}{\sqrt{3}}(x+2)$.

Commonly Observed Responses:

Candidate A		Candidate B	
$m = \tan\frac{\pi}{3}$	$\bullet^1$ ✗	$m = \frac{1}{\sqrt{3}}$ (with or without a diagram)	$\bullet^1$ ^ $\bullet^2$ ✓ 2
$m = \sqrt{3}$	$\bullet^2$ ✓ 1	$y = \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$	
$y = \sqrt{3}x + 2\sqrt{3}$	$\bullet^3$ ✓ 1		$\bullet^3$ ✓ 1

Candidate C		Candidate D	
$m = \tan\theta$ (with or without a diagram)	$\bullet^1$ ^	$m = \tan\theta$ (with or without a diagram)	$\bullet^1$ ^
$m = \frac{1}{\sqrt{3}}$	$\bullet^2$ ✓ 1	$m = \sqrt{3}$	$\bullet^2$ ✗
		$y = \sqrt{3}x + 2\sqrt{3}$	$\bullet^3$ ✓ 1

Candidate E		Candidate F	
$m = \tan\theta = \frac{\pi}{6}$	$\bullet^1$ ✗	$m = \tan\frac{\pi}{3}$	$\bullet^1$ ✗
$m = \frac{1}{\sqrt{3}}$	$\bullet^2$ ✓ 1	$m = 60$	$\bullet^2$ ✗
		$y = 60(x+2)$	$\bullet^3$ ✗

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			Evaluate $\int_{-5}^{2}(10-3x)^{-\frac{1}{2}}\,dx$.		
			•1 start to integrate	•1 $\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\ldots$	4
			•2 complete integration	•2 $\ldots\times-\dfrac{1}{3}$	
			•3 process limits	•3 $-\dfrac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\dfrac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$	
			•4 evaluate integral	•4 2	

Notes:

1. For candidates who **differentiate throughout** or make **no attempt to integrate**, award 0/4.
2. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
3. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after •1.
4. •3 is only available for substitution into an expression which is equivalent to the integrand obtained at •2.
5. The integral obtained must contain a non-integer power for •4 to be available.
6. •4 is only available to candidates who deal with the coefficient of x at the •2 stage. See Candidate A.

Commonly Observed Responses:

Candidate A

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$ •1 ✓ •2 ^

$2(10-3(2))^{\frac{1}{2}}-2(10-3(-5))^{\frac{1}{2}}$ •3 $\boxed{\checkmark\ 1}$

-6 •4 $\boxed{\checkmark\ 2}$ Note 6

Candidate B - NOT differentiating throughout

$-\dfrac{1}{2}(10-3x)^{-\frac{3}{2}}\times-\dfrac{1}{3}$ •1 ✗ •2 ✓

$\dfrac{1}{6}(10-3(2))^{-\frac{3}{2}}-\dfrac{1}{6}(10-3(-5))^{-\frac{3}{2}}$ •3 $\boxed{\checkmark\ 1}$

$\dfrac{39}{2000}$ •4 $\boxed{\checkmark\ 1}$

Candidate C

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times-3$ •1 ✓ •2 ✗

$-6(10-3(2))^{\frac{1}{2}}-\left(-6(10-3(-5))^{\frac{1}{2}}\right)$ •3 $\boxed{\checkmark\ 1}$

18 •4 $\boxed{\checkmark\ 1}$

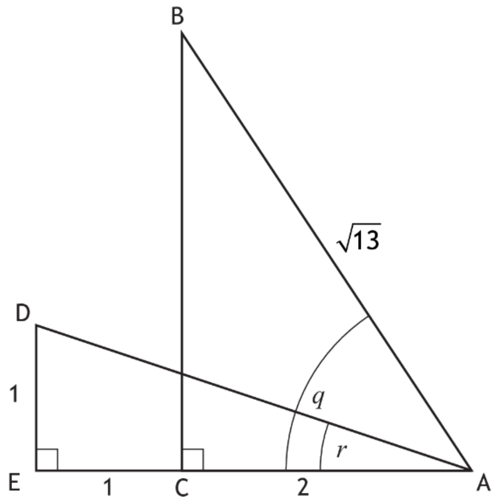
Candidate D - integrating over two lines

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}$

$\dfrac{(10-3x)^{\frac{1}{2}}}{\frac{1}{2}}\times-\dfrac{1}{3}$ •1 ✓ •2 ^

$-\dfrac{2}{3}(10-3(2))^{\frac{1}{2}}-\left(-\dfrac{2}{3}(10-3(-5))^{\frac{1}{2}}\right)$ •3 $\boxed{\checkmark\ 1}$

2 •4 $\boxed{\checkmark\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		Triangles ABC and ADE are both right angled. Angle BAC $= q$ and angle DAE $= r$ as shown in the diagram.  (a) Determine the value of: (i) $\sin r$ (ii) $\sin q$.		
		(i)	•[1] determine $\sin r$	•[1] $\dfrac{1}{\sqrt{10}}$	1
		(ii)	•[2] determine $\sin q$	•[2] $\dfrac{3}{\sqrt{13}}$	1

Notes:

1. In (a)(ii), where candidates do not simplify the perfect square see Candidates A and B.

Commonly Observed Responses:

Candidate A	**Candidate B** - simplification in part (b)
$\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •[2] $\boxed{✓ 2}$	(a)(ii) $\sin q = \dfrac{\sqrt{9}}{\sqrt{13}}$ •[2] ✓ (b) $\sin(q-r) = \dfrac{7}{\ldots}$ Roots have been simplified in (b)

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(b)		(b) Hence determine the value of $\sin(q-r)$.		
			•³ select appropriate formula and express in terms of p and q •⁴ substitute into addition formula •⁵ evaluate $\sin(q-r)$	•³ $\sin q\cos r - \cos q\sin r$ stated or implied by •⁴ •⁴ $\frac{3}{\sqrt{13}}\times\frac{3}{\sqrt{10}}-\frac{2}{\sqrt{13}}\times\frac{1}{\sqrt{10}}$ •⁵ $\frac{7}{\sqrt{130}}$	3

Notes:

2. Award •³ for candidates who write $\sin\left(\frac{3}{\sqrt{13}}\right)\times\cos\left(\frac{3}{\sqrt{10}}\right)-\sin\left(\frac{2}{\sqrt{13}}\right)\times\cos\left(\frac{1}{\sqrt{10}}\right)$. •⁴ and •⁵ are unavailable.
3. For any attempt to use $\sin(q-r)=\sin q-\sin r$, •⁴ and •⁵ are unavailable.
4. At •⁵, the answer must be given as a single fraction. Accept $\frac{7}{\sqrt{13}\sqrt{10}}$, $\frac{7\sqrt{10}}{10\sqrt{13}}$ and $\frac{7\sqrt{13}}{13\sqrt{10}}$.
5. Do not penalise trigonometric ratios which are less than –1 or greater than 1.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			Solve $\log_6 x + \log_6 (x+5) = 2$, where $x > 0$.		
			Method 1 •¹ apply $\log_6 x + \log_6 y = \log_6 xy$ •² write in exponential form •³ express in standard quadratic form •⁴ solve quadratic and state solution of log equation	**Method 1** •¹ $\log_6 (x(x+5)) = \ldots$ •² $x(x+5) = 6^2$ •³ $x^2 + 5x - 36 = 0$ •⁴ $-9,\ 4$ and $x > 0 \Rightarrow x = 4$	4
			Method 2 •¹ apply $\log_6 x + \log_6 y = \log_6 xy$ •² apply $m \log_6 x = \log_6 x^m$ •³ express in standard quadratic form •⁴ solve quadratic and state solution of log equation	**Method 2** •¹ $\log_6 (x(x+5)) = \ldots$ •² $\ldots = \log_6 6^2$ •³ $x^2 + 5x - 36 = 0$ •⁴ $-9,\ 4$ and $x > 0 \Rightarrow x = 4$	

Notes:

1. Accept $\log_6 x(x+5) = \ldots$ for $\bullet^1$.
2. $\bullet^2$ is not available for $x(x+5) = 2^6$; however candidates may still gain $\bullet^3$ and $\bullet^4$.
3. $\bullet^3$ and $\bullet^4$ are only available if the quadratic reached at $\bullet^3$ is obtained by applying the rules in $\bullet^1$ and $\bullet^2$.
4. $\bullet^4$ is only available for solving a polynomial of degree two or higher.
5. At $\bullet^4$, accept any indication that -9 has been discarded. For example, scoring out $x = -9$ or underlining $x = 4$.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_6 (x(x+5)) = 2$	$\bullet^1$ ✓	$\log_6 (x(x+5)) = 2$	$\bullet^1$ ✓
$x(x+5) = 12$	$\bullet^2$ ✗	$x(x+5) = 64$	$\bullet^2$ ✗
$x^2 + 5x - 12 = 0$	$\bullet^3$ $\boxed{✓ 1}$	$x^2 + 5x - 64 = 0$	$\bullet^3$ $\boxed{✓ 1}$
$\dfrac{-5 \pm \sqrt{73}}{2}$ and $x > 0 \Rightarrow x = \dfrac{-5 + \sqrt{73}}{2}$	$\bullet^4$ $\boxed{✓ 1}$	$\dfrac{-5 \pm \sqrt{281}}{2}$ and $x > 0 \Rightarrow x = \dfrac{-5 + \sqrt{281}}{2}$	$\bullet^4$ $\boxed{✓ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			Solve the equation $\cos 2x° = 5\cos x° - 3$ for $0 \leq x < 360$.		
			•1 substitute for $\cos 2x°$ into equation •2 express in standard quadratic form •3 factorise •4 solve for $\cos x°$ •5 solve for x	•1 $2\cos^2 x° - 1...$ •2 $2\cos^2 x° - 5\cos x° + 2 = 0$ •3 $(2\cos x° - 1)(\cos x° - 2) = 0$ •4 $\cos x° = \frac{1}{2}$ •5 $\cos x° = 2$ •5 $x = 60, 300$ 'no solutions'	5

Notes:

1. •1 is not available for simply stating $\cos 2x° = 2\cos^2 x° - 1$ with no further working.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $1 - 2\sin^2 x°$ being substituted for $\cos 2x°$, •1 cannot be awarded until the equation reduces to a quadratic in $\cos x°$.
3. Substituting $2\cos^2 A - 1$ or $2\cos^2 \alpha - 1$ for $\cos 2x°$ at the •1 stage should be treated as bad form provided the equation is written in terms of x at •2 stage. Otherwise, •1 is not available.
4. Do not penalise the omission of degree signs.
5. '$=0$' must appear by •3 stage for •2 to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •2 stage for •2 to be awarded.
6. $\cos x° = \dfrac{5 \pm \sqrt{9}}{4}$ gains •3.
7. Candidates may express the equation obtained at •2 in the form $2c^2 - 5c + 2 = 0$ or $2x^2 - 5x + 2 = 0$. In these cases, award •3 for $(2c-1)(c-2) = 0$ or $(2x-1)(x-2) = 0$. However, •4 is only available if $\cos x°$ appears explicitly at this stage. See Candidate A.
8. The equation $2 + 2\cos^2 x° - 5\cos x° = 0$ does not gain •2 unless •3 has been awarded.
9. •4 and •5 are only available as a consequence of trying to solve a quadratic equation. See Candidate B. However, •5 is not available if the quadratic equation has repeated roots.
10. •3, •4 and •5 are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$. See Candidate C.
11. •5 is only available for 2 valid solutions within the stated range. Ignore 'solutions' outwith the range. However, see Candidate E.
12. Accept $\cancel{\cos x° = 2}$ for •5. See Candidate A.

Commonly Observed Responses:

Candidate A

$\vdots$	$\bullet^1$ ✓ $\bullet^2$ ✓
$2c^2-5c+2=0$	$\bullet^3$ ✓
$c=\frac{1}{2},\ c=2$	$\bullet^4$ ✗
$x=60,\ 300\quad \cancel{\cos x^\circ=2}$	$\bullet^5$ $\boxed{✓\ 1}$

Candidate B - not solving a quadratic

$\vdots$	$\bullet^1$ ✓
$2\cos^2 x^\circ-5\cos x^\circ+2=0$	$\bullet^2$ ✓
$-3\cos x^\circ+2=0$	$\bullet^3$ ✗
$\cos x^\circ=\frac{2}{3}$	$\bullet^4$ $\boxed{✓\ 2}$ $\bullet^5$ ^

Candidate C - not in standard quadratic form

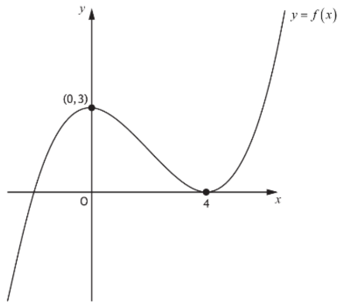
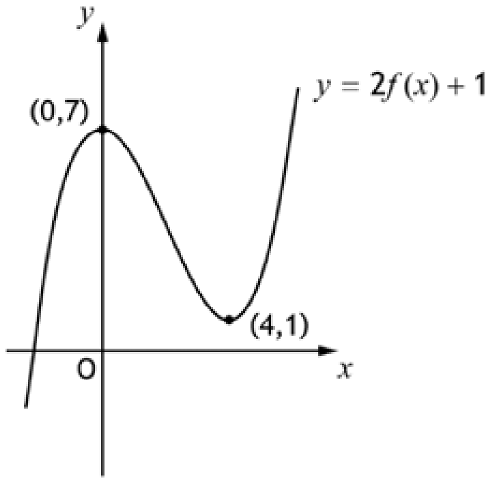
$2\cos^2 x^\circ-1=5\cos x^\circ-3$	$\bullet^1$ ✓
$2\cos^2 x^\circ-5\cos x^\circ=-2$	$\bullet^2$ $\boxed{✓\ 2}$
$\cos x^\circ\left(2\cos x^\circ-5\right)=-2$	$\bullet^3$ $\boxed{✓\ 2}$
$\cos x^\circ=-2,\ 2\cos x^\circ-5=-2$	
$\Rightarrow \cos x=\frac{3}{2}$	$\bullet^4$ ✗
No solutions	$\bullet^5$ ✗

Candidate D - reading $\cos 2x^\circ$ as $\cos^2 x^\circ$

$\cos^2 x^\circ=5\cos x^\circ-3$	$\bullet^1$ ✗
$\cos^2 x^\circ-5\cos x^\circ+3=0$	$\bullet^2$ $\boxed{✓\ 1}$
$\cos x^\circ=\frac{5\pm\sqrt{13}}{2}$	$\bullet^3$ $\boxed{✓\ 1}$
	$\bullet^4$ ^ $\bullet^5$ ^

Candidate E

$\vdots$	$\bullet^1$ ✓ $\bullet^2$ ✓
$\left(\cos x^\circ-1\right)\left(\cos x^\circ-2\right)=0$	$\bullet^3$ ✗
$\cos x^\circ=1,\qquad \cos x^\circ=2$	$\bullet^4$ $\boxed{✓\ 1}$
$x=0\qquad$ No solutions	$\bullet^5$ $\boxed{✓\ 1}$

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(a)		The diagram shows the graph of a cubic function with equation $y = f(x)$. The curve has stationary points at (0, 3) and (4, 0). (a) Sketch the graph of $y = 2f(x) + 1$. Use the diagram provided in the answer booklet.  		
			$\bullet^1$ vertical scaling by a factor of 2 identifiable from graph $\bullet^2$ vertical translation of '+1' units identifiable from graph $\bullet^3$ transformations applied in correct order	$\bullet^1$ $\bullet^2$ $\bullet^3$  	**3**

Notes:

1. $\bullet^1$, $\bullet^2$ and $\bullet^3$ are only available for a 'cubic' with a maximum and minimum turning point.
2. Ignore intersections (or lack of intersections) with the original graph.

Commonly Observed Responses:

Where the image of (4,0) is not (4,1), that point must be annotated (or drawn to within tolerance). In the following table, the images of the given points must be stationary points for the marks to be awarded.

Image of (0,3)	Image of (4,0)	Award…	
(0,8)	(4,2)	2/3	Transformation in wrong order
(0,4)	(8,1)	1/3	Only vertical translation correct
(0,4)	(4,1)	1/3	
(0,4)	(2,1)	1/3	
(0,5)	(4,−1)	2/3	Evidence of vertical scaling and transformation in correct order
(0,6)	(4,0)	1/3	Evidence of vertical scaling
(0,7)	any incorrect point	1/3	
(1,6)	(5,0)	1/3	
(−1,6)	(3,0)	1/3	
(0,−2)	(4,1)	1/3	Evidence of vertical translation
(0,4)	(−4,1)	1/3	
(0,5)	any other point	0/3	Insufficient evidence of scaling/translation
(0,2)	any other point	0/3	

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.	(b)		(b) State the coordinates of the stationary points on the graph of $y = f\left(\frac{1}{2}x\right)$.		
			•⁴ state coordinates of stationary points	•⁴ $(0,3)$ and $(8,0)$	**1**

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.			Express $2x^2+12x+23$ in the form $p(x+q)^2+r$.		
			Method 1 •¹ identify common factor •² complete the square •³ process for r and write in required form	**Method 1** •¹ $2(x^2+6x\ldots$ stated or implied by •² •² $2(x+3)^2\ldots$ •³ $2(x+3)^2+5$	3
			Method 2 •¹ expand completed square form •² equate coefficients •³ process for q and r and write in required form	**Method 2** •¹ $px^2+2pqx+pq^2+r$ •² $p=2,\ 2pq=12,\ pq^2+r=23$ •³ $2(x+3)^2+5$	

Notes:

1. $2(x+3)^2+5$ with no working gains •¹ and •² only. However, see Candidate E.

Commonly Observed Responses:

Candidate A $2(x^2+6)+23$ $2((x+3)^2-9)+23$ •¹ ✓ •² ✓ $2(x+3)^2+5$ •³ ✓ See the exception to marking principle (h)	**Candidate B** $px^2+2pqx+pq^2+r$ •¹ ✓ $p=2,\ 2pq=12,\ pq^2+r=23$ •² ✓ $q=3,\ r=5$ •³ ^ •³ is lost as answer is not in completed square form
Candidate C $2(x^2+12x)+23$ •¹ ✗ $2((x+6)^2-36)+23$ •² ✓1 $2(x+6)^2-49$ •³ ✓1	**Candidate D** $2((x+6)^2-36)+23$ •¹ ✗ •² ✗ $2(x+6)^2-49$ •³ ✓1
Candidate E $2(x+3)^2+5$ •¹ ✓ •² ✓ Check: $=2(x^2+6x+9)+5$ $=2x^2+12x+18+5$ $=2x^2+12x+23$ •³ ✓	

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.			Given that $f(x) = 4\sin\left(3x - \frac{\pi}{3}\right)$, evaluate $f'\left(\frac{\pi}{6}\right)$.		
			•¹ start to differentiate •² complete differentiation •³ evaluate derivative	•¹ $4\cos\left(3x - \frac{\pi}{3}\right)\ldots$ •² $\ldots \times 3$ •³ $6\sqrt{3}$	**3**

Notes:

1. Where candidates make no attempt to differentiate or use another invalid approach, •² and •³ are not available.
2. At the •¹ and •² stage, candidates who work in degrees cannot gain •¹. However •² and •³ are still available.
3. At the •³ stage, do not penalise candidates who work in degrees or in radians and degrees.
4. Ignore the appearance of $+c$ at any stage.

Commonly Observed Responses:

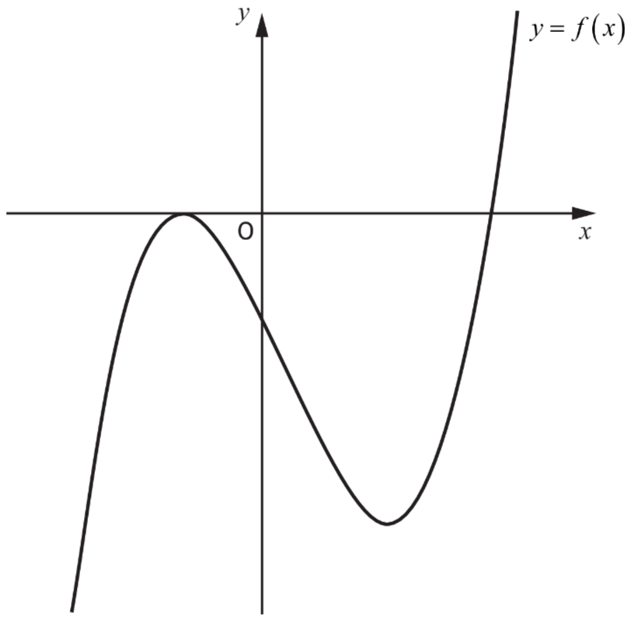
Candidate A Differentiating over two lines	Candidate B	Candidate C
$f'(x) = 4\cos\left(3x - \frac{\pi}{3}\right)$ •¹ ✓ $f'(x) = 12\cos\left(3x - \frac{\pi}{3}\right)$ •² ^ $6\sqrt{3}$ •³ ✓ 1	$4\cos\left(3x - \frac{\pi}{3}\right) \times \frac{1}{3}$ •¹ ✓ •² ✗ $\frac{2\sqrt{3}}{3}$ •³ ✓ 1	$4\cos\left(3x - \frac{\pi}{3}\right)$ •¹ ✓ •² ^ $2\sqrt{3}$ •³ ✓ 1
Candidate D	**Candidate E**	**Candidate F**
$\pm 12\sin\left(3x - \frac{\pi}{3}\right)$ •¹ ✗ •² ✗ ± 6 •³ ✓ 1	$\pm 4\sin\left(3x - \frac{\pi}{3}\right)\ldots$ •¹ ✗ $\ldots \times 3$ •² ✓ 1 ± 6 •³ ✓ 1	$-12\cos\left(3x - \frac{\pi}{3}\right)$ •¹ ✗ •² ✓ $-6\sqrt{3}$ •³ ✓ 1

Question			Generic Scheme	Illustrative Scheme	Max Mark
13.	(a)		(a) (i) Show that $(x+2)$ is a factor of $f(x)=x^3-2x^2-20x-24$. (ii) Hence, or otherwise, solve $f(x)=0$.		
		(i)	•¹ use −2 in synthetic division or evaluation of the cubic •² complete division/evaluation and interpret result	•¹ $\begin{array}{r\|rrrr} -2 & 1 & -2 & -20 & -24 \\ & & & & \\ \hline & 1 & & & \end{array}$ or $(-2)^3-2(-2)^2-20(-2)-24$ •² $\begin{array}{r\|rrr\|r} -2 & 1 & -2 & -20 & -24 \\ & & -2 & 8 & 24 \\ \hline & 1 & -4 & -12 & 0 \end{array}$ Remainder $=0 \therefore (x+2)$ is a factor or $f(-2)=0 \therefore (x+2)$ is a factor	**2**
		(ii)	•³ state quadratic factor •⁴ find remaining factors or apply the quadratic formula •⁵ state solution	•³ $x^2-4x-12$ •⁴ $(x+2)$ and $(x-6)$ or $\dfrac{4\pm\sqrt{(-4)^2-4(1)(-12)}}{2(1)}$ •⁵ $-2, 6$	**3**

Notes:

1. Communication at •² must be consistent with working at that stage - a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •²:
    - '$f(-2)=0$ so $(x+2)$ is a factor'
    - 'since remainder $=0$, it is a factor'
    - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore, \rightarrow, \Rightarrow$ etc.
3. Do not accept any of the following for •²:
    - double underlining the '0' or boxing the '0' without comment
    - '$x=-2$ is a factor', '... is a root'
    - the word 'factor' only, with no link.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		The diagram shows the graph of $y = f(x)$.  (b) The graph of $y = f(x-k)$, $k > 0$ has a stationary point at $(1, 0)$. State the value of k.		
			$\bullet^6$ state value of k	$\bullet^6$ 3	**1**

Notes:

1. Accept $y = f(x-3)$ or $f(x-3)$ for $\bullet^6$.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.	(a)		C_1 is the circle with equation $(x-7)^2+(y+5)^2=100$. (a) (i) State the centre and radius of C_1. (ii) Hence, or otherwise, show that the point $P(-2,7)$ lies outside C_1. C_2 is a circle with centre P and radius r.		
		(i)	•¹ state coordinates of centre •² state radius	•¹ $(7,-5)$ •² 10	2
		(ii)	•³ substitute coordinates of P and evaluate •⁴ communicate result	•³ $(-2-7)^2+(7+5)^2=225$ •⁴ $225>100 \therefore$ P lies outside	2

Notes:

1. Accept $x=7,\ y=-5$ for •¹.
2. Do not accept $g=7,\ f=-5$ or $7,\ -5$ for •¹.

Commonly Observed Responses:

Candidate A $d=15$ •³ ✓ $15>10 \therefore$ P lies outside •⁴ ✓	**Candidate B** $d=\sqrt{225}$ •³ ✓ $\sqrt{225}>10 \therefore$ P lies outside •⁴ ✓
Candidate C $d=15$ •³ ✓ $r=10$ $d>r \therefore$ P lies outside •⁴ ✓	

Question			Generic Scheme	Illustrative Scheme	Max Mark
	(b)		(b) Determine the value(s) of r for which circles C_1 and C_2 have exactly one point of intersection.		
			•⁵ determine first value of r •⁶ determine second value of r	•⁵ 5 •⁶ 25	2

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]