# Sets + Functions

① a) $x \neq 0$ b) $x \neq 4$ c) $x \neq \pm 2$ d) $x \geq -2$

e) $x \geqslant \frac{5}{3}$ f) $x > -6$

② a) $f(x)^2$
$= \underline{x^2+1}$

b) $f(x-1)$
$= (x-1)^2$
$= \underline{x^2-2x+1}$

c) $f(2x-1)$
$= (2x-1)^2+3$
$= 4x^2-4x+1+3$
$= \underline{4x^2-4x+4}$

d) $f(5-2x)$
$= \underline{\frac{1}{5-2x}}$

e) $f(1-x)$
$= 2(1-x)^2-(1-x)$
$= 2(1-2x+x^2)-1+x$
$= 2x^2-4x+2-1+x$
$= \underline{2x^2-3x+1}$

f) $f\left(\frac{1}{x}\right)$
$= \dfrac{\frac{1}{x}}{\frac{1}{x}+1}$
$= \dfrac{\frac{1}{x}}{\frac{x+1}{x}}$
$= \underline{\underline{\frac{1}{x+1}}}$

$\frac{1}{x}+1$
$= \frac{1}{x}+\frac{x}{x}$
$= \frac{x+1}{x}$

$\frac{1}{x} \div \frac{x+1}{x}$
$= \frac{1}{\cancel{x}} \times \frac{\cancel{x}}{x+1}$
$= \frac{1}{x+1}$

③ a) $f(x^2-m)$
$= \underline{2(x^2-m)}$

b) $g(2x)$
$= (2x)^2-m$
$= \underline{4x^2-m}$

$g(f(x))-f(g(x))$
$= 4x^2-m-2(x^2-m)$
$= 4x^2-m-2x^2+2m$
$= \underline{2x^2+m}$ (as required)

④ a) (i) $p(x) = f(x^2-2)$
$= 3(x^2-2)+1$
$= \underline{3x^2-5}$

(ii) $q(x) = g(3x+1)$
$= (3x+1)^2-2$
$= 9x^2+6x+1-2$
$= \underline{9x^2+6x-1}$

b) $p'(x) = 6x$
$q'(x) = 18x+6$

$\therefore 18x+6 = 6x$
$12x = -6$
$x = \underline{\underline{-\frac{1}{2}}}$

5 a) $k(x) = f(3-2x)$
$= 2(3-2x)-1$
$= 6-4x-1$
$= \underline{5-4x}$

b) $h(5-4x)$
$= \frac{1}{4}(5-(5-4x))$
$= \frac{1}{4}(5-5+4x)$
$= \frac{1}{4}(4x)$
$= \underline{\underline{x}}$

c) $h(x) = k^{-1}(x)$ (INVERSE functions)

6 a) $y = 2x+4$
$2x = y-4$
$x = \frac{y-4}{2}$
$\therefore f^{-1}(x) = \frac{x-4}{2}$

b) $y = 6-2x$
$2x = 6-y$
$x = \frac{6-y}{2}$
$\therefore f^{-1}(x) = \frac{6-x}{2}$

c) $y = \sqrt[3]{2x+5}$
$y^3 = 2x+5$
$2x = y^3-5$
$x = \frac{y^3-5}{2}$
$\therefore f^{-1}(x) = \frac{x^3-5}{2}$

d) $y = \frac{1}{x-4}$
$x-4 = \frac{1}{y}$
$x = \frac{1}{y}+4$
$\therefore f^{-1}(x) = \frac{1}{x}+4$

e) $y = 2-\frac{1}{x}$
$\frac{1}{x} = 2-y$
$\frac{1}{2-y} = x$
$\therefore f^{-1}(x) = \frac{1}{2-x}$

f) $y = \sqrt[3]{\frac{2}{x+1}}$
$y^3 = \frac{2}{x+1}$
$x+1 = \frac{2}{y^3}$
$x = \frac{2}{y^3}-1$
$\therefore f^{-1}(x) = \frac{2}{x^3}-1$