1. Evaluate $2\frac{1}{3}+\frac{4}{5}$.   2

$$2\frac{1}{3}+\frac{4}{5} = 2\frac{5}{15}+\frac{12}{15} = 2\frac{17}{15} = 3\frac{2}{15}$$

2. Expand and simplify $(3x+1)(x-1)+2(x^2-5)$.   3

$$3x(x-1)+1(x-1)+2(x^2-5)$$
$$= 3x^2-3x+x-1+2x^2-10$$
$$= 5x^2-2x-11$$

3. Solve, algebraically, the system of equations

$$4x+5y=-3 \quad ①$$
$$6x-2y=5. \quad ②$$

                                                                                       3

①×2 $8x+10y=-6$ ③

②×5 $30x-10y=25$ ④

③+④ $38x = 19$

$$x=\frac{1}{2}$$

Sub $\frac{1}{2}$ for x in ①

$$2+5y=-3$$
$$5y=-5$$
$$y=-1$$

4. Two vectors are given by $\mathbf{u}=\begin{pmatrix}1\\5\\1\end{pmatrix}$ and $\mathbf{u}+\mathbf{v}=\begin{pmatrix}6\\-4\\3\end{pmatrix}$.

Find vector $\mathbf{v}$.

Express your answer in component form.   2

$$\mathbf{u}+\mathbf{v}-\mathbf{u} = \begin{pmatrix}6\\-4\\3\end{pmatrix}-\begin{pmatrix}1\\5\\1\end{pmatrix} = \begin{pmatrix}5\\-9\\2\end{pmatrix}$$
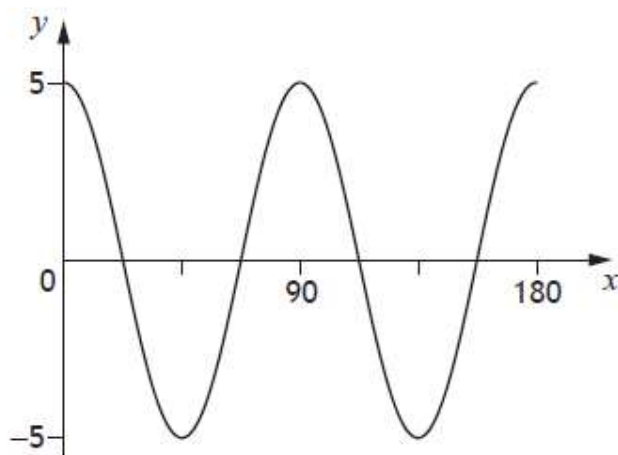
5. Solve

$$x^2 - 11x + 24 = 0.$$

2

$$x^2 - 8x - 3x + 24 = 0$$
$$x(x-8) - 3(x-8) = 0$$
$$(x-8)(x-3) = 0 \checkmark$$
$$x - 8 = 0 \qquad x - 3 = 0$$
$$x = 8 \qquad x = 3 \checkmark$$

$$\frac{24}{\begin{array}{c|c} 1 & 24 \\ 2 & 12 \\ 3 & 8 \\ 4 & 6 \end{array}}$$

6. Part of the graph of $y = a \cos bx°$ is shown in the diagram.

State the values of a and b.

2

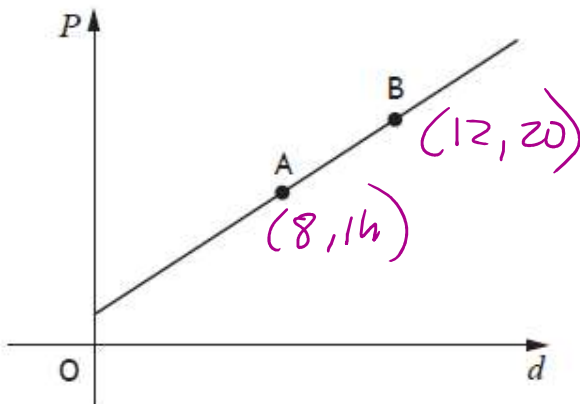$$\text{Max/Min} = 5/-5 \qquad \rightarrow a = 5 \checkmark$$
$$2 \text{ waves in } 180° \qquad \rightarrow 4 \text{ waves in } 360°$$
$$\rightarrow b = 4 \checkmark$$

7. The cost of a journey with Tom's Taxis depends on the distance travelled.

The graph below shows the cost, P pounds, of a journey with Tom's Taxis against the distance travelled, d miles.

Point A represents a journey of 8 miles which costs £14.

Point B represents a journey of 12 miles which costs £20.

(a) Find the equation of the line in terms of P and d.

Give the equation in its simplest form.   3

$$M = \frac{20 - 14}{12 - 8}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$

$$y - b = m(x - a)$$
$$y - 14 = \frac{3}{2}(x - 8)$$
$$2y - 28 = 3x - 24$$
$$2y = 3x + 4$$
$$y = \frac{3}{2}x + 2$$

$$\therefore P = \frac{3}{2}d + 2$$

7. (continued)

(b) Calculate the cost of a journey of 5 miles.   1

$$P = \frac{3}{2}(5) + 2$$
$$= 9.5$$

$$\therefore £9.50$$

8. Determine the nature of the roots of the function $f(x) = 2x^2 + 4x + 5$.   **2**

$a = 2$

$b = 4$
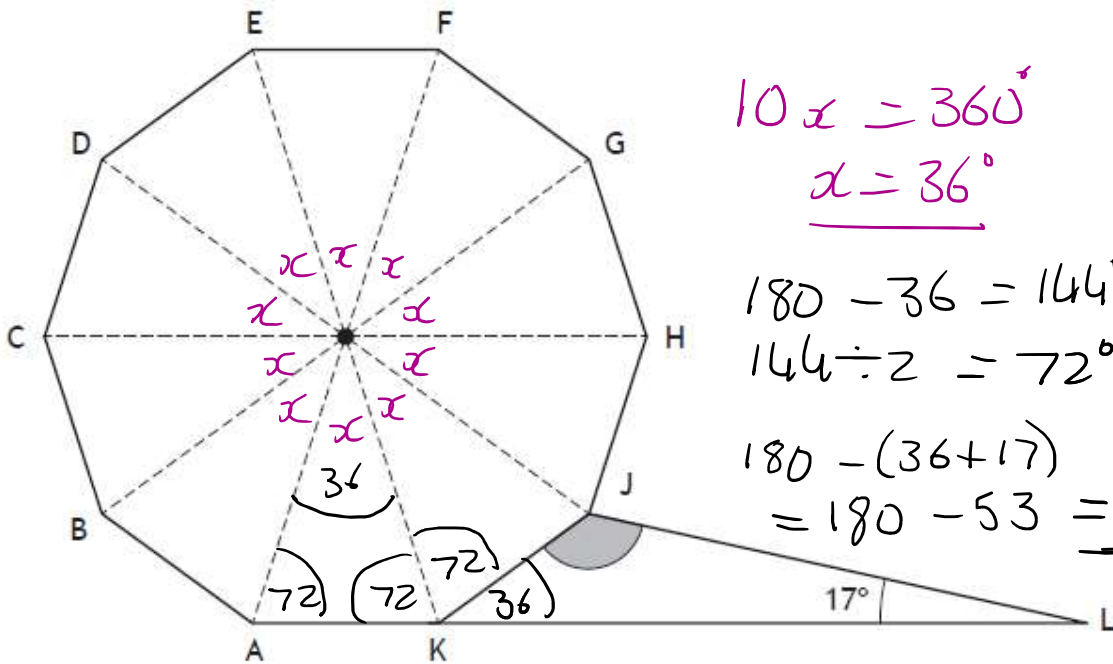
$c = 5$

$b^2 - 4ac$

$= 4^2 - 4(2)(5)$

$= 16 - 20$

$= -4$ ✓

$b^2 - 4ac < 0$

$\therefore$ No real roots. ✓ ²

9. In the diagram shown below, ABCDEFGHJK is a regular decagon.

- Angle KLJ is 17°.
- AKL is a straight line.

$10x = 360°$

$x = 36°$

$180 - 36 = 144°$

$144 \div 2 = 72°$ ✓ ¹

$180 - (36 + 17)$

$= 180 - 53 = 127°$ ✓ ²

Calculate the size of shaded angle KJL.   **2**

10. In triangle XYZ:

- XZ = 10 centimetres
- YZ = 8 centimetres
- $\cos Z = \dfrac{1}{8}$.
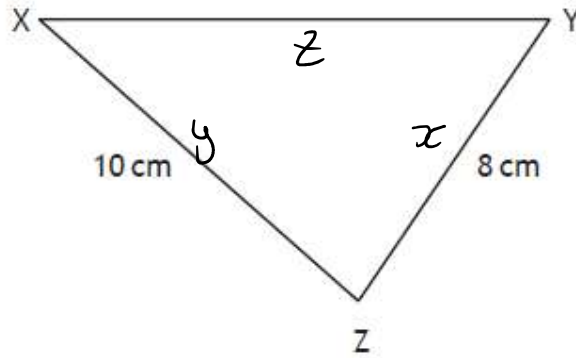
Calculate the length of XY. **3**

$$z^2 = x^2 + y^2 - 2xy\cos Z$$
$$z^2 = 8^2 + 10^2 - 2(8)(10)\left(\tfrac{1}{8}\right)\ \checkmark$$
$$z^2 = 164 - 20$$
$$z^2 = 144\ \checkmark{}^2$$
$$z = 12\,cm\ \checkmark{}^3$$

11. Express $\dfrac{9}{\sqrt{6}}$ with a rational denominator.

Give your answer in its simplest form. **2**

$$\frac{9}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}\ \checkmark = \frac{9\sqrt{6}}{6} = \frac{3\sqrt{6}}{2}\ \checkmark{}^2$$

12. Given that $\cos 60° = 0{\cdot}5$, state the value of $\cos 240°$. **1**

240° is in quadrant <u>3</u>

$$\cos 240° = -\cos 60°$$
$$= -0{\cdot}5\ \checkmark$$

$$\frac{S\ |\ A}{T\ |\ C}$$

$180 + 60 = \underline{240}$

13. The diagram shows a triangular prism, ABCDEF, relative to the coordinate axes.
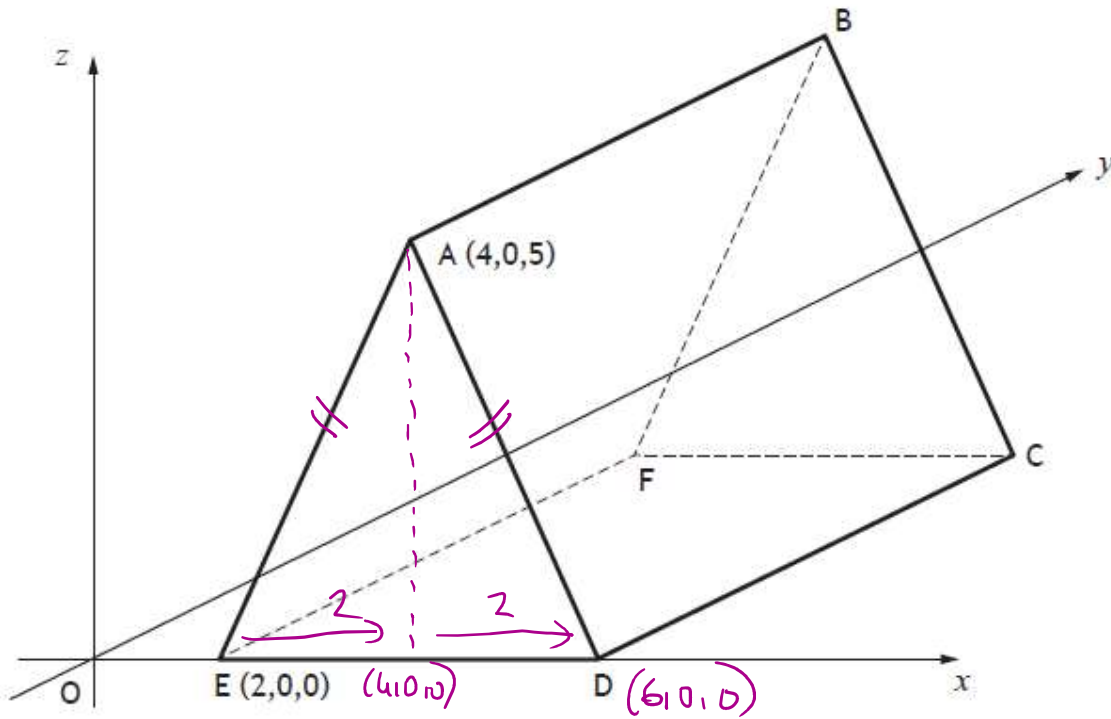
- AD = AE. → Triangle ADE IS ISOSCELES
- DC = 8 units.
- Edges EF, DC and AB are parallel to the y-axis.

Write down the coordinates of B and C.   2

$$B = (4,8,5) \qquad C = (6,8,0)$$

14. Change the subject of the formula $y = g\sqrt{x} + h$ to x.   3

$$g\sqrt{x} + h = y$$
$$g\sqrt{x} = y - h \checkmark$$
$$\sqrt{x} = \frac{y - h}{g} \quad 2\checkmark$$
$$x = \left(\frac{y - h}{g}\right)^2 \quad 3\checkmark$$

15. Remove the brackets and simplify $\left(\frac{2}{3}p^4\right)^2$. **2**

$$\left(\frac{2}{3}p^4\right)^2 = \frac{4}{9}p^8$$

$\frac{4}{9}$ ✓ p^8 ✓

16. Sketch the graph of $y=(x-6)(x+4)$.

On your sketch, show clearly the points of intersection with the x-axis and the y-axis, and the coordinates of the turning point. **3**

$\underline{x-axis}$ $(x-6)(x+4)=0$ $(6, 0)$

$x-6=0 \qquad x+4=0$

$\underline{x=6} \qquad \underline{x=-4}$ ✓ $(-4, 0)$

$\underline{y-axis} \qquad y=(0-6)(0+4)$
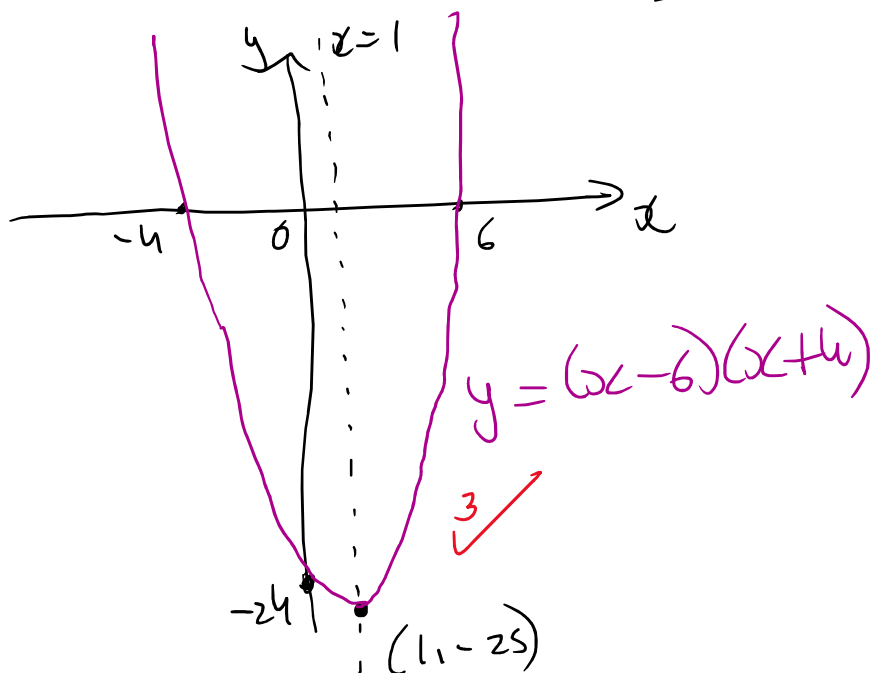
$= (-6)(4)$

$= \underline{-24}$ $(0, -24)$ ✓

$\underline{TP} \quad \frac{1}{2}$ way between roots $\therefore x=\frac{6+(-4)}{2}=\underline{1}$
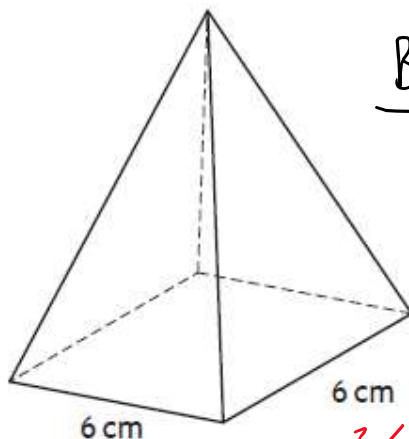
If $x=1$, $y=(1-6)(1+4)$

$= (-5)(5)$

$= \underline{-25}$ $(1, -25)$

$y=(x-6)(x+4)$

3 ✓

$(1, -25)$

17. A square based pyramid is shown in the diagram below.

6 cm

6 cm

The square base has length 6 centimetres.

The volume is 138 cubic centimetres.

Calculate the height of the pyramid.

Base $A = 6 \times 6$
$= 36 \, cm^2$

$V = \frac{1}{3} A h$

$138 = \frac{1}{3}(36)h$ ✓

$138 = 12h$

$h = \frac{138}{12}$ 3

$h = 11\frac{6}{12}$ 3 ✓

$h = 11.5 \, cm$

18. Express $\sin x° \cos x° \tan x°$ in its simplest form.

Show your working.   2

$\sin x \cos x \tan x = \sin x \cos x \left(\frac{\sin x}{\cos x} \right)$ $\frac{\sin x}{\cos x}$ ✓

$= \sin x (\sin x)$

$= \sin^2 x$ ✓

19. (a) (i) Express $x^2 - 6x - 81$ in the form $(x-p)^2 + q$. ✓ 1/ ² 2/ 2

$$(x-3)^2 - 81 - 9 \quad = (x-3)^2 - 90$$

(ii) Hence state the equation of the axis of symmetry of the graph of
$y = x^2 - 6x - 81$. 1

$$TP = (3, -90) \Rightarrow \underline{\underline{x = 3}} \; ^3/$$

19. (continued)

(b) The roots of the equation $x^2 - 6x - 81 = 0$ can be expressed in the form
$x = d \pm d\sqrt{e}$.

Find, algebraically, the values of d and e. 4

$a = 1$
$b = -6$
$c = -81$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-81)}}{2(1)} \quad ✓$$

$$= \frac{6 \pm \sqrt{36 + 324}}{2}$$

$$= \frac{6 \pm \sqrt{360}}{2} \qquad 360 \; ^2/$$

$$= \frac{6 \pm \sqrt{36}\sqrt{10}}{2} \qquad 6\sqrt{10} \; ^3/$$

$$= \frac{6 \pm 6\sqrt{10}}{2} \qquad \therefore d = 3 \; ^4/$$

$$= \underline{\underline{3 \pm 3\sqrt{10}}} \qquad e = 10$$

4. Solve, algebraically, the inequation

$$3x < 6(x-1) - 12.$$ 3

$$3x < 6x - 6 - 12 \checkmark^1$$
$$3x < 6x - 18$$
$$-3x < -18 \checkmark^2$$
$$\underline{\underline{x > 6}} \checkmark^3$$

5. A farmers' market took place one weekend.

Stallholders were asked to record the number of customers who visited their stall.

The number of customers who visited six of the stalls on Saturday were as follows:

120   126   125   131   130   124

(a) Calculate the mean and standard deviation of the number of customers.   4

x	$x - \bar{x}$	$(x - \bar{x})^2$
120	−6	36
126	0	0
125	−1	1
131	5	25
130	4	16
124	−2	4
756	0	82

$\checkmark^2$

$$\bar{x} = 756 \div 6$$
$$= \underline{\underline{126}} \checkmark^1$$

$$S = \sqrt{\frac{82}{5}} \checkmark^3$$

$$= \underline{\underline{4 \cdot 05}} \checkmark^4$$

5. (continued)

The mean number of customers who visited these six stalls on Sunday was 117 and the standard deviation was 6·2.

(b) Make two valid comments comparing the number of customers who visited these stalls on Saturday and Sunday. **2**

There were fewer customers on Sunday as $117 < 126$. ✓[5]

The number of customers per stall was ✓ less consistent on Sunday as $6.2 > 4.05$.

6. A function is defined as $f(x) = 5 + 4x$.

Given that $f(a) = 73$, calculate a. **2**

$$f(a) = 5 + 4a \quad \Rightarrow \quad 5 + 4a = 73 \;\checkmark^{[1]}$$
$$4a = 68$$
$$a = 17 \;\checkmark^{[2]}$$

7. A toy company makes juggling balls in the shape of a sphere with a diameter of 6·4 centimetres.

6·4 cm

$$V = \frac{4}{3}\pi r^3 \;\checkmark^{[1]}$$
$$= \frac{4}{3}\pi (3.2)^3$$
$$= 137.258\ldots \;\checkmark^{[2]}$$
$$= 140 \text{ cm}^2 \;\checkmark^{[3]}$$
$$(\text{to 2 s.f.})$$

Calculate the volume of one juggling ball.

Give your answer correct to 2 significant figures. **3**

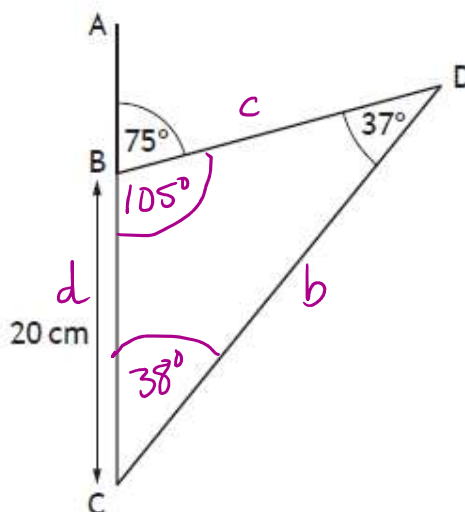8. Solve the equation $7\sin x° + 2 = 3$, for $0 \le x < 360$.   3

$$7\sin x = 1$$
$$\sin x = \frac{1}{7}$$
$$x = \sin^{-1}\left(\frac{1}{7}\right) \checkmark$$
$$x = 8.2°, 171.8°$$

(margin notes)
S A
T C
180 − 8.2
= 171.8°

9. In this diagram:

- angle ABD = 75°
- angle BDC = 37°
- BC = 20 centimetres.

Calculate the length of DC.   3

$$\frac{b}{\sin 105} = \frac{20}{\sin 37} \checkmark$$

$$b = \frac{20\sin 105}{\sin 37} \checkmark$$

$$b = 32.1 \text{ cm} \checkmark$$

10. In the diagram below, $\vec{AB}$ and $\vec{EA}$ represent the vectors **u** and **w** respectively.

A   **u**   B

w

E   C

D
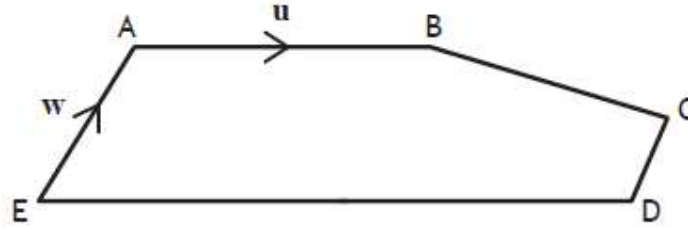
- $\vec{ED} = 2\vec{AB}$
- $\vec{EA} = 2\vec{DC}$

Express $\vec{BC}$ in terms of **u** and **w**.

Give your answer in its simplest form.

$$\vec{BC} = \vec{BA} + \vec{AE} + \vec{ED} + \vec{DC}$$
$$= -\vec{AB} - 2\vec{DC} + 2\vec{AB} + \vec{DC}$$
$$= \vec{AB} - \vec{DC}$$
$$= \underline{\underline{u - \frac{1}{2}w}}$$

2

11. Venus and Earth are two planets within our solar system.

Venus   Earth

The volume of Venus is approximately $9 \cdot 3 \times 10^{11}$ cubic kilometres.
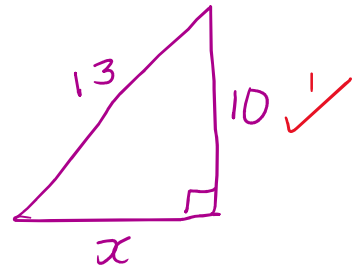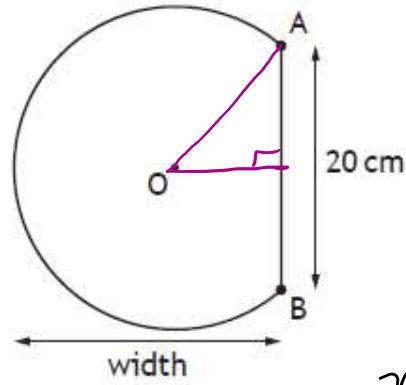
This is 85% of the volume of Earth.

Calculate the volume of Earth.

3

%	km^3
85	$9 \cdot 3 \times 10^{11}$
1	$1 \cdot 094 \times 10^{10}$
100	$1 \cdot 094 \times 10^{12}$

$$1 \cdot 1 \times 10^{12} \, km^3$$

12. The shape below is part of a circle, centre O.

A

20 cm

B

width

The circle has radius 13 centimetres.

AB is a chord of length 20 centimetres.

Calculate the width of the shape.

13

10 ✓¹

x

$x^2 = 13^2 - 10^2$ ✓²

$x^2 = 69$

$x = 8.3 cm$ ✓³

$width = 13 + 8.3 \quad = 21.3 cm$ ✓⁴

4

13. A ferry and a trawler receive a request for help from a stranded yacht.

On the diagram the points F, T and Y show the positions of the ferry, the trawler and the yacht respectively.
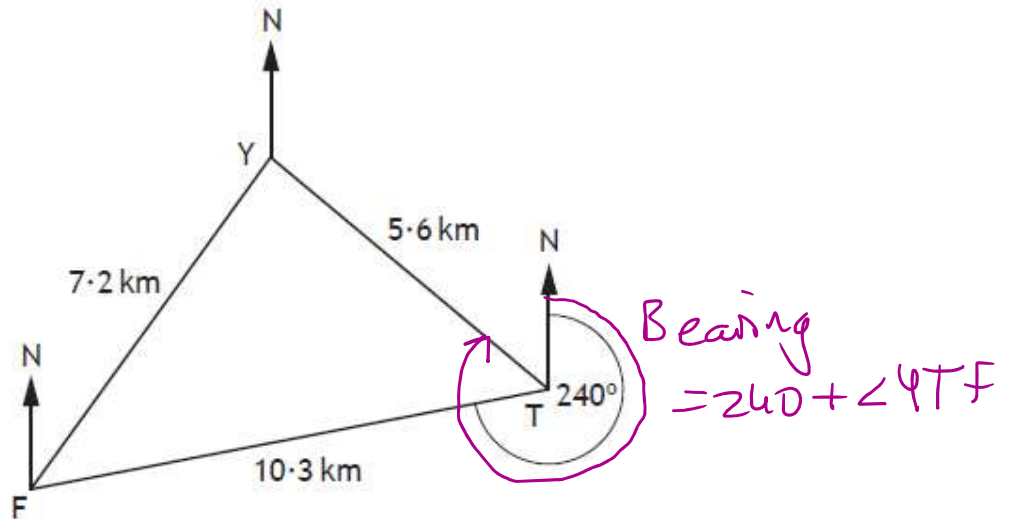
Bearing
$= 240 + \angle YTF$

- FY is 7·2 kilometres.
- TY is 5·6 kilometres.
- FT is 10·3 kilometres.
- F is on a bearing of 240° from T.

Calculate the bearing of the yacht from the trawler.   4

$$\cos T = \frac{f^2 + y^2 - t^2}{2fy}$$

$$\cos T = \frac{5\cdot6^2 + 10\cdot3^2 - 7\cdot2^2}{2(5\cdot6)(10\cdot3)} \checkmark$$

$$T = \cos^{-1}\left(\frac{85\cdot61}{115\cdot36}\right) \checkmark^{2}$$

$$T = 42° \checkmark^{3} \qquad \therefore \text{Bearing} = 282° \qquad ^{4}\checkmark$$

14. A straight line has equation $2x - 5y = 20$.

    Find the coordinates of the point where this line crosses the y-axis. ✓   2

    y – axis $x = 0$ $\therefore$ $2(0) - 5y = 20$ $(0, -4)$ ✓

    $-5y = 20$

    $y = -4$

15. Express

$$\frac{n}{n^2 - 4} \div \frac{3}{n - 2}, \quad n \neq -2, n \neq 2$$
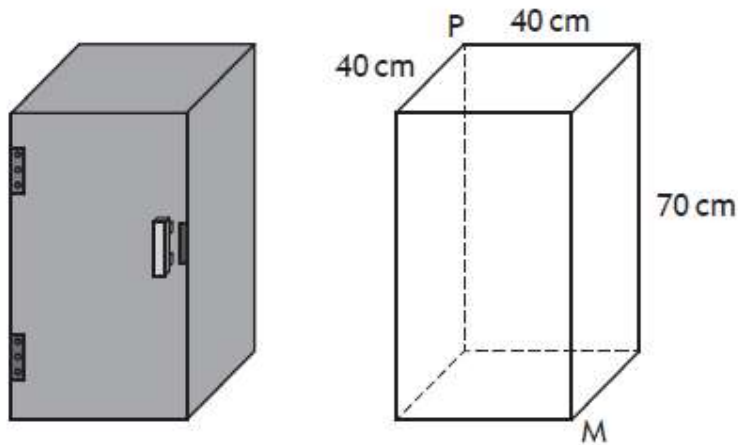
as a single fraction in its simplest form.   3

$$\frac{n}{n^2 - 4} \times \frac{n - 2}{3} \checkmark = \frac{n(n - 2)}{3(n^2 - 4)}$$

$$= \frac{n(n - 2)}{3(n + 2)(n - 2)} \checkmark$$

$$= \frac{n}{3(n + 2)} \checkmark$$

16. Chris wants to store his umbrella in a locker.

The locker is a cuboid with internal dimensions of length 40 centimetres, breadth 40 centimetres and height 70 centimetres.

P 40 cm
40 cm
70 cm
M

The umbrella is 85 centimetres long.

He thinks it will fit into the locker from corner P to corner M.

Is he correct?

Justify your answer. 4

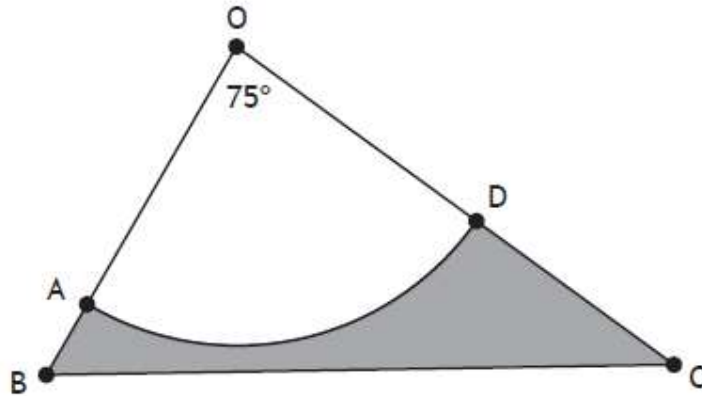$(PM)^2 = 40^2 + 40^2 + 70^2$ ✓

$(PM)^2 = 8100$

$\therefore PM = 90\,cm$ ✓

Yes it will fit, as
$85\,cm < 90\,cm$ ✓

17. In the diagram below AOD is a sector of a circle, with centre O, and BOC is a triangle.

In sector AOD:

- radius = 30 centimetres
- angle AOD = 75°.

In triangle OBC:

- OB = 38 centimetres
- OC = 55 centimetres.

Calculate the area of the shaded region, ABCD.   5

SECTOR $\dfrac{\text{Area}}{\pi r^2} = \dfrac{75}{360}$ ✓ ¹

$\text{Area} = \dfrac{75}{360} \times \pi (30)^2$ ✓ ²

$\text{Area} = 589.048\ldots$

TRIANGLE

$A = \dfrac{1}{2} ab \sin C$

$= \dfrac{1}{2}(38)(55) \sin 75$

$= 1009.392\ldots$ ✓ ³

∴ Shaded area $= 1009.392\ldots - 589.048\ldots$ ✓ ⁴

$= 420.344\ldots$

∴ Area $= 420 \text{ cm}^2$ ✓ ⁵

18. A cinema sells popcorn in two different sized cartons.

The small carton is 16 centimetres deep and has a volume of 576 cubic centimetres.

The large carton is 24 centimetres deep and has a volume of 1125 cubic centimetres.

(a) Show that the two cartons are not mathematically similar. 3

Vol SF $=$ (length SF)^3

$= \left(\frac{24}{16}\right)^3$

$= \left(\frac{3}{2}\right)^3$

$= \frac{27}{4}$

$\frac{24}{16}$ ✓

$\frac{1125}{576} = \frac{125}{64}$ ✓

As $\frac{1125}{576} \neq \frac{27}{4}$, cartons are NOT similar. ✓

18. (continued)

The large carton is redesigned so that the two cartons are now mathematically similar.

The volume of the redesigned large carton is 1500 cubic centimetres.

(b) Calculate the depth of the redesigned large carton. 2

Length SF $= \sqrt[3]{\frac{1500}{576}}$ ✓ ∴ Depth $= 16 \times \sqrt[3]{\frac{1500}{576}}$

$= 22 cm$ ✓