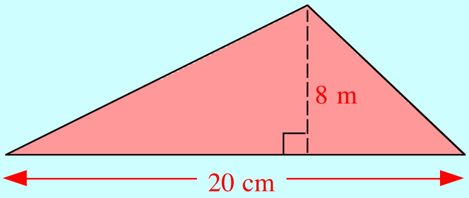
# The Area of a Triangle

**Reminder :-**

You should already know a formula for finding the area of a triangle given the length of its **base** and its **height**.



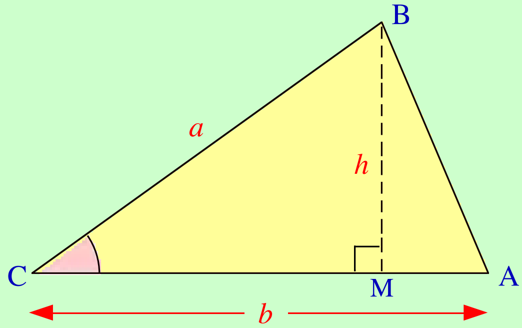

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

$$= \tfrac{1}{2} \times 20 \times 8$$

$$= \mathbf{80\ cm^2}$$

What happens when you are **NOT** told the height of the triangle ?

In this case, you are given **two of the sides** and the **angle** between those sides.




Draw in height BM to make two RAT's.
Let BM = $h$ units.

Using **SOH**CAHTOA in triangle BCM

$$\Rightarrow \sin C = \frac{h}{a} \quad \Rightarrow \quad h = a\sin C$$

$$\Rightarrow \text{AREA of triangle ABC} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

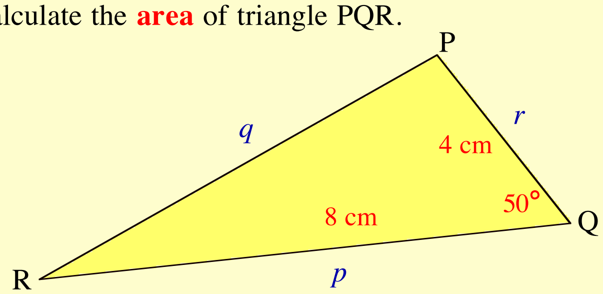
$$\Rightarrow \text{Area} = \tfrac{1}{2}ab\sin C.$$

Generally :- If given **2 sides** of a triangle and the **included angle**, whether acute or obtuse, then :-

$$\boxed{\text{Area of a Triangle} = \tfrac{1}{2}ab\sin C}$$

**Example :-**

Calculate the **area** of triangle PQR.




$$\text{Area} = \tfrac{1}{2}pr\sin Q$$

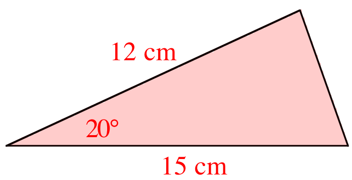
$$= \tfrac{1}{2} \times 4 \times 8 \times \sin 50°$$

$$= \mathbf{12{\cdot}3\ cm^2} \quad \text{(to 3 sig. figs.)}$$

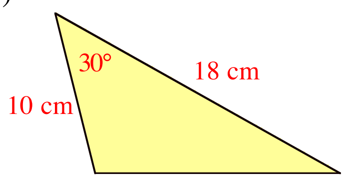
## Exercise 8·2

1. Calculate the area of each of these triangles, (*to 3 sig. figs.*) :-

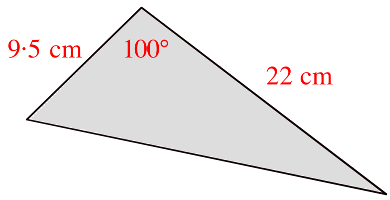
(a)




(b)



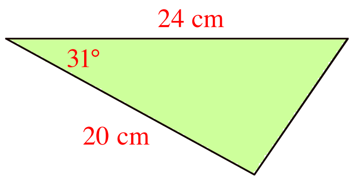

(c)



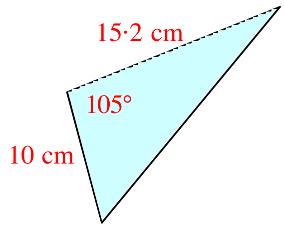
*Answer to 3 significant figures unless stated* :-

2. Calculate the **area** of each of these triangles :-

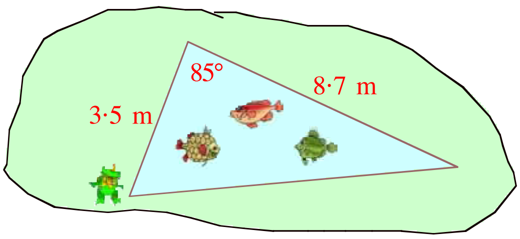
(a)




(b)

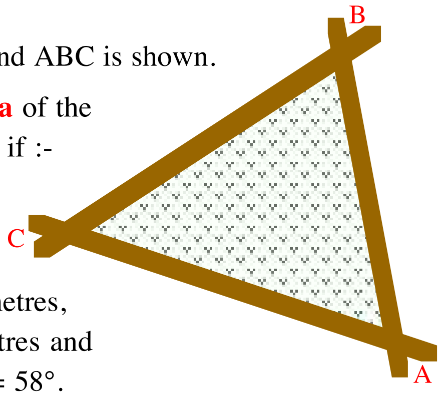



3. Calculate the **area** of the triangular tropical fish pond in the garden.



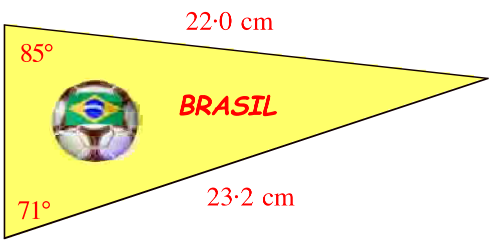

4. A traffic island ABC is shown.

Find the **area** of the traffic island if :-




AB = 12·6 metres,
AC = 10 metres and
angle BAC = 58°.

5. This is a replica of Brazil's World Cup soccer pennant.

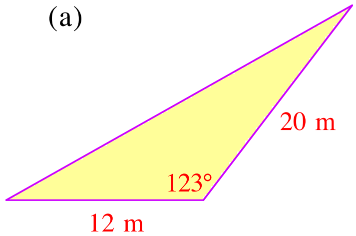



(a) Write down the size of the **third** angle in the triangular pennant.

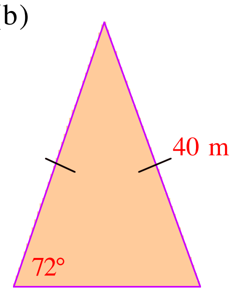
(b) Calculate the **area** of the pennant.

6. Calculate the **area** of each of these triangles :-

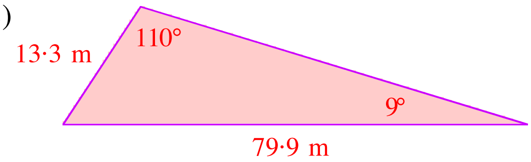
(a)



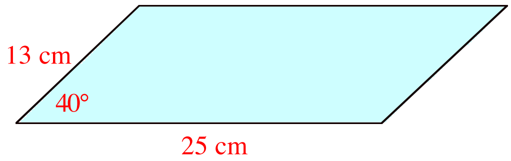

(b)



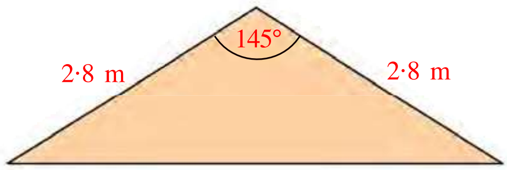

(c)




7. Calculate the area of this **parallelogram** :-

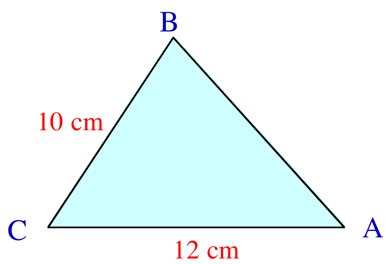



8. After a very damp winter, the owner of this bungalow decided to protect the brickwork at the front of his garage by coating it with an orange all-weather waterproof sealant.




If £15 worth of sealant covers 1 $m^2$ of brickwork, calculate how much it will cost him to coat this part of the garage wall with paint.

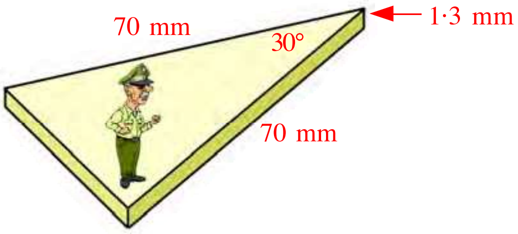
9. The **area** of this triangle ABC is **54 $cm^2$**.




AC = 12 cm and BC = 10 cm.

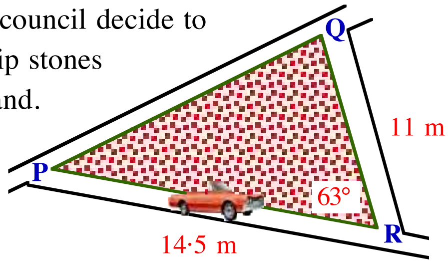
Calculate the size of **acute** angle ACB.

10. An identification tag, made of plastic, is in the form of an isosceles triangle, with dimensions as shown.
The badge is 1·3 millimetres thick.




Calculate the **volume** of plastic required to make one tag.

11. Another traffic island, PQR, is shown.
The town council decide to lay red chip stones on the island.

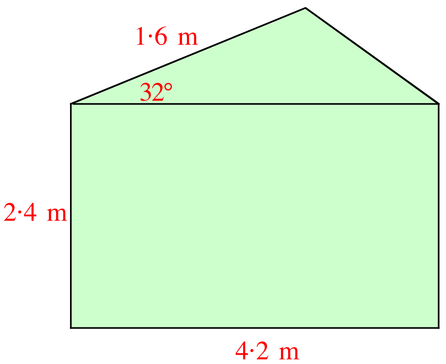



The price of the red chips is shown on this sign.

| |
|---|
| **RED CHIPS** |
| **1 lorry-load - £15·00.** |
| **Covers 25 square metres** |

Will £45 be enough to cover the entire traffic island ? *Explain fully with working*.

12. The side wall of a hut, with measurements shown, requires to be painted with green creosote.




The wall consists of a rectangular base with a triangular top.

A litre of paint will cover (*on average*), 3 square metres.

A painter guesses that he will require 4 litres of paint.

Will he have enough paint ?
*Justify your answer*.

13. (a) Use your calculator to look up each of the following pairs of **sine** values :-

(i) sin 30° and sin 150°.

(ii) sin 50° and sin 130°

(iii) sin 10° and sin 170°

(iv) sin 105° and sin 75°

(v) sin 175° and sin 5°

(vi) sin 63° and sin 117°.

(b) What did you notice ?

Copy and complete :-

"for any acute angle $a$°,

=> **sin $a$° = sin (180 – ...)°**".

14. For each angle below, state its pair using 13(b).

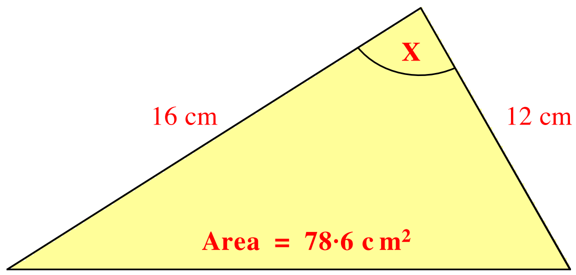
(a) 60° (b) 45° (c) 110°

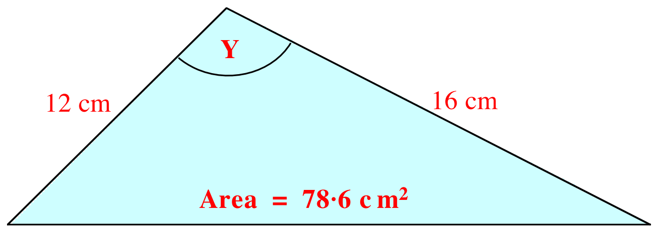
(d) 12° (e) 177° (f) 1°.

Check each of these on your calculator.

> Can you see that if you now know the value of the sine of an angle,
> then there are **two possible values** for the actual size of the angle ?
> (*This will be studied later in the course*).

15. The area of **both** triangles below is 78·6 cm².




Calculate the sizes of acute angle **X** and obtuse angle **Y**.

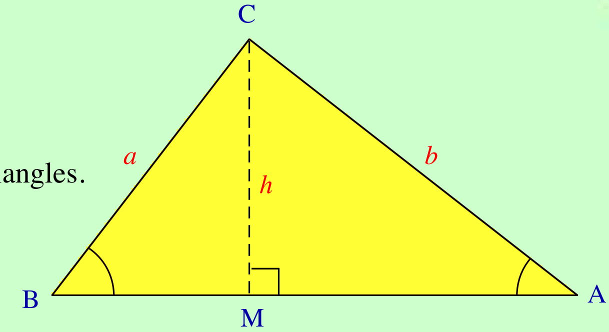
## The Sine Rule - Missing Sides

Look at the (*non right angled*) triangle ABC.

We **cannot** use **SOHCAHTOA** in $\Delta$ABC since it is not a right angle triangle.

We can draw in altitude CM to create 2 right angled triangles.

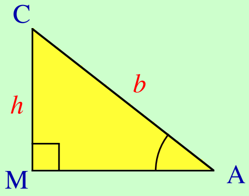
Let CM = $h$ units.




In **$\Delta$ACM**,

$$\sin A = \frac{h}{b}$$

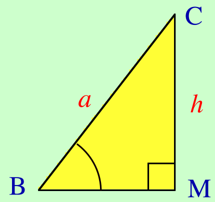
$$\Rightarrow \quad h = b\sin A$$




In **$\Delta$BCM**,

$$\sin B = \frac{h}{a}$$

$$\Rightarrow \quad h = a\sin B$$




$$\Rightarrow \quad b\sin A = a\sin B$$

÷ both sides by sinAsinB

$$\Rightarrow \quad \frac{b\sin A}{\sin A\sin B} = \frac{a\sin B}{\sin A\sin B}$$

$$\Rightarrow \quad \frac{a}{\sin A} = \frac{b}{\sin B}$$

By symmetry, it can also be shown that :-

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

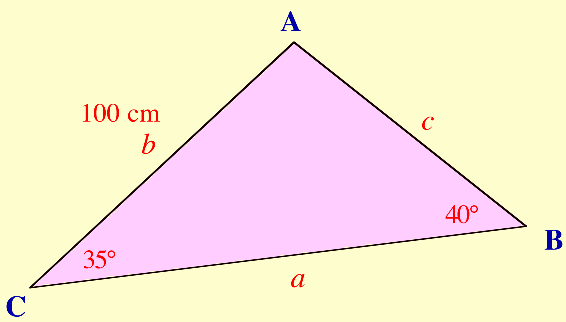
We now have a tremendously powerful formula that enables us to find missing sides and angles in non-right angled triangles - **the Sine Rule**.

**The Sine Rule**

in any $\Delta$ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

### Example :-

Calculate the length of side AB in triangle ABC.




$$\frac{a}{\sin A} = \frac{b\,\checkmark}{\sin B\,\checkmark} = \frac{c\,\checkmark}{\sin C\,\checkmark}$$

$$\frac{100}{\sin 40^\circ} = \frac{c}{\sin 35^\circ}$$

* note

$$\Rightarrow \quad c = \frac{100\sin 35^\circ}{\sin 40^\circ}$$

$$= \mathbf{89{\cdot}2\ cm}$$

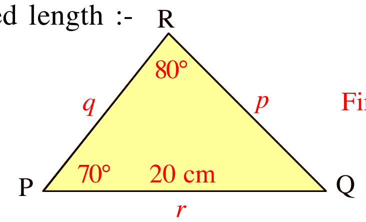
- Write down all 3 ratios $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- tick the 2 angles and side you are given.
- tick the side you are asked to calculate.
- score out the 1 ratio not required.

## Exercise 8·3

*Answer to 3 significant figures unless otherwise asked*

1. Copy and complete the following to find the required length :-

(a)




Find RQ

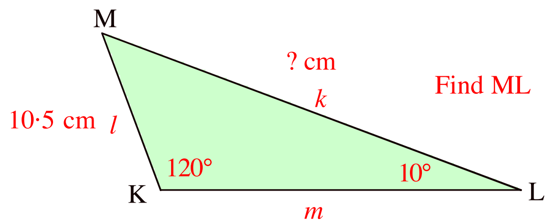
$$\frac{p\checkmark}{\sin P\checkmark} = \frac{q}{\sin Q} = \frac{r\checkmark}{\sin R\checkmark}$$

$$\frac{p}{\sin 70°} = \frac{20}{\sin 80°}$$

$$p = \frac{20\sin ...°}{\sin ....°}$$

$$\mathbf{p = ...... \text{ cm}}$$

(b)




Find ML

$$\frac{k\checkmark}{\sin K\checkmark} = \frac{l\checkmark}{\sin L\checkmark} = \frac{m}{\sin M}$$

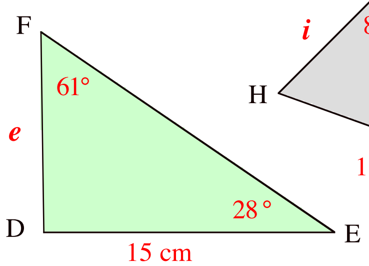
$$\frac{k}{\sin 120°} = \frac{10\cdot 5}{\sin 10°}$$

$$k = \frac{.....\sin ...°}{\sin ....°}$$

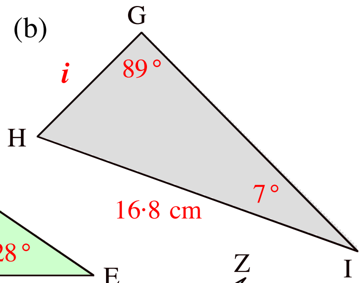
$$\mathbf{k = ...... \text{ cm}}$$

2. Calculate the length of the marked side in each of the following triangles.

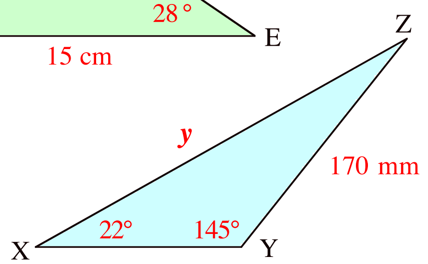
(a)




(b)

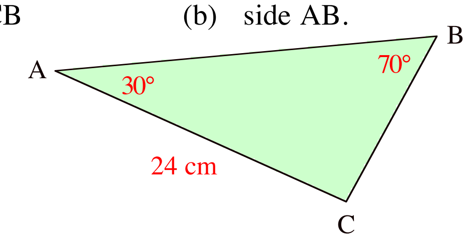



(c)



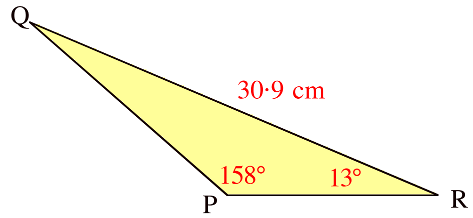

3. In ΔABC, calculate the size of :-

(a) ∠ACB (b) side AB.

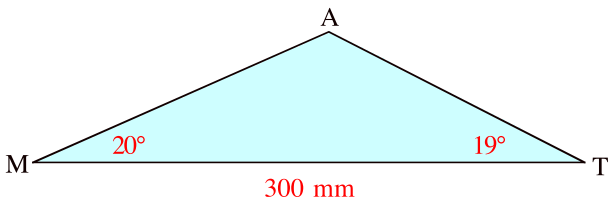



4. In ΔPQR, calculate the size of :-

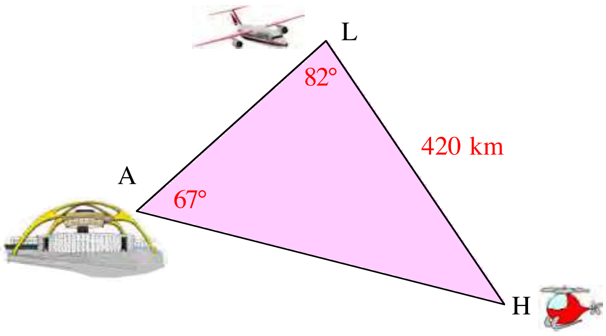
(a) ∠PQR (b) side PR.




5. In ΔMAT, calculate the length of the **shortest** side.




6. The diagram shows the positions of an airport (A), a light jet aircraft (L) and a helicopter (H).

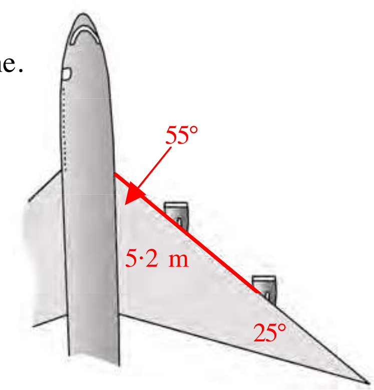



(a) How far away is H from A ?

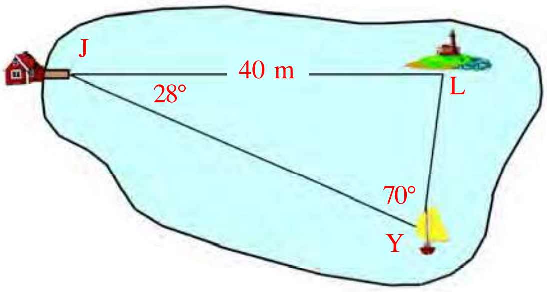
(b) How far away is A from L ?

7. Shown is the wing of a passenger plane.

Calculate the length of the leading edge of the wing.



8. A yacht sets sail from a jetty, 40 metres from the lighthouse.



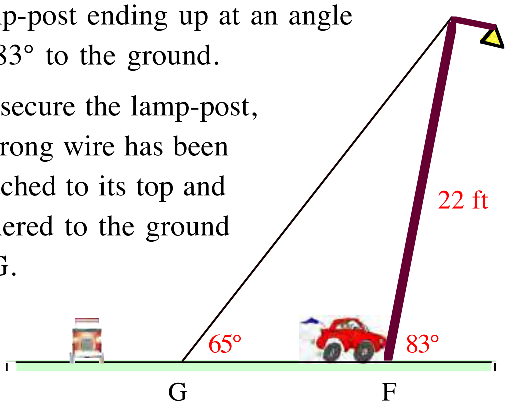

Its course makes an angle of 28° to the coast.

Find, (*to the nearest metre*), the distance :-

(a) from L to Y (b) from J to Y.

9. A road traffic accident resulted in a 22 foot lamp-post ending up at an angle of 83° to the ground.

To secure the lamp-post, a strong wire has been attached to its top and tethered to the ground at G.




The wire makes an angle of 65° with the ground.

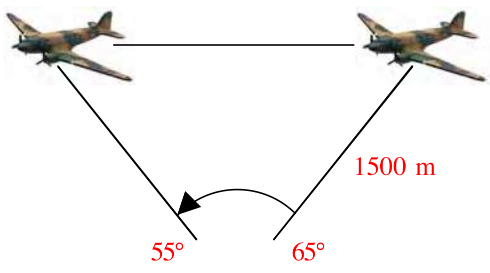
(a) Calculate how long the wire is.

(b) Calculate the distance from G to the foot of the lamp-post.

(c) One week later, the Lighting Department restores the lamp-post to its vertical position but leaves a shortened wire (still attached at G) for a few more days.

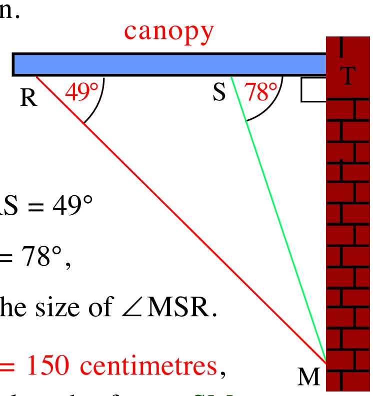
What is the length of the shortened wire ?

10. During a raid, a search-light follows a bomber as it flies at a constant height across the sky.




Calculate how far the bomber had flown.

11. A canopy is built over the front door of a house. To support it, two metal struts, MR and MS, are attached as shown.




(a) Given ∠MRS = 49°

and ∠MST = 78°,

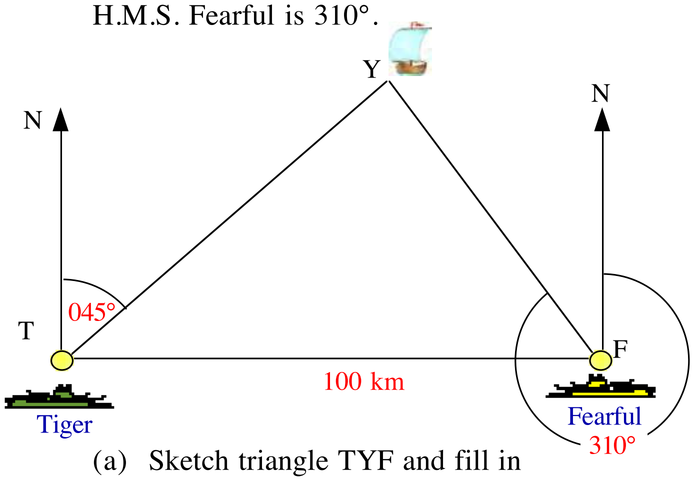
write down the size of ∠MSR.

(b) If strut RM = 150 centimetres, calculate the length of strut SM.

12. H.M.S. Tiger is positioned 100 kilometres **west** of H.M.S. Fearful when they both receive a distress signal from a yacht (at point Y).

The bearing of the yacht from H.M.S. Tiger is 045°.

The bearing of the yacht from H.M.S. Fearful is 310°.




(a) Sketch triangle TYF and fill in the sizes of all three angles.

(b) Which ship will be closer to the yacht ?

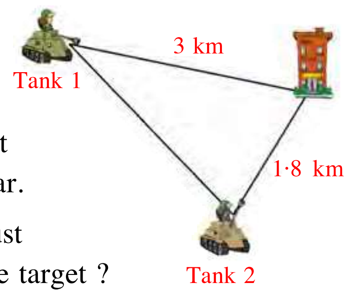
(c) Calculate the distance from this ship to the yacht.

13. ***Difficult***. Two tanks are on a firing range.

Tank 2 is South East of Tank 1.

Both have the same target.

Tank 1 sees the target 3 km away on a bearing of 100°.




Tank 2 is 1·8 km away from the target according to its radar.

On what bearing must Tank 2 fire to hit the target ?

# The Sine Rule - Finding an Angle

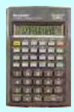
**Example :-**

In $\Delta$ABC, find the size of $\angle$ACB.

$$\frac{a\checkmark}{\sin A\checkmark} = \frac{b}{\sin B} = \frac{c\checkmark}{\sin C\checkmark}$$

=> $\dfrac{25}{\sin 60°} = \dfrac{20}{\sin C}$ Now Rearrange

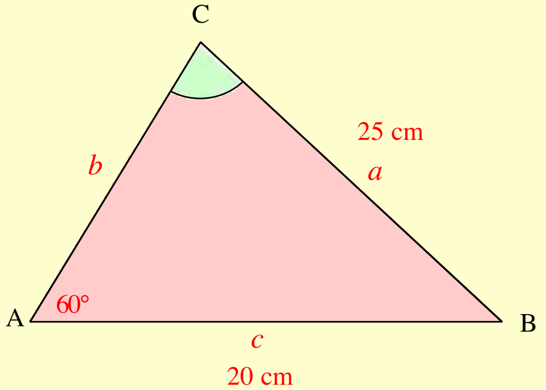
=> $\sin C = \dfrac{20 \sin 60°}{25}$



INV sin

=> $\sin C = 0{\cdot}6928$

=> $\angle C = \boxed{43{\cdot}9°}$ to 3 sig. figs.



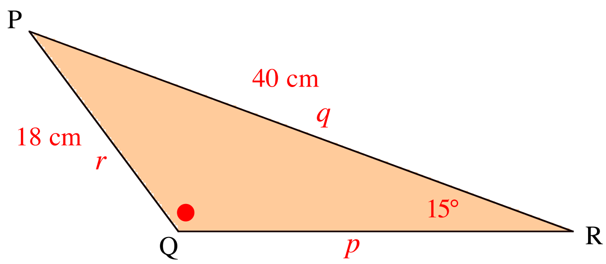

* You will find later in the course that $\angle$C could also, in theory, be (180 – 43·9) = 136·1°, (*The reason being that if sinC = 0·6928, there are 2 possible solutions*).

This is not the case here, since angle C is **acute**.

## Exercise 8·4

*Give your answer to 3 significant figures from now on.*

1. Copy and complete the following to find the size of **obtuse** angle PQR.




$$\frac{p}{\sin P} = \frac{q\checkmark}{\sin Q\checkmark} = \frac{r\checkmark}{\sin R\checkmark}$$

=> $\dfrac{40}{\sin Q} = \dfrac{18}{\sin 15°}$

=> $\sin Q = \dfrac{40 \sin 15°}{\ldots\ldots}$

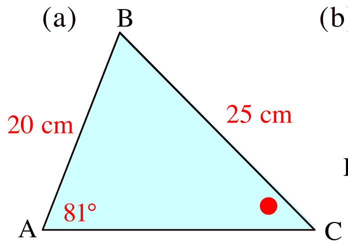
=> $\sin Q = 0{\cdot}\ldots\ldots$ => $Q = \ldots\ldots$ or

=> $\angle Q = (180 - \ldots)° = \ldots\ldots°$.

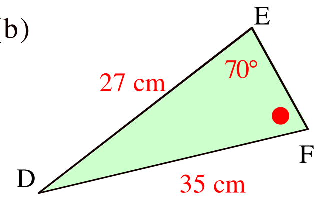
CAREFUL - ANGLE Q IS OBTUSE

2. Copy and complete the following to find the marked angle in each case :-

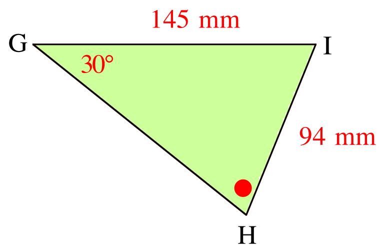
(a)



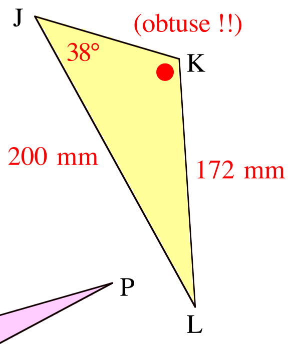

(b)



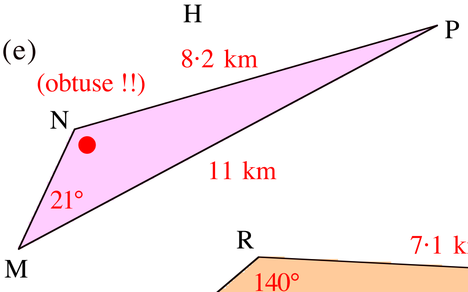

(c)



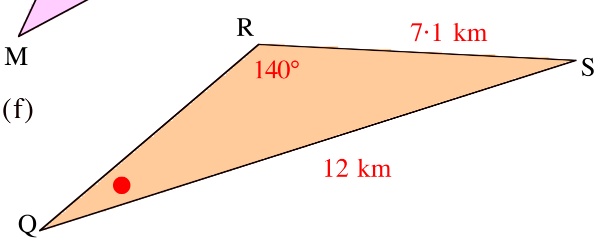

(d)



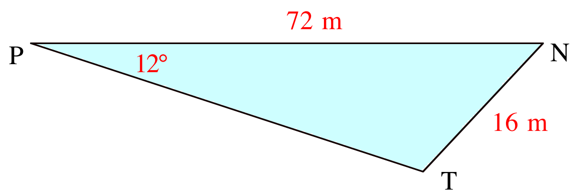

(e)



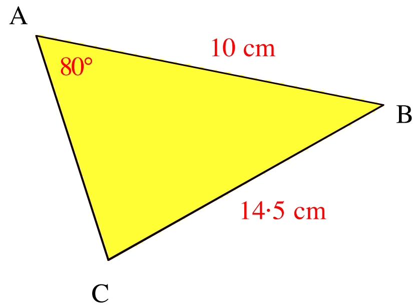

(f)



3. Find the size of **obtuse** ∠PTN in ΔPNT.



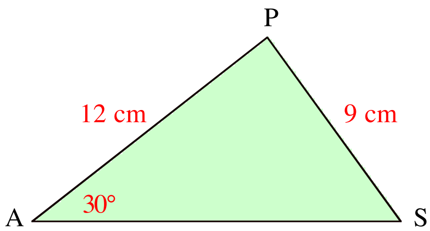

4. In ΔABC, calculate the size of the other **TWO** angles.



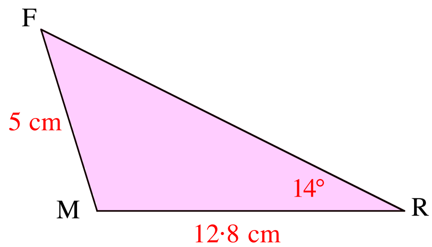

5. Calculate the sizes of :-

(i) ∠APS (ii) ∠FMR (iii) ∠UDY.

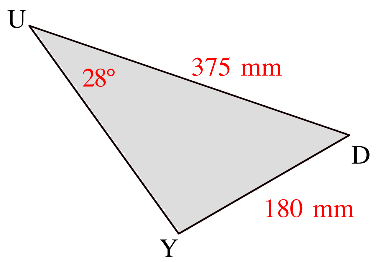
(a)



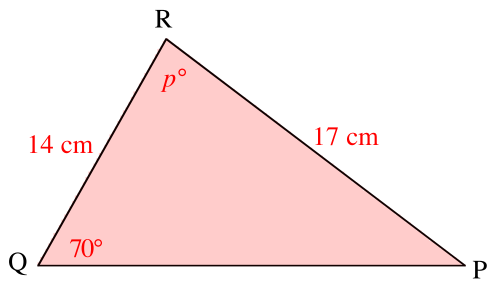

(b)



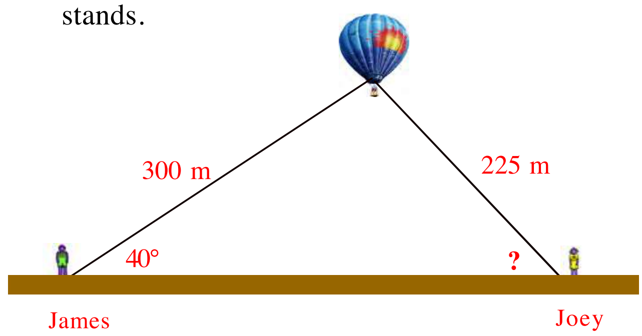

(c)




6. Calculate the size of the angle marked $p°$.

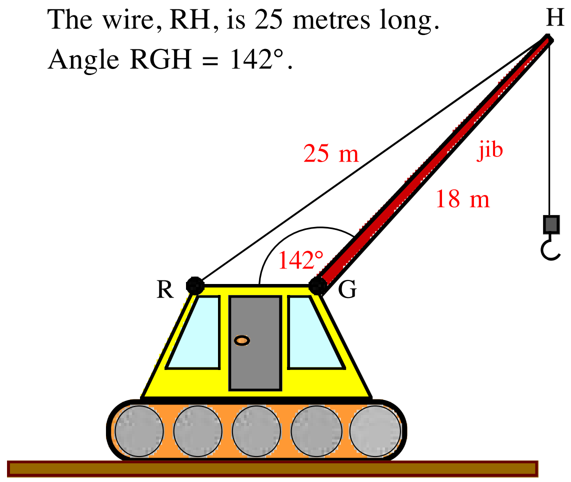



7. A hot air balloon is hovering above the ground. From James, the balloon is 300 metres away and its angle of elevation is 40°.
The balloon is 225 metres from where Joey stands.




What is the angle of elevation of the balloon from Joey ?

8. The jib, GH, of a crane is 18 metres long.
The wire, RH, is 25 metres long.
Angle RGH = 142°.

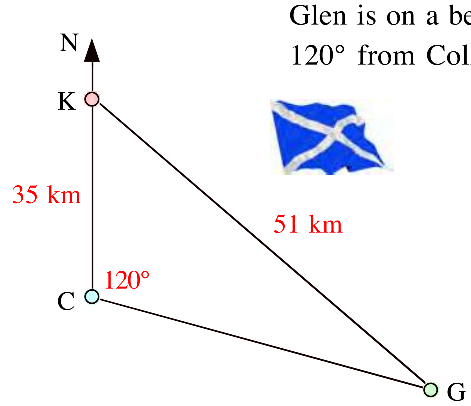



Calculate the sizes of angles GRH and RHG.

9. Three radio masts, Colligan (C), Kelty (K) and Glen (G) are situated in the Scottish Highlands.

Colligan is 35 km due south of Kelty.
Kelty is 51 km from Glen.

Glen is on a bearing of 120° from Colligan.



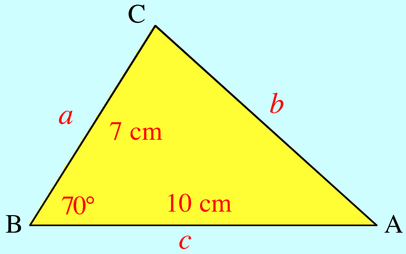

Calculate the bearing of Glen from Kelty.

# The Cosine Rule - Missing Sides

**Calculating the Length of a Side of a Triangle given Two Sides and the Included Angle.**

**Example :-**

Calculate the length of AC.



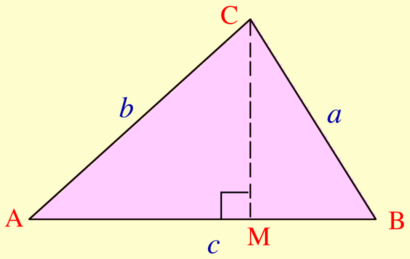

$$\frac{a\checkmark}{\sin A} = \frac{b\checkmark}{\sin B\checkmark} = \frac{c\checkmark}{\sin C}$$

We **don't** have a group of **FOUR**.

=> We **cannot** use the **Sine Rule**.

We need a new rule to calculate a missing side in a triangle like this when the **Sine Rule** won't work.

Consider right angled $\Delta$CAM formed in $\Delta$ABC by drawing the perpendicular line from C to AB.



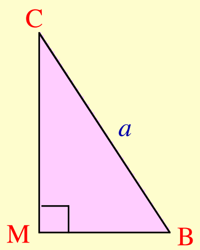

$$\sin A = \frac{CM}{b} \qquad \cos A = \frac{AM}{b}$$

$$\Rightarrow \quad CM = b\sin A \qquad \Rightarrow \quad AM = b\cos A$$

Can you see :- $\quad MB = c - AM$

$$= c - b\cos A$$

By using **Pythagoras' Theorem** in $\Delta$CMB,




$$BC^2 = CM^2 + MB^2$$

$$\Rightarrow \quad a^2 = (b\sin A)^2 + (c - b\cos A)^2$$

$$\Rightarrow \quad a^2 = b^2\sin^2 A + c^2 - 2bc\cos A + b^2\cos^2 A$$

$$\Rightarrow \quad a^2 = b^2(\sin^2 A + \cos^2 A)^* + c^2 - 2bc\cos A$$

$$\Rightarrow \quad a^2 = b^2 + c^2 - 2bc\cos A$$ ...... which is known as the **Cosine Rule**.

Whenever two sides of a triangle and the angle between these two sides are given, the third side can be calculated using the **Cosine Rule** :-

$$a^2 = b^2 + c^2 - 2bc\cos A$$

* It is known that $\sin^2 A + \cos^2 A = 1$ .

This proof shown above is probably beyond your understanding at this stage - it will be explained later on in the course, when you have more background knowledge.

**Example :-** Calculate the length of BC in $\Delta$ABC.

Two sides and included angle given => use **Cosine Rule**.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

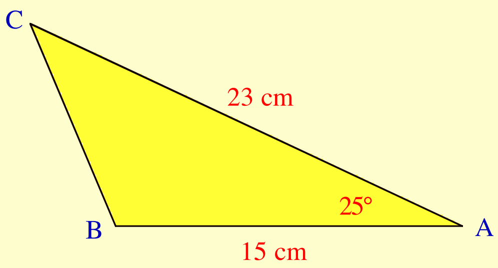
$$\Rightarrow \quad a^2 = 23^2 + 15^2 - 2 \times 23 \times 15 \times \cos 25°$$

$$\Rightarrow \quad a^2 = 128 \cdot 648$$

$$\Rightarrow \quad a = 11 \cdot 3 \text{ to 3 sig. figs.}$$

Remember to press $\sqrt{\phantom{x}}$

BC = **11·3 cm**

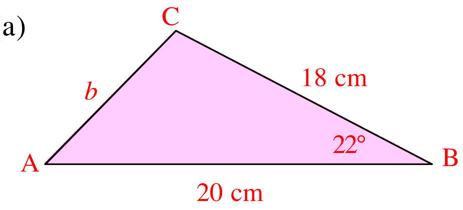


## Exercise 8·5

*Answer to 3 significant figures.*

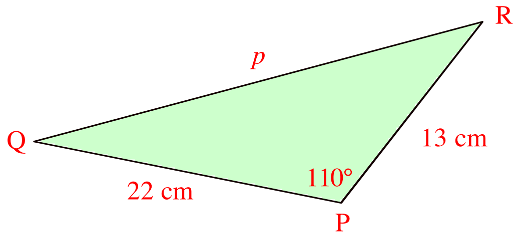
1. **Copy and complete** the following to find the length of the third side :-

(a)




$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\Rightarrow \quad b^2 = 18^2 + \;.....^2 - 2 \times \;..... \times \;.....\cos 22°$$
$$\Rightarrow \quad b^2 = \;...........$$
$$\Rightarrow \quad b = \;...... \quad \Rightarrow \quad AC = \;......\ \text{cm}$$

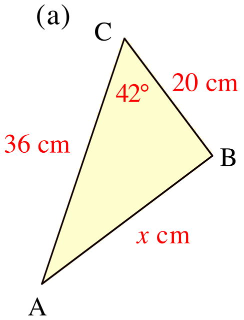
(b)



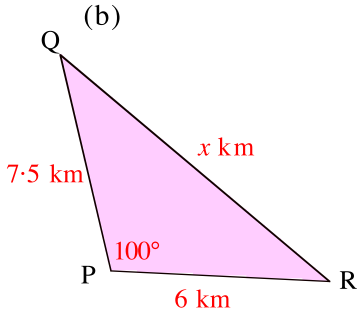

$$p^2 = q^2 + r^2 - 2qr\cos P$$
$$\Rightarrow \quad p^2 = 13^2 + \;.....^2 - 2 \times \;..... \times \;....\cos....$$
$$\Rightarrow \quad p^2 = \;...........$$
$$\Rightarrow \quad p = \;...... \quad \Rightarrow \quad QR = \;......\ \text{cm}$$

2. Calculate the length of the unknown side in each of the following triangles :-

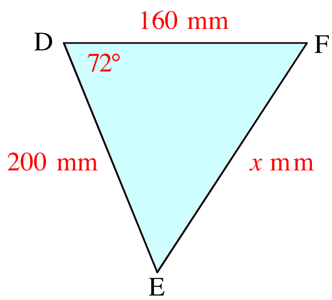
(a)



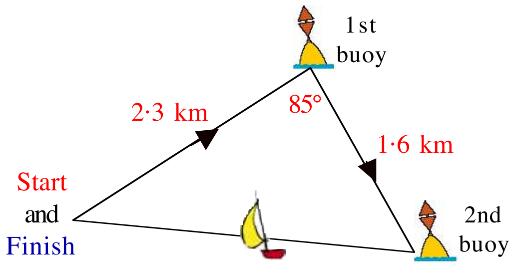

(b)




(c)

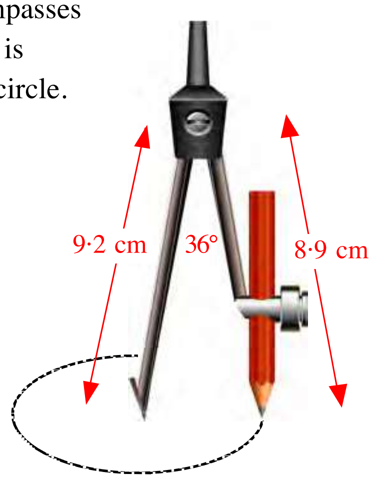



3. A yacht takes part in a race over a triangular course.




Calculate the length of the final stage of the race, from the 2nd buoy to the finishing line.

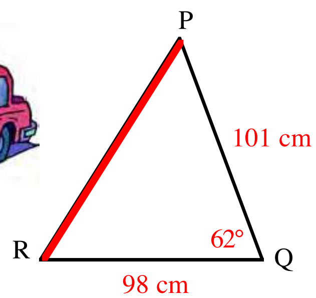
4. The pair of compasses shown opposite is used to draw a circle.




Calculate the **radius** of the circle.

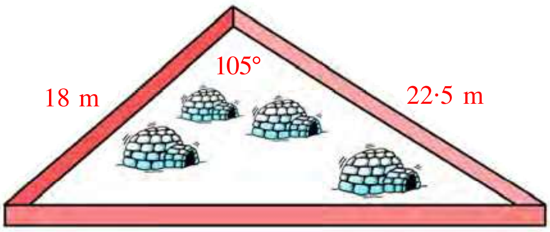
5. The bonnet of a car is held open at an angle of 62°, by a metal rod.

PQ represents the bonnet, PR represents the metal rod and QR represents the distance from the base of the bonnet to the front of the car.



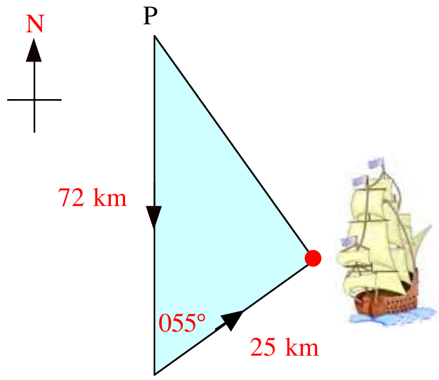

Calculate the length of the metal rod.

6. A triangular wall has been built round a compound of igloos.
It has sides measuring 18 metres and 22·5 metres.
The angle between these sides is 105°.




Calculate the total length of the **perimeter** wall.

7. A ship sailed south from a port (P) for a distance of 72 kilometres.
It then sailed on a bearing of 055° for 25 kilometres.




How far is the ship now from port ?

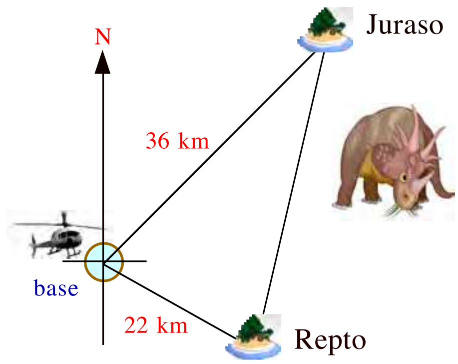
8. The town of Port Greenick is 20 miles north of Longbank and the town of Donburton lies 15 miles north-west of Longbank.

(a) Make a (*rough*) sketch, showing the relative positions of the 3 towns.

(b) Calculate how far it is from Donburton to Port Greenick.

9. The computer game "Dinosaur Islands" indicates the position of a helicopter base in relation to two islands, Juraso and Repto, inhabited by dinosaurs.

From the helicopter base, the island of Juraso is 36 km away on a bearing of 050°.
From the same base, the island of Repto is 22 km away on a bearing of 135°.




Calculate the distance between Juraso and Repto.

## The Cosine Rule - Calculating an Angle

The **Cosine Rule** formula, used to calculate the length of a missing side, can be re-arranged to allow you to calculate the size of a missing angle.
(*As long as you know the lengths of all 3 sides*).

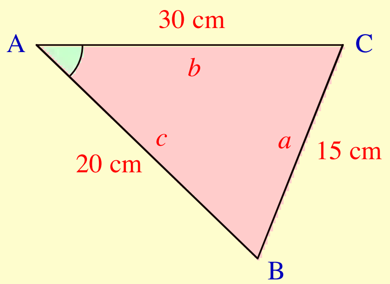
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Rightarrow \quad 2bc\cos A = b^2 + c^2 - a^2$$

$$\Rightarrow \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

**Example :-**

Calculate the size of $\angle BAC$.




$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \quad \cos A = \frac{20^2 + 30^2 - 15^2}{2 \times 20 \times 30}$$

$$= 0{\cdot}895833...$$

INV cos

$\angle BAC$ = **26·4°** to 3 sig. figs.

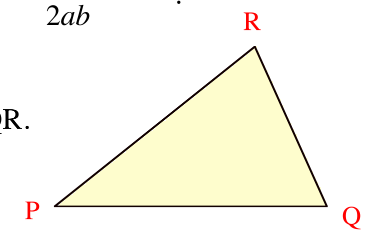
## Exercise 8·6

*Answer to 3 significant figures.*

1. In a $\Delta$ABC, to find $\angle$C, we can use the formula

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

This time we are dealing with $\Delta$PQR.



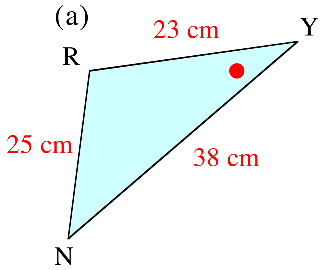

Using the Cosine Rule, write down a formula for calculating the size of each of the following angles :-

(a) $\angle$Q ($\cos Q = .....$) (b) $\angle$P

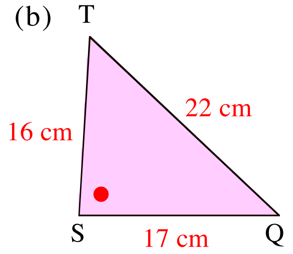
(c) $\angle$R.

2. Calculate the size of the marked angle in each of the following triangles :-

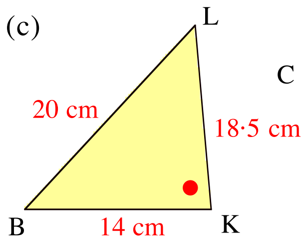
(a)



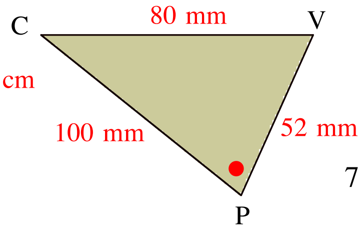

(b)



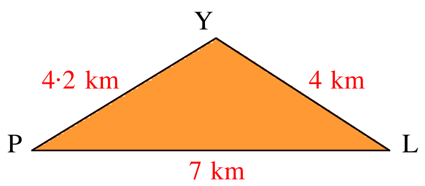

(c)




(d)

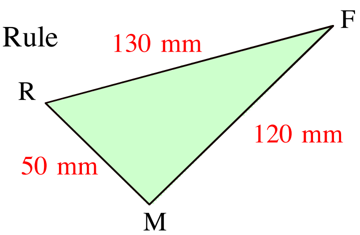



3. Calculate the size of the **largest** angle here.




*note - if the cosine of an angle turns out to be **negative**, we will discover later that the angle is **obtuse**.

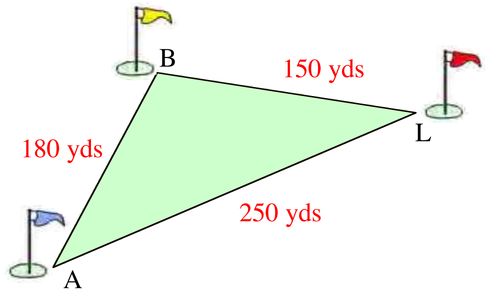
4. (a) Use the Cosine Rule to show that $\angle RMF = 90°$.




4. (b) Use the Converse of Pythagoras' Theorem to **confirm** that $\angle$RMF **is** a right angle.

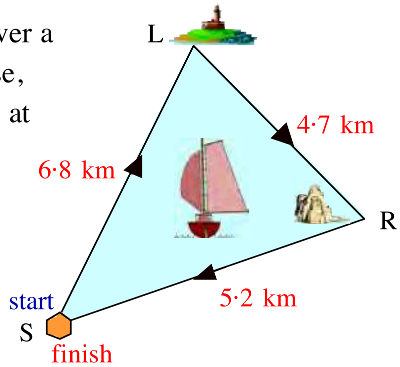
5. The diagram shows part of a pitch & putt golf course.

The lengths of the holes are shown below.



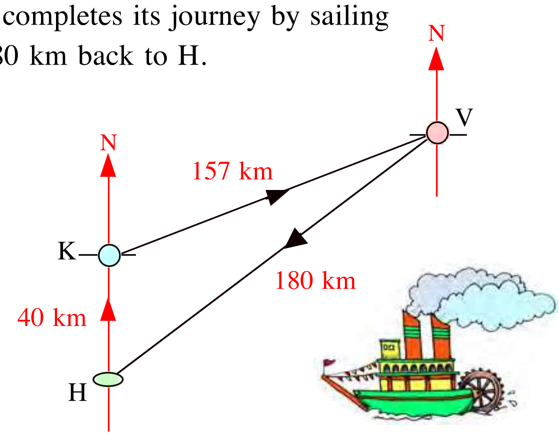

Calculate the size of obtuse angle ABL.

6. A yacht race, over a triangular course, starts at S, turns at the lighthouse, sails around the rocks and finishes back at the starting point.




Calculate the size of the angle LSR.

7. A steamboat leaves H and sails 40 km due north to K.
It then turns and sails 157 km to V.
It completes its journey by sailing 180 km back to H.




(a) Calculate the **bearing** of V from H.

(b) Calculate the **bearing** of V from K.

## The Sine Rule, The Cosine Rule and SOHCAHTOA

Sometimes, a "**SOHCAHTOA**" question is disguised behind the Sine Rule or the Cosine Rule. The following exercise gives you some practice at these types of questions.

**Example :-**

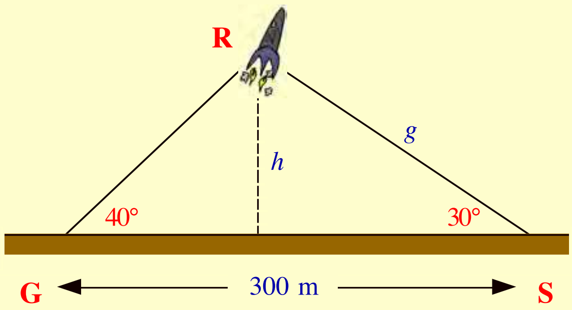
Two girls, who live 300 metres apart, are looking up at what they believe to be a space rocket.

The angle of elevation of the rocket is 40° from Gemma and 30° from Sammi.

Calculate :-

(a) the distance from Sammi to the rocket.

(b) the height the rocket is above the ground.



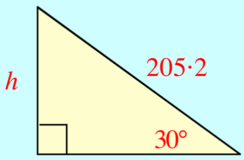

∠GRS is not given, but can be found easily.

(a) $\dfrac{g}{\sin G} = \dfrac{s}{\sin S} = \dfrac{r}{\sin R}$

$\Rightarrow \dfrac{g}{\sin 40°} = \dfrac{300}{\sin 110°}$ R = 180 – 40 – 30

$\Rightarrow g = \dfrac{300\sin 40°}{\sin 110°}$

$g = $ **205·2 m**




(b) $\sin 30° = \dfrac{h}{205{\cdot}2}$

$h = 205{\cdot}2 \times \sin 30°$

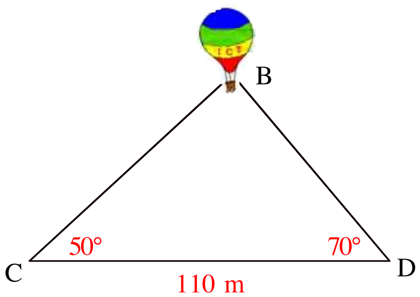
$=$ **102·6 m**

The rocket is 102·6 m above the ground.

## Exercise 8·7

1. Inverness Caley Thistle advertise the return to their football stadium on a helium balloon.

   The distance between the two points C and D on the ground is 110 metres and the angle of elevation from each point is shown on the diagram below.

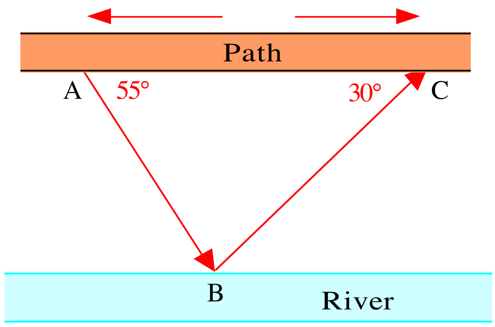



   From the base of the balloon (B), two holding cables are attached to the ground at C and D.

   (a) Calculate length of the cable BC.

   (b) Calculate the height of the balloon.

2. The path in the diagram below runs parallel to the river.

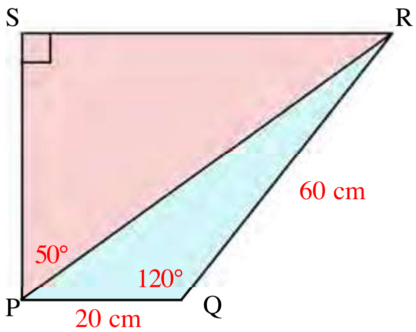



   Colin leaves the path at A, walks to the river for a paddle (B) and rejoins the path further on at C.

   (a) Calculate the distance from A to B.

   (b) Calculate the (*shortest*) distance between the river and the path.

3. In the diagram shown, PQRS has been split into two triangles, one of which is right angled.



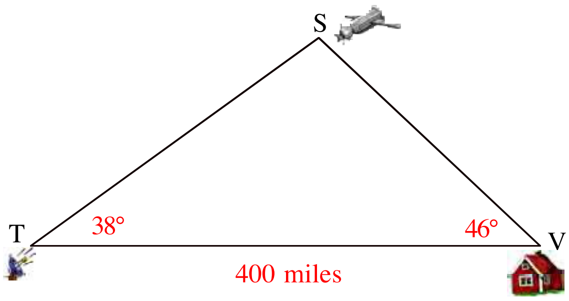

PQ = 20 cm and QR = 60 cm.

∠PQR = 120° and ∠SPR = 50°.

(a) Calculate the length of the line PR.

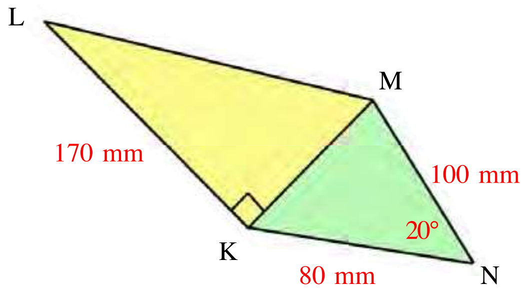
(b) Calculate the length of the line SP.

4. A TV signal is sent from a transmitter T, via a satellite S, to a village V.
The village is 400 miles from the transmitter.
The signal is sent out at an angle of 38° and is received in the village at an angle of 46°.




Calculate the height of the satellite above the ground.

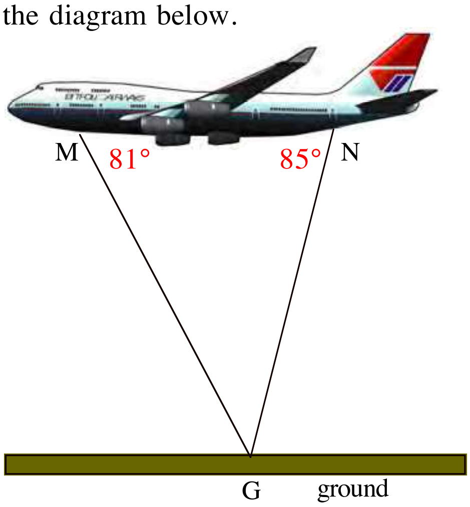
5. Two triangles are formed into a composite shape, as shown.




(a) Find the length of KM.

(b) Find the size of ∠KLM.

6. An aeroplane is flying parallel to the ground. Lights have been fitted at M and N as shown in the diagram below.




When the aeroplane is flying at a certain height, the beams from these lights meet exactly on the ground at G.

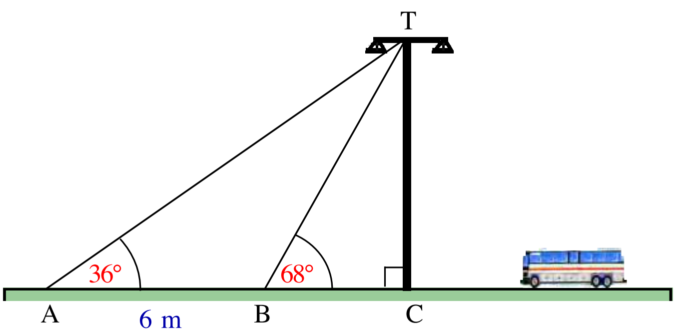
- The angle of depression of the beam of light from M to G is 81°.
- The angle of depression of the beam of light from N to G is 85°.
- The distance MN is 15 metres.

(a) Sketch triangle MGN and mark on all the sizes.

(b) Calculate the **height** of the aeroplane above G.

7. Two support cables, from the top (T) of a motorway light, are attached to the ground at A and B. A is 6 metres away from B.

The angles of elevation are 36° and 68°.

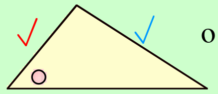
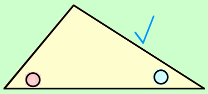
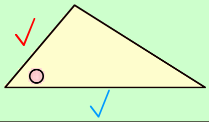
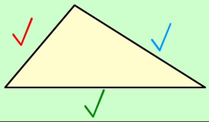
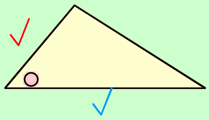



(a) Calculate the sizes of ∠ABT and ∠ATB.

(b) Calculate the length of wire BT.

(c) Calculate the height of pole TC.

## Which Formula should I use ?

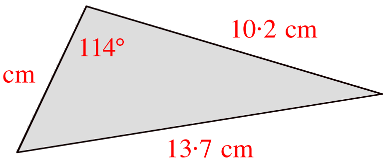
| *What you are given* | | *What you should use* |
|---|---|---|
| **A side & the angle opposite this side** |  or  | **the Sine Rule** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ |
| **Two sides and the angle between the two sides** |  | **the Cosine Rule** $a^2 = b^2 + c^2 - 2bc\cos A$ |
| **All three sides** |  | **the Cosine Rule** $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ |
| **Two sides and the angle between the two sides (area required)** |  | **Area of a Triangle** $\text{Area} = \frac{1}{2}ab\sin C$ |

## Exercise 8·8

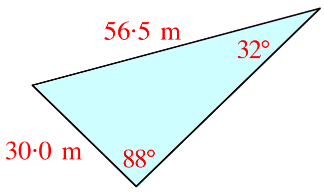
*Answer to 3 significant figures each time here.*

1. Calculate the **area** of these triangles :-

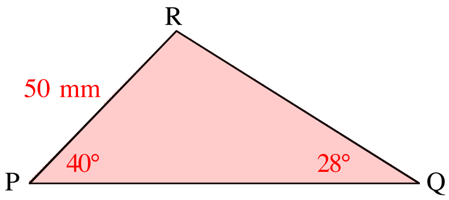
(a)



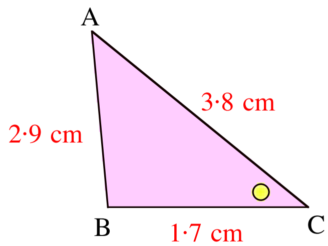

(b)



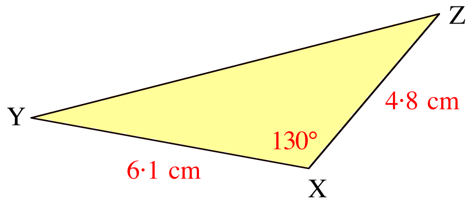

2. In $\Delta$PQR, find the length of the line **PQ**.



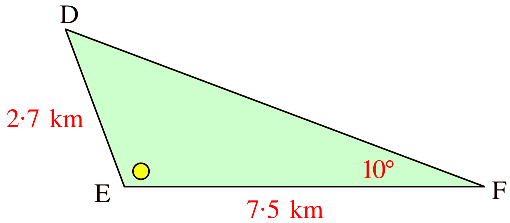

3. Calculate the size of **∠BCA**.




4. Calculate the length of the line **YZ**.

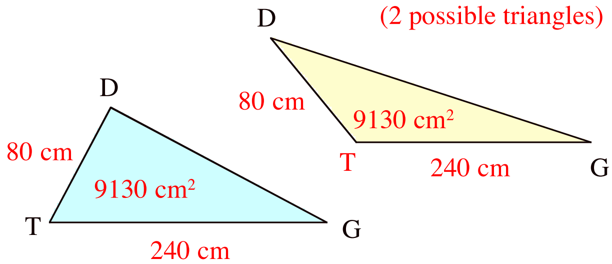



5. Calculate the size of **∠DEF**.
(*Think carefully about this one*).




6. The area of a triangle GTD is 9130 cm². GT = 240 cm and TD = 80 cm.

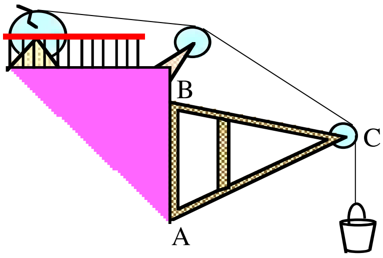
(2 possible triangles)



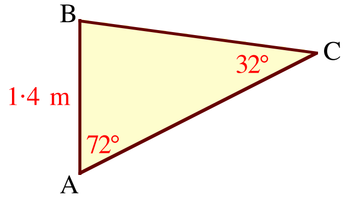

(a) Calculate the size of **acute** angle GTD.

(b) If angle GTD is **obtuse**, calculate its size.

7. A pulley system is used to raise objects up to the top of a high building.



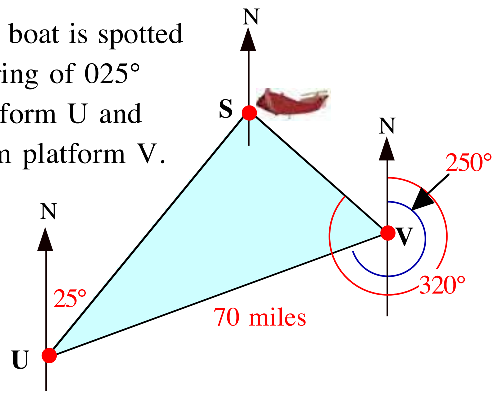

The triangular metal structure, ABC, is used to support the small pulley wheel.




Calculate the length of the bar AC.

8. Two oil platforms in the North Sea are 70 miles apart. Platform U is on a bearing of 250° from platform V.

A rowing boat is spotted on a bearing of 025° from platform U and 320° from platform V.

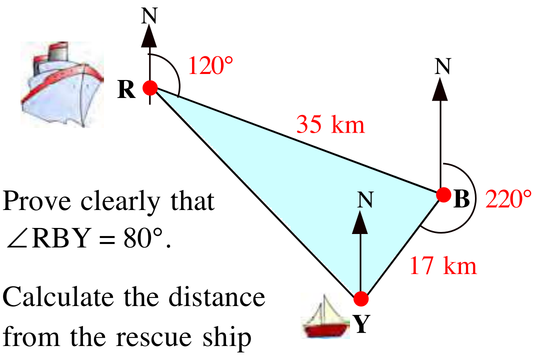



(a) Show that $\angle$USV = 65°.

(b) Now calculate how far the boat is from V.

9. A rescue boat, at R, picks up a distress call from a boat B, 35 km away, on a bearing of 120°.

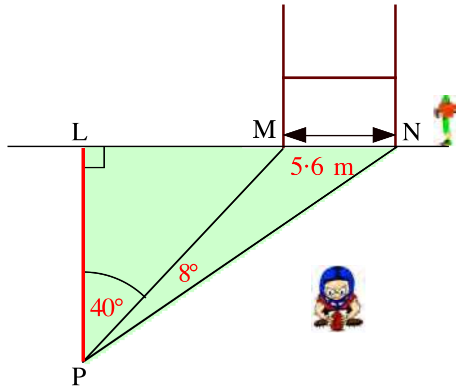
At the same time, another distress call comes from a yacht Y, which is 17 km away from B and on a bearing of 220° from B.




(a) Prove clearly that $\angle$RBY = 80°.

(b) Calculate the distance from the rescue ship to the yacht.

10. The diagram below shows the goalposts on an American Football field.
LP is perpendicular to the touchline, LN.




$\angle$LPM = 40° and $\angle$MPN = 8°.
The distance MN between the goalposts is 5·6 metres.
To kick for goal, the kicker walks straight out from L to P.

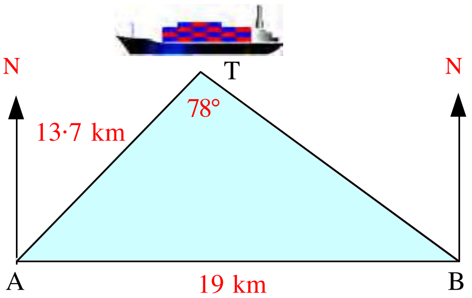
Calculate the distance LP.

(*Hint - find $\angle$PMN and the side PM first*).

11. A coastguard at A is 19 kilometres due west of a coastguard at B.

In relation to the two coastguards, a tanker is spotted at T, such that $\angle$ATB = 78°.

The tanker is 13·7 km away from point A.




(a) Calculate the size of $\angle$TBA, then $\angle$TAB.

(b) Calculate the bearing of the tanker from A.

(c) Calculate the bearing of the tanker from B.

12. Two ships leave port together.

One sails on a course of 030° at 9 mph.

The other sails on a course of 090° at 12 mph.

Make a neat sketch and calculate how far apart they will be after 5 hours.

## Answers to Chapter 8 Page 70

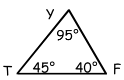
### Exercise 8·1 page 70

1. a 4 cm  b 18·5 cm  c 19·5 cm
   d 14·9 cm  e 4·0 cm  f 36·9 cm
2. a 61·9°  b 46·2°  c 22·0°
   d 47·4°  e 49·0°  f 34·7°
3. 24·6 m
4. 69·9 m
5. 14·0°
6. a sketch of rectangle 20 cm by 8 cm
   b 21·8°, 90° and 68·2°  c 21·5 cm
7. 30°
8. 36·9°
9. 41·8°
10. a 25·9°  b 21·1 m
11. 2·29°
12. a 11·6 m  b 9·58 m
13. 108 cm$^2$
14. a 32·7 cm  b 52·4 cm$^2$  c 12·8 cm
15. 58·5°
16. a 9·17 cm  b 39·8°
17. a 67·8 cm  b 55·3°
18. a 146·3°  b 315°

### Exercise 8·2 page 73

1. a 30·8 cm$^2$  b 45 cm$^2$  c 102·9 cm$^2$
2. a 124 cm$^2$  b 73·4 cm$^2$
3. 15·2 m$^2$
4. 53·4 m$^2$
5. a 24°  b 104 cm$^2$
6. a 101 m$^2$  b 470 m$^2$  c 465 m$^2$
7. 209 cm$^2$
8. £33·73
9. 64·2°
10. 1590 mm$^2$
11. Yes - area = 71 m$^2$ and £45 buys 75 m$^2$
12. Yes - Area = 11·9 m$^2$
13. a (i) 0·5  (ii) 0·766  (iii) 0·174
    (iv) 0·966  (v) 0·087  (vi) 0·891
    b Sin a = sin (180 - a)
14. a 120°  b 135°  c 70°
    d 168°  e 3°  f 179°
15. X = 55°  Y = 125°

### Exercise 8·3 page 76

1. a 19·1 cm  b 52·4 cm
2. a 8·05 cm  b 2·05 cm  c 260 mm
3. a 80°  b 25·2 cm
4. a 9°  b 12·9 cm
5. AM = 155 mm
6. a 452 km  b 235 km
7. 12·1 m
8. a 20·0 m  b 42·2 m
9. a 24·1 ft  b 7·50 ft  c 23·2 ft
10. 1586 m
11. a 102°  b 116 cm
12. a 

    b Tiger  c 64·5 km
13. 028°

### Exercise 8·4 page 79

1. 145°
2. a 52·2°  b 46·5°  c 50·5°
   d 134°  e 151°  f 22·4°
3. 111°
4. ∠ACB = 42·8°, ∠CBA = 57·2°
5. a 108°  b 127·7°  c 74°
6. 59°  7. 59°
8. 11·7° and 26·3°
9. 156°

### Exercise 8·5 page 81

1. a 7·51 cm  b 29·1 cm
2. a 25·0 cm  b 10·4 km  c 214 mm
3. 2·68 km
4. 5·60 cm
5. 103 cm
6. 72·7 m
7. 61·2 km
8. a see sketch  b 14·2 miles
9. 40·5 km

### Exercise 8·6 page 83

1. a $CosQ = \dfrac{p^2 + r^2 - q^2}{2pr}$  b $CosP = \dfrac{q^2 + r^2 - p^2}{2qr}$
   c $CosR = \dfrac{p^2 + q^2 - r^2}{2pq}$
2. a 39·5°  b 83·6°  c 74·5°  d 52·7°
3. 117°
4. a Proof  b Proof
5. 98·1°
6. 43·6°
7. a 049·5°  b 60·6°

### Exercise 8·7 page 85

1. a 119 m  b 91·4 m
2. a 35·1 m  b 28·8 m
3. a 72·1 km  b 46·4 cm
4. 178 miles
5. a 36·9 mm  b 12·3°
6. a see sketch  b 61·5 m
7. a 112° & 32°  b 6·66 m  c 6·18 m

### Exercise 8·8 page 87

1. a 27·5 m$^2$  b 734 m$^2$
2. 98·7 mm
3. 46·3°
4. 9·89 cm
5. 141°
6. a 72°  b 108°
7. 2·56 m
8. a Proof  b 54·6 m
9. a Proof  b 36·2 km
10. 20·6 m
11. a ∠TBA = 44·9°, ∠TAB = 57·1°
    b 032·9°  c 314·9°
12. 54·1 miles

### *Remember Remember 8 page 89*

1. a 74·4 cm$^2$  b 16 700 mm$^2$
2. 30° or 150°
3. 129°
4. 13·2 km
5. 12·7 km
6. 43·8°
7. a 32·6 m  b 13·8 m
8. a Proof  b 154 km

## Answers to Chapter 9 Page 90

### Exercise 9·1 page 90

1. a $^3/_4$  b $^4/_5$  c $^5/_9$  d $^2/_5$
   e $^1/_3$  f $^3/_8$  g $^3/_{10}$  h $^1/_{19}$
2. a $y$  b $a^2$  c $^1/_b$  d 1
   e $^1/_{p^3}$  f $^1/_{q^3}$  g $^1/_{g^6}$  h $t^3$
3. a 3  b 2  c $4x$  d 11
   e $^{3x}/_4$  f $^1/_{2k}$  g $^1/_5$  h $^{3m}/_{13}$
4. a $6a$  b $^9/_5$  c $^3/_{2x}$  d $p$
   e $xy$  f $y^2$  g $^{pq}/_2$  h $8b$
5. a $pq$  b $p^2$  c $a^2b^2$  d $2e$
   e 1  f $gh$  g $m^2n$  h $^{2x}/_3$
6. a $(a + 3)$  b $^1/_{(b - 2)}$
   c $(c - 4)^3$  d $d + 1$
   e $(e - 6)^2$  f $^1/_{(f + 7)^3}$
   g $^1/_{(2a - 1)}$  h 1
   i $(5 - w)$  j $^1/_{(3 - 4v)^2}$
   k $(9 + t)^2$  l $^1/_{(a^2 + 1)}$
   m $^1/_{(4 - 3x^2)^2}$  n $p^2 - 2p + 1$
   o $^2/_q$  p $^1/(_{h + j})$
7. a $(x + 2)$  b $(p + 3)$
   c $^1/_{(a + 3)}$  d $^{(2q + 1)}/_{(q - 5)}$
   e $^{(m + 2)}/_{(m + 1)}$  f 1
   g $^{(3x + 7)}/_{(3x + 6)}$  h $^{(1 + 3p)}/_{(1 - 2p)}$
   i $^{2(1 - x)}/_{3(1 + x)}$

### Exercise 9·2 page 92

1. a $a(a - 6)$  b $(p + 3)(p - 3)$
   c $(y + 1)(y + 8)$  d $6(2q - 3)$
   e $(x - 4)(x - 4)$  f $(k - 2)(k + 3)$
   g $(2v + 1)(v - 4)$  h $4(d + 5)(d - 5)$
2. a $^{(a - 2)}/_3$  b $^1/_{(b - 5)}$
   c $^1/_{(p + 1)}$  d $^{(q - 5)}/_{(q + 5)}$
3. a $^1/_4$  b $^1/_2$  c 3  d $^a/_7$
4. a $^1/_k$  b $^1/_{(c + 3)}$  c $^3/_{(g - 5)}$  d $x + 1$
5. a $^1/_{(x - 1)}$  b $x + 3$  c $^x/_{(x - 1)}$
6. a $4x - 1$  b $^{(x - 1)}/_{(x + 7)}$
   c $^{3(x + y)}/_{(x - y)}$
7. a $^{(x - 3)}/_5$  b $^{(3x - 1)}/_2$  c $^5/_{(x - 1)}$
8. a $^{(x - 2)}/_{(x + 1)}$  b $^{(2x + 3)}/_{(x - 1)}$
   c $^p/_{(x - y)}$
9. a $-^1/_m$  b 1  c $u^2 + 1$

### Exercise 9·3 page 93

1. a $^4/_7$  b $^{13}/_{24}$  c $^7/_{12}$  d $^{23}/_{24}$
2. a $^8/_p$  b $^6/_m$
   c $^{(6y - x)}/_{xy}$  d $^{(4w + 11v)}/_{vw}$
   e $^{(3n + 7m)}/_{mn}$  f $^{(9d - 8c)}/_{cd}$
   g $^{11}/_{3d}$  h $^{(4v - 6w)}/_{12}$
3. a $^{(15n - 4m)}/_{10mn}$  b $^{(8y + 5x)}/_{10xy}$
   c $^{(15s - 16a)}/_{24as}$  d $^{(9h + 7e)}/_{12eh}$
4. a $^{(r^2 + s^2)}/_{rs}$  b $^{(b^2 - c^2)}/_{bc}$
   c $^{(5bx + 3ax)}/_{ab}$  d $^{(2p - q)}/_{4a}$
5. a $^{(2 + 3a)}/_{a^2}$  b $^{(1 - g)}/_{g^2}$
   c $^{(7x - 2)}/_{x^2}$  d $^{(5 - 5t)}/_{t^2}$
   e $^{(20 - 2m)}/_{5m^2}$  f $^{(45b - 24)}/_{20b^2}$
   g $^{(8x^2 - 15y^2)}/_{12xy}$  h $^{(3g^2 + 10h^2)}/_{18gh}$
6. a $^{(5a - 1)}/_6$  b $^{(7p + 1)}/_{12}$
   c $^{(3w + 6)}/_8$  d $^{(9x + 14)}/_{20}$
   e $^{(5g + 7)}/_{18}$  f $^{(3h - 5)}/_4$
7. a $^{(x + 10)}/_6$  b $^{(3w + 31)}/_{10}$
   c $^{(p - 7)}/_{12}$  d $^1/_2$
   e $^1/_{12}$  f $^{(22 - k)}/_{18}$