

Expressions & Formulae 1.3: Algebraic Fractions

Exercise 1: Reducing an Algebraic Fraction to its Simplest Form

1. Express these fractions in their simplest form:

(a) $\frac{3}{6}$ (b) $\frac{8}{12}$ (c) $\frac{30}{16}$ (d) $\frac{54}{72}$ (e) $\frac{10a}{5}$ (f) $\frac{9b}{6}$

(g) $\frac{18}{12x}$ (h) $\frac{25}{15y}$ (i) $\frac{4c}{16c^2}$ (j) $\frac{32a}{8a^3}$ (k) $\frac{13p^2}{52p^3}$ (l) $\frac{36ab}{6bc}$

(m) $\frac{4a}{2a^2}$ (n) $\frac{10x^2}{12xy}$ (o) $\frac{3v^2t}{9vt^2}$ (p) $\frac{10ab^3}{2a^2b}$ (q) $\frac{30p^2q}{25pq^2}$ (r) $\frac{81x^2y^2}{6y^2}$

(s) $\frac{42mn^2}{56mn}$ (t) $\frac{8def^2}{10e^2f}$ (u) $\frac{3ab^2c}{4a^2c}$ (v) $\frac{4k^2m}{28km^2}$ (w) $\frac{5efg^2}{10e^2fg^3}$ (x) $\frac{21xy^2}{36x^3}$

2. Simplify by first finding the common factor:

(a) $\frac{3a+6b}{6}$ (b) $\frac{4x+12y}{2}$ (c) $\frac{3a+a^2}{ab}$ (d) $\frac{xy+y^2}{2y}$

(e) $\frac{xy+x^2}{6x+xy}$ (f) $\frac{3ab+6b^2}{9b^2}$ (g) $\frac{25b^2+15b^3}{10b}$ (h) $\frac{14p+10q}{2s}$

(i) $\frac{3a}{2ab-ac}$ (j) $\frac{6x}{9x+9y}$ (k) $\frac{2st}{6rs-2st}$ (l) $\frac{5c}{10ac+15bc}$

(m) $\frac{14p+28p^2}{8+16p}$ (n) $\frac{8c+4d}{6ac+3ad}$ (o) $\frac{8n^2-2n}{12n-3}$ (p) $\frac{15x^2+6xy}{10x+4y}$

3. Simplify the following by first factorising the numerator and/or denominator:

(a) $\frac{b^2-4}{b+2}$ (b) $\frac{x^2-81}{x-9}$ (c) $\frac{a^2-25}{a+5}$ (d) $\frac{y^2-36}{y+6}$ (e) $\frac{c^2-49}{2c-14}$

(f) $\frac{a^2-64}{2a+16}$ (g) $\frac{p^2-1}{5p-5}$ (h) $\frac{q^2-9}{3q+9}$ (i) $\frac{a^2-b^2}{3a+3b}$ (j) $\frac{x^2-y^2}{5x-5y}$

(k) $\frac{2m^2-18}{2m+6}$ (l) $\frac{3d^2-48}{12d-48}$ (m) $\frac{x^2+3x+2}{x+1}$ (n) $\frac{p-1}{p^2-2p+1}$

(o) $\frac{ax-5a}{x^2-25}$ (p) $\frac{a^2-1}{a^2+2a+1}$ (q) $\frac{b^2+6p-9}{b^2-9}$ (r) $\frac{c^2+2c-15}{c^2-25}$

$$(s) \frac{3x^2 + 5x - 2}{x^2 - 4} \quad (t) \frac{y^2 + 6y + 8}{y^2 + y - 12} \quad (u) \frac{p^2 - 4p - 5}{p^2 + 2p + 1} \quad (v) \frac{c^2 + 4c - 32}{c^2 + c - 56}$$

$$(w) \frac{2x^2 + 13x + 6}{x^2 + 9x + 18} \quad (x) \frac{6a^2 - 13a - 5}{3a^2 - 11a - 4} \quad (y) \frac{10b^2 - 33b - 7}{10b^2 - 37b + 7}$$

Exercise 2: Addition, Subtraction, Multiplication and Division of Algebraic Fractions

1. Express each sum as a fraction in its simplest form:

$$(a) \frac{a}{5} + \frac{a}{5} \quad (b) \frac{2b}{5} + \frac{b}{10} \quad (c) \frac{3x}{4} + \frac{x}{8} \quad (d) \frac{p}{6} + \frac{2p}{3} \quad (e) \frac{y}{9} + \frac{2y}{3} \quad (f) \frac{3}{m} + \frac{2}{m}$$

$$(g) \frac{5}{x} + \frac{1}{x} \quad (h) \frac{2}{a} + \frac{5}{2a} \quad (i) \frac{4}{3y} + \frac{3}{y} \quad (j) \frac{8}{p} + \frac{3}{5p} \quad (k) \frac{3}{a} + \frac{2}{b} \quad (l) \frac{5}{x} + \frac{3}{y}$$

$$(m) \frac{2}{m} + \frac{7}{n} \quad (n) \frac{4}{p} + \frac{3}{q} \quad (o) \frac{9}{c} + \frac{7}{d} \quad (p) \frac{3}{2x} + \frac{2}{3y} \quad (q) \frac{4}{3a} + \frac{5}{2b} \quad (r) \frac{2}{3a} + \frac{9}{3b}$$

$$(s) \frac{5}{4m} + \frac{3}{2n} \quad (t) \frac{7}{3p} + \frac{2}{6q} \quad (u) \frac{1}{a} + \frac{2}{a^2} \quad (v) \frac{1}{a} + \frac{2}{a^2} \quad (w) \frac{3}{3b} + \frac{4}{b^2} \quad (x) \frac{8}{2m} + \frac{5}{3m^2}$$

2. Express each difference as a fraction in its simplest form:

$$(a) \frac{3a}{5} - \frac{a}{5} \quad (b) \frac{2b}{5} - \frac{b}{10} \quad (c) \frac{3x}{4} - \frac{x}{8} \quad (d) \frac{5p}{6} - \frac{2p}{3} \quad (e) \frac{8y}{9} + \frac{2y}{3} \quad (f) \frac{5}{m} - \frac{2}{m}$$

$$(g) \frac{7}{x} - \frac{3}{x} \quad (h) \frac{5}{a} - \frac{1}{2a} \quad (i) \frac{8}{3y} - \frac{2}{y} \quad (j) \frac{8}{p} - \frac{3}{5p} \quad (k) \frac{3}{a} - \frac{2}{b} \quad (l) \frac{5}{x} - \frac{3}{y}$$

$$(m) \frac{7}{m} - \frac{2}{n} \quad (n) \frac{4}{p} - \frac{3}{q} \quad (o) \frac{9}{c} - \frac{7}{d} \quad (p) \frac{3}{2x} - \frac{2}{3y} \quad (q) \frac{5}{3a} - \frac{3}{2b} \quad (r) \frac{5}{3a} - \frac{2}{3b}$$

$$(s) \frac{5}{4m} - \frac{3}{2n} \quad (t) \frac{7}{3p} - \frac{2}{6q} \quad (u) \frac{1}{a} - \frac{2}{a^2} \quad (v) \frac{7}{x^2} - \frac{3}{x} \quad (w) \frac{4}{3b} - \frac{3}{b^2} \quad (x) \frac{7}{2p^2} - \frac{4}{3p}$$

3. Simplify the following:

$$(a) \frac{x+2}{3} + \frac{x+3}{6} \quad (b) \frac{a+6}{4} + \frac{a-2}{3} \quad (c) \frac{d-3}{2} - \frac{d+2}{6} \quad (d) \frac{2a-1}{4} - \frac{a+2}{5}$$

$$(e) \frac{a+3b}{2} + \frac{a-2b}{4} \quad (f) \frac{2u+v}{3} - \frac{u-v}{4} \quad (g) \frac{2}{x+3} + \frac{3}{x+2} \quad (h) \frac{4}{x+5} + \frac{5}{x+1}$$

$$(i) \frac{7}{x-3} + \frac{4}{x+2} \quad (j) \frac{2}{x+4} - \frac{3}{x-3} \quad (k) \frac{1}{x-3} - \frac{5}{x-2} \quad (l) \frac{2}{x-5} - \frac{3}{x-4}$$

4. Express each product as a fraction in its simplest form

(a) $\frac{x}{3} \times \frac{x}{6}$ (b) $\frac{y}{2} \times \frac{y}{4}$ (c) $\frac{a}{2} \times \frac{b}{7}$ (d) $\frac{p}{3} \times \frac{q}{8}$ (e) $\frac{c^2}{5} \times \frac{c}{6}$ (f) $\frac{6}{a} \times \frac{2}{a}$

(g) $\frac{3}{x} \times \frac{10}{y}$ (h) $\frac{3}{p} \times \frac{4}{p}$ (i) $\frac{2}{3m} \times \frac{4}{5m}$ (j) $\frac{1}{b} \times \frac{11}{3c}$ (k) $\frac{5m}{6} \times \frac{3}{2m}$ (l) $\frac{5}{7x} \times \frac{4x}{3}$

(m) $\frac{2y}{9} \times \frac{12}{5y^2}$ (n) $\frac{2}{3a} \times \frac{3}{7a^2}$ (o) $\frac{5}{3p} \times \frac{2}{p^3}$ (p) $\frac{3t^2}{5s} \times \frac{2s^2}{6t^3}$

(q) $\frac{5pq}{2} \times \frac{3}{4pq^2}$ (r) $\frac{7ab^2}{6c} \times \frac{2c^3}{3a^2}$ (s) $\frac{4}{5mn} \times \frac{2m^4}{n^2}$ (t) $\frac{4yz}{9x} \times \frac{3xz}{2y^3}$

(u) $\frac{5ab^3}{3c} \times \frac{3a}{2bc^2}$ (v) $\frac{2cd}{7a} \times \frac{3a^2}{4cd^2}$ (w) $\frac{10xy^2}{3} \times \frac{12xy}{5y^2}$ (x) $\frac{3}{8s^3} \times \frac{4st}{t^3}$

5. Express as a single fraction:

(a) $\frac{a}{4} \div \frac{a}{2}$ (b) $\frac{x}{2} \div \frac{y}{2}$ (c) $\frac{ab}{5} \div \frac{a}{2}$ (d) $\frac{p^2}{10} \div \frac{p}{5}$ (e) $\frac{2c}{3} \div \frac{c^2}{6}$ (f) $\frac{3}{t} \div \frac{6}{t}$

(g) $\frac{2}{k} \div \frac{4}{m}$ (h) $\frac{3}{y} \div \frac{9}{y^2}$ (i) $\frac{4}{bc} \div \frac{2}{c}$ (j) $\frac{3}{2x} \div \frac{12}{x^2}$ (k) $\frac{24xy}{35z} \div \frac{20xy}{21z}$

(l) $\frac{6q^2}{25p} \div \frac{9q}{20p^2}$ (m) $\frac{8ab}{21c} \div \frac{9b}{14ac}$ (n) $\frac{10m}{21n^2} \div \frac{8mn}{9}$ (o) $\frac{20ax}{33y} \div \frac{15x}{44ay^2}$

Exercise 1, Page 30

1. (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{15}{8}$ (d) $\frac{3}{4}$ (e) $2a$ (f) $\frac{3b}{2}$ (g) $\frac{3}{2x}$ (h) $\frac{5}{3y}$
 (i) $\frac{1}{4c}$ (j) $\frac{4}{a^2}$ (k) $\frac{1}{4p}$ (l) $\frac{6a}{c}$ (m) $\frac{2}{a}$ (n) $\frac{5x}{6y}$ (o) $\frac{v}{3t}$ (p) $\frac{5b^2}{a}$
 (q) $\frac{6p}{5q}$ (r) $\frac{27x^2}{2}$ (s) $\frac{3n}{4}$ (t) $\frac{4df}{5e}$ (u) $\frac{3b^2}{4a}$ (v) $\frac{k}{7m}$ (w) $\frac{1}{2eg}$ (x) $\frac{7y^2}{12x^2}$
2. (a) $\frac{a+2b}{2}$ (b) $2(2x+3y)$ (c) $\frac{3+a}{b}$ (d) $\frac{x+y}{2}$ (e) $\frac{y+x}{6+y}$
 (f) $\frac{a+2b}{3b}$ (g) $\frac{5b+3b^2}{2}$ (h) $\frac{7p+5q}{s}$ (i) $\frac{3}{2b-c}$ (j) $\frac{2x}{3(x+y)}$
 (k) $\frac{t}{3r-t}$ (l) $\frac{1}{2a+3b}$ (m) $\frac{7p}{4}$ (n) $\frac{4}{3a}$ (o) $\frac{2n}{3}$ (p) $\frac{3x}{2}$
3. (a) $b-2$ (b) $x+9$ (c) $a-5$ (d) $y-6$ (e) $\frac{c+7}{2}$ (f) $\frac{a-8}{2}$ (g) $\frac{p+1}{5}$
 (h) $\frac{q-3}{3}$ (i) $\frac{a-b}{3}$ (j) $\frac{x+y}{5}$ (k) $m-3$ (l) $\frac{d+4}{4}$ (m) $x+2$ (n) $\frac{1}{p-1}$
 (o) $\frac{a}{x+5}$ (p) $\frac{a-1}{a+1}$ (q) $\frac{b-3}{b+3}$ (r) $\frac{c-3}{c-5}$ (s) $\frac{3x-1}{x-2}$ (t) $\frac{y+2}{y-3}$ (u) $\frac{p-5}{p+1}$
 (v) $\frac{c-4}{c-7}$ (w) $\frac{2x+1}{x+3}$ (x) $\frac{2a-5}{a-4}$ (y) $\frac{5b+1}{5b-1}$

Exercise 2, Page 37

1. (a) $\frac{2a}{5}$ (b) $\frac{b}{2}$ (c) $\frac{7x}{8}$ (d) $\frac{5p}{6}$ (e) $\frac{7y}{9}$ (f) $\frac{5}{m}$ (g) $\frac{6}{x}$
 (h) $\frac{9}{2a}$ (i) $\frac{13}{3y}$ (j) $\frac{43}{5p}$ (k) $\frac{3b+2a}{ab}$ (l) $\frac{5y+3x}{xy}$ (m) $\frac{2n+7m}{mn}$
 (n) $\frac{4q+3p}{pq}$ (o) $\frac{9d+7c}{cd}$ (p) $\frac{9y+4x}{6xy}$ (q) $\frac{8b+15a}{6ab}$ (r) $\frac{2b+9a}{3ab}$ (s) $\frac{5n+6m}{4mn}$
 (t) $\frac{7q+p}{3pq}$ (u) $\frac{2+a}{a^2}$ (v) $\frac{5+3x}{x^2}$ (w) $\frac{b+4}{b^2}$ (x) $\frac{12m+5}{3m^2}$
2. (a) $\frac{2a}{5}$ (b) $\frac{3b}{10}$ (c) $\frac{5x}{8}$ (d) $\frac{p}{6}$ (e) $\frac{2y}{9}$ (f) $\frac{3}{m}$ (g) $\frac{4}{x}$
 (h) $\frac{9}{2a}$ (i) $\frac{2}{3y}$ (j) $\frac{37}{5p}$ (k) $\frac{3b-2a}{ab}$ (l) $\frac{5y-3x}{xy}$ (m) $\frac{7n-2m}{mn}$
 (n) $\frac{4q-3p}{pq}$ (o) $\frac{9d-7c}{cd}$ (p) $\frac{9y-4x}{6xy}$ (q) $\frac{10b-9a}{6ab}$ (r) $\frac{5b-2a}{3ab}$ (s) $\frac{5n-6m}{4mn}$

$$(t) \frac{7q-p}{3pq} \quad (u) \frac{a-2}{a^2} \quad (v) \frac{7-3x}{x^2} \quad (w) \frac{4b-9}{3b^2} \quad (x) \frac{21-8p}{6p^2}$$

$$3. \quad (a) \frac{3x+7}{6} \quad (b) \frac{7a+10}{12} \quad (c) \frac{2d-11}{6} \quad (d) \frac{6a-13}{20} \quad (e) \frac{3a+4b}{4} \quad (f) \frac{5u+7v}{12}$$

$$(g) \frac{5x+13}{(x+3)(x+2)} \quad (h) \frac{9x+29}{(x+5)(x+1)} \quad (i) \frac{11x+2}{(x-3)(x+2)} \quad (j) \frac{-x-18}{(x+4)(x-3)} \quad (k) \frac{13-4x}{(x-3)(x-2)}$$

$$(l) \frac{7-x}{(x-5)(x-4)}$$

$$4. \quad (a) \frac{x^2}{18} \quad (b) \frac{y^2}{8} \quad (c) \frac{ab}{14} \quad (d) \frac{pq}{24} \quad (e) \frac{c^3}{30} \quad (f) \frac{12}{a^2} \quad (g) \frac{30}{xy}$$

$$(h) \frac{12}{p^2} \quad (i) \frac{8}{15m^2} \quad (j) \frac{11}{3bc} \quad (k) \frac{5}{4} \quad (l) \frac{20}{21} \quad (m) \frac{8}{15y} \quad (n) \frac{2}{7a^3}$$

$$(o) \frac{10}{3p^4} \quad (p) \frac{s}{5t} \quad (q) \frac{15}{8q} \quad (r) \frac{7b^2c^2}{9a} \quad (s) \frac{8m^3}{5n^3} \quad (t) \frac{2z^2}{3y^2} \quad (u) \frac{5a^2b^2}{2p^3}$$

$$(v) \frac{3a}{14d} \quad (w) 8x^2y \quad (x) \frac{3}{2s^2t^2}$$

$$5. \quad (a) \frac{1}{2} \quad (b) \frac{x}{y} \quad (c) \frac{2b}{5} \quad (d) \frac{p}{2} \quad (e) \frac{4}{c} \quad (f) \frac{1}{2} \quad (g) \frac{m}{2k} \quad (h) \frac{y}{3}$$

$$(i) \frac{2}{b} \quad (j) \frac{x}{8} \quad (k) \frac{18}{25} \quad (l) \frac{8pq}{15} \quad (m) \frac{16a^2}{27} \quad (n) \frac{15}{28n^3} \quad (o) \frac{16a^2y}{9}$$