

Factorising Algebraic Expressions - "The Difference of Two Squares"

When expanding brackets, we discovered that ... $(a-b)(a+b) = a^2 + ab - ab - b^2 = a^2 - b^2$.

In reverse, when we factorise $a^2 - b^2$ we obtain the answer $(a+b)(a-b)$.

An algebraic expression of the form $a^2 - b^2$ is known as "a **Difference of Two Squares**" - obviously because both terms are **squares** and also the appearance of a **minus** sign.

Examples :-

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|--------------|----------------|----------------|---------------------|
| Factorise :- | 1. $x^2 - 9$ | 2. $49 - x^2$ | 3. $4x^2 - 25y^2$ |
| | $= (x-3)(x+3)$ | $= (7-x)(7+x)$ | $= (2x)^2 - (5y)^2$ |
| | | | $= (2x-5y)(2x+5y)$ |

Exercise 9.6

1. Factorise, using the difference of two squares :-

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|------------------|--------------------|-----------------------|---------------------|
| (a) $x^2 - 4$ | (b) $a^2 - 16$ | (c) $b^2 - 25$ | (d) $x^2 - 1$ |
| (e) $1 - k^2$ | (f) $49 - w^2$ | (g) $64 - h^2$ | (h) $100 - x^2$ |
| (i) $a^2 - b^2$ | (j) $w^2 - v^2$ | (k) $4a^2 - 1$ | (l) $x^2 - 25y^2$ |
| (m) $36 - 49p^2$ | (n) $81a^2 - 4b^2$ | (o) $121v^2 - 100w^2$ | (p) $64p^2 - 81q^2$ |
| (q) $1 - 16a^2$ | (r) $25 - 81x^2$ | (s) $49 - 4k^2$ | (t) $1 - 144y^2$ |

Consider this **example** :-

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| Factorise $3x^2 - 48$ $= 3(x^2 - 16)$ $= 3(x-4)(x+4)$ | It is a "difference", but NOT of two squares ! By removing the common factor , we now have a difference of two squares . |
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2. Factorise these fully :- (*difficult* !)

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| (a) $2x^2 - 18$ $= 2(x^2 - 9)$ $= \dots\dots\dots$ | (b) $3p^2 - 3$ $= 3(p^2 - \dots)$ $= \dots\dots\dots$ | (c) $5a^2 - 80$ $= 5(\dots^2 - \dots)$ $= \dots\dots\dots$ | (d) $6v^2 - 24$ $= 6(\dots - \dots)$ $= \dots\dots\dots$ |
| (e) $4g^2 - 16$ | (f) $7x^2 - 7y^2$ | (g) $6v^2 - 150u^2$ | (h) $10a^2 - 90b^2$ |
| (i) $19x^2 - 19y^2$ | (j) $aw^2 - av^2$ | (k) $\pi m^2 - \pi n^2$ | (l) $kp^2 - 36kq^2$ |
| (m) $kp^2 - 9kq^2$ | (n) $d^3 - 4d$ | (o) $27x^3 - 48x$ | (p) $a^4 - 1$ |
| (q) $1 - k^4$ | (r) $p^4 - q^4$ | (s) $1 - 16y^4$ | (t) $3d^4 - 48$ |

3. Shown is a square with side 5 centimetres inside a square of side k centimetres.

- (a) Prove that the **pink** area can be expressed as :- $(k-5)(k+5) \text{ cm}^2$.
- (b) Find the area when $k = 8.5$.

