

### Exercise 1: Simplifying surds

1. Express each of the following in its simplest form:

- (a)  $\sqrt{8}$  (b)  $\sqrt{12}$  (c)  $\sqrt{50}$  (d)  $\sqrt{20}$  (e)  $\sqrt{24}$  (f)  $\sqrt{108}$   
(g)  $\sqrt{60}$  (h)  $\sqrt{72}$  (i)  $\sqrt{300}$  (j)  $\sqrt{27}$  (k)  $\sqrt{96}$  (l)  $\sqrt{48}$   
(m)  $\sqrt{45}$  (n)  $\sqrt{98}$  (o)  $\sqrt{90}$  (p)  $\sqrt{18}$  (q)  $\sqrt{28}$  (r)  $\sqrt{80}$   
(s)  $\sqrt{32}$  (t)  $\sqrt{160}$  (u)  $\sqrt{150}$  (v)  $\sqrt{44}$  (w)  $\sqrt{63}$  (x)  $\sqrt{175}$

2. Simplify:

- (a)  $5\sqrt{8}$  (b)  $3\sqrt{32}$  (c)  $5\sqrt{40}$  (d)  $2\sqrt{12}$  (e)  $4\sqrt{18}$  (f)  $3\sqrt{24}$   
(g)  $3\sqrt{27}$  (h)  $10\sqrt{48}$  (i)  $2\sqrt{108}$  (j)  $3\sqrt{45}$  (k)  $2\sqrt{63}$  (l)  $4\sqrt{20}$

3. Express each of the following in its simplest form:

- (a)  $5\sqrt{2} + 3\sqrt{2}$  (b)  $3\sqrt{7} - \sqrt{7}$  (c)  $4\sqrt{3} + 2\sqrt{3} - 3\sqrt{3}$   
(d)  $5\sqrt{6} - 2\sqrt{6} + \sqrt{6}$  (e)  $4\sqrt{3} + 5\sqrt{3}$  (f)  $8\sqrt{6} - 2\sqrt{6}$   
(g)  $\sqrt{2} + 2\sqrt{2}$  (h)  $3\sqrt{7} - 9\sqrt{7}$  (i)  $5\sqrt{10} - 5\sqrt{10}$   
(j)  $\sqrt{5} + 5\sqrt{5} - 3\sqrt{5}$  (k)  $2\sqrt{3} + \sqrt{3} - 5\sqrt{3}$  (l)  $5\sqrt{11} + 7\sqrt{11} - \sqrt{11}$

4. Express each of the following in its simplest form:

- (a)  $\sqrt{12} + \sqrt{27}$  (b)  $\sqrt{32} - \sqrt{8}$  (c)  $\sqrt{72} - \sqrt{50}$  (d)  $\sqrt{2} + \sqrt{98}$   
(e)  $\sqrt{80} + \sqrt{20}$  (f)  $\sqrt{24} + \sqrt{54}$  (g)  $\sqrt{180} - \sqrt{45}$  (h)  $\sqrt{1000} - \sqrt{90}$   
(i)  $\sqrt{50} - \sqrt{8}$  (j)  $\sqrt{3} - \sqrt{12}$  (k)  $\frac{\sqrt{75} + \sqrt{108}}{\sqrt{3}}$  (l)  $\frac{\sqrt{5} + \sqrt{20} + \sqrt{80}}{\sqrt{80}}$   
(m)  $\sqrt{108} + \sqrt{12}$  (n)  $\sqrt{32} - \sqrt{8}$  (o)  $\sqrt{72} - \sqrt{50}$  (p)  $\sqrt{2} + \sqrt{98}$   
(q)  $\sqrt{80} + \sqrt{20}$  (r)  $\sqrt{24} + \sqrt{54}$  (s)  $3\sqrt{12} + \sqrt{27}$  (t)  $\frac{3\sqrt{2} + 2\sqrt{8} - \sqrt{18}}{\sqrt{18}}$

5. Simplify:

(a)  $\sqrt{5} \times \sqrt{5}$  (b)  $\sqrt{2} \times \sqrt{2}$  (c)  $\sqrt[11]{11} \times \sqrt[11]{11}$  (d)  $\sqrt{a} \times \sqrt{a}$  (e)  $\sqrt{6} \times \sqrt{6}$   
(f)  $\sqrt{c} \times \sqrt{c}$  (g)  $\sqrt{k} \times \sqrt{k}$  (h)  $\sqrt{3} \times \sqrt{6}$  (i)  $\sqrt{8} \times \sqrt{2}$  (j)  $\sqrt{6} \times \sqrt{2}$   
(k)  $\sqrt{3} \times \sqrt{5}$  (l)  $\sqrt{x} \times \sqrt{y}$  (m)  $\sqrt{2} \times \sqrt{8}$  (n)  $\sqrt{12} \times \sqrt{3}$  (o)  $\sqrt{5} \times \sqrt{20}$   
(p)  $\sqrt{2} \times \sqrt{32}$  (q)  $\sqrt{a} \times \sqrt{b}$  (r)  $\sqrt{10} \times \sqrt{x}$  (s)  $\sqrt{p} \times \sqrt{q}$  (t)  $\sqrt{k} \times \sqrt{6}$   
(u)  $\sqrt{2} \times \sqrt{10}$  (v)  $\sqrt{24} \times \sqrt{3}$  (w)  $\sqrt{5} \times \sqrt{10}$  (x)  $\sqrt{6} \times \sqrt{12}$  (y)  $\sqrt{20} \times \sqrt{3}$   
(z)  $\sqrt{4} \times \sqrt{8}$

6. Simplify:

(a)  $3\sqrt{2} \times \sqrt{2}$  (b)  $2\sqrt{5} \times 3\sqrt{5}$  (c)  $3\sqrt{2} \times 2\sqrt{7}$  (d)  $4\sqrt{3} \times 2\sqrt{3}$   
(e)  $\sqrt{5} \times 3\sqrt{2}$  (f)  $2\sqrt{6} \times 3\sqrt{3}$  (g)  $8\sqrt{2} \times \sqrt{12}$  (h)  $5\sqrt{3} \times 3\sqrt{5}$

7. Simplify:

(a)  $\frac{\sqrt{8}}{\sqrt{2}}$  (b)  $\frac{\sqrt{27}}{\sqrt{12}}$  (c)  $\frac{\sqrt{2}}{\sqrt{32}}$  (d)  $\frac{\sqrt{3}}{\sqrt{27}}$  (e)  $\frac{\sqrt{20}}{\sqrt{5}}$  (f)  $\frac{\sqrt{12}}{\sqrt{48}}$   
(g)  $\frac{\sqrt{54}}{\sqrt{24}}$  (h)  $\frac{\sqrt{175}}{\sqrt{63}}$  (i)  $\frac{\sqrt{18}}{\sqrt{72}}$  (j)  $\frac{\sqrt{6}}{\sqrt{54}}$  (k)  $\frac{\sqrt{288}}{\sqrt{8}}$  (l)  $\frac{\sqrt{1000}}{\sqrt{90}}$   
(m)  $\frac{\sqrt{48}}{\sqrt{6}}$  (n)  $\frac{\sqrt{3}}{\sqrt{24}}$  (o)  $\frac{\sqrt{98}}{\sqrt{7}}$  (p)  $\frac{\sqrt{50}}{\sqrt{250}}$

8. Expand and simplify:

(a)  $\sqrt{2}(1 - \sqrt{2})$  (b)  $\sqrt{3}(\sqrt{3} + 1)$  (c)  $\sqrt{5}(\sqrt{5} - 1)$  (d)  $\sqrt{2}(5 + \sqrt{2})$   
(e)  $\sqrt{2}(3 + \sqrt{6})$  (f)  $2\sqrt{3}(\sqrt{8} + 1)$  (g)  $\sqrt{3}(\sqrt{6} - 2\sqrt{8})$  (h)  $\sqrt{5}(\sqrt{5} + 2)$   
(i)  $4\sqrt{6}(2\sqrt{6} - \sqrt{8})$  (j)  $\sqrt{8}(\sqrt{2} + 4)$  (k)  $2\sqrt{12}(\sqrt{3} + \sqrt{6})$  (l)  $\sqrt{5}(\sqrt{200} + \sqrt{50})$   
(m)  $\sqrt{3}(\sqrt{2} + 1)$  (n)  $\sqrt{2}(\sqrt{8} + \sqrt{2})$  (o)  $\sqrt{3}(\sqrt{2} + \sqrt{6})$  (p)  $\sqrt{5}(3 - \sqrt{5})$

9. Expand and simplify where possible:

(a)  $(\sqrt{2} + 3)(\sqrt{2} - 1)$  (b)  $(\sqrt{5} + 1)(2\sqrt{5} - 4)$  (c)  $(2\sqrt{2} + 3)(\sqrt{2} + 4)$  (d)  $(\sqrt{3} + 1)(\sqrt{3} - 1)$   
(e)  $(2 + \sqrt{5})(2 - \sqrt{5})$  (f)  $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$  (g)  $(\sqrt{2} - 4)(3\sqrt{2} - 1)$  (h)  $(\sqrt{8} + 2)(\sqrt{8} - 1)$

(i) $(\sqrt{2} + 3)^2$	(j) $(2\sqrt{3} + \sqrt{2})(\sqrt{3} + 3\sqrt{2})$	(k) $(\sqrt{2} + \sqrt{3})^2$	(l) $(2\sqrt{3} - 1)^2$
(m) $(2\sqrt{7} - \sqrt{2})^2$	(n) $(5 - 2\sqrt{3})^2$	(o) $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$	(p) $(\sqrt{7} + 1)^2$
(q) $(\sqrt{6} + \sqrt{2})^2$	(r) $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$		

10. Express each of the following with a *rational denominator* and simplify where possible:

(a) $\frac{1}{\sqrt{2}}$	(b) $\frac{1}{\sqrt{3}}$	(c) $\frac{1}{\sqrt{5}}$	(d) $\frac{6}{\sqrt{3}}$	(e) $\frac{10}{\sqrt{5}}$	(f) $\frac{2}{\sqrt{3}}$
(g) $\frac{3}{\sqrt{5}}$	(h) $\frac{20}{\sqrt{2}}$	(i) $\frac{2}{\sqrt{2}}$	(j) $\frac{12}{\sqrt{3}}$	(k) $\frac{3}{\sqrt{6}}$	(l) $\frac{4}{\sqrt{5}}$
(m) $\frac{10}{\sqrt{2}}$	(n) $\frac{35}{\sqrt{7}}$	(o) $\frac{1}{2\sqrt{5}}$	(p) $\frac{4}{5\sqrt{2}}$	(q) $\frac{3}{3\sqrt{2}}$	(r) $\frac{12}{5\sqrt{6}}$
(s) $\frac{8}{3\sqrt{2}}$	(t) $\frac{20}{7\sqrt{5}}$	(u) $\frac{50}{3\sqrt{10}}$	(v) $\frac{10}{3\sqrt{2}}$		

11. Express each of the following in its simplest form with a rational denominator.

(a) $\frac{\sqrt{3}}{\sqrt{2}}$	(b) $\frac{\sqrt{2}}{\sqrt{5}}$	(c) $\frac{\sqrt{8}}{\sqrt{2}}$	(d) $\frac{\sqrt{18}}{\sqrt{3}}$	(e) $\frac{\sqrt{5}}{\sqrt{20}}$	(f) $\frac{\sqrt{2}}{\sqrt{12}}$
(g) $\frac{\sqrt{15}}{\sqrt{5}}$	(h) $\frac{\sqrt{8}}{\sqrt{6}}$	(i) $\frac{\sqrt{5}}{\sqrt{2}}$	(j) $\frac{\sqrt{11}}{\sqrt{2}}$	(k) $\frac{\sqrt{7}}{\sqrt{3}}$	(l) $\frac{\sqrt{13}}{\sqrt{5}}$
(m) $\frac{\sqrt{8}}{3\sqrt{2}}$	(n) $\frac{2\sqrt{3}}{3\sqrt{2}}$	(o) $\frac{5\sqrt{3}}{3\sqrt{5}}$	(p) $\frac{4\sqrt{5}}{5\sqrt{3}}$	(q) $\frac{\sqrt{6}}{\sqrt{18}}$	(r) $\frac{\sqrt{50}}{\sqrt{10}}$
(s) $\sqrt{\frac{3}{12}}$	(t) $\sqrt{\frac{5}{2}}$				

12. Express each of the following with a *rational denominator* and simplify where possible:

(a)  $\frac{1}{\sqrt{50}}$    (b)  $\frac{18}{\sqrt{27}}$    (c)  $\frac{5}{\sqrt{50}}$    (d)  $\frac{3}{\sqrt{20}}$    (e)  $\frac{6}{\sqrt{18}}$    (f)  $\frac{2}{\sqrt{8}}$   
 (g)  $\frac{10}{\sqrt{12}}$    (h)  $\frac{3}{\sqrt{50}}$    (i)  $\frac{4}{\sqrt{32}}$    (j)  $\frac{2\sqrt{3}}{\sqrt{54}}$    (k)  $\frac{3\sqrt{2}}{\sqrt{24}}$    (l)  $\frac{2\sqrt{5}}{\sqrt{45}}$

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1. (a)  $2\sqrt{2}$    (b)  $2\sqrt{3}$    (c)  $5\sqrt{2}$    (d)  $2\sqrt{5}$    (e)  $2\sqrt{6}$    (f)  $6\sqrt{3}$   
 (g)  $2\sqrt{15}$    (h)  $6\sqrt{2}$    (i)  $10\sqrt{3}$    (j)  $3\sqrt{3}$    (k)  $4\sqrt{6}$    (l)  $4\sqrt{3}$   
 (m)  $3\sqrt{5}$    (n)  $7\sqrt{2}$    (o)  $3\sqrt{10}$    (p)  $3\sqrt{2}$    (q)  $2\sqrt{7}$    (r)  $4\sqrt{5}$   
 (s)  $4\sqrt{2}$    (t)  $4\sqrt{10}$    (u)  $5\sqrt{6}$    (v)  $2\sqrt{11}$    (w)  $3\sqrt{7}$    (x)  $5\sqrt{7}$
2. (a)  $10\sqrt{2}$    (b)  $12\sqrt{2}$    (c)  $10\sqrt{10}$    (d)  $4\sqrt{3}$    (e)  $12\sqrt{2}$    (f)  $6\sqrt{6}$   
 (g)  $9\sqrt{3}$    (h)  $40\sqrt{3}$    (i)  $12\sqrt{3}$    (j)  $9\sqrt{5}$    (k)  $6\sqrt{7}$    (l)  $8\sqrt{5}$
3. (a)  $8\sqrt{2}$    (b)  $2\sqrt{7}$    (c)  $3\sqrt{3}$    (d)  $4\sqrt{6}$    (e)  $9\sqrt{3}$    (f)  $6\sqrt{6}$   
 (g)  $3\sqrt{2}$    (h)  $-6\sqrt{7}$    (i)  $0$    (j)  $3\sqrt{5}$    (k)  $-2\sqrt{3}$    (l)  $11\sqrt{11}$
4. (a)  $5\sqrt{3}$    (b)  $2\sqrt{2}$    (c)  $\sqrt{2}$    (d)  $8\sqrt{2}$    (e)  $6\sqrt{5}$    (f)  $5\sqrt{6}$   
 (g)  $3\sqrt{5}$    (h)  $7\sqrt{10}$    (i)  $3\sqrt{2}$    (j)  $-\sqrt{3}$    (k)  $10\sqrt{3}$    (l)  $7\sqrt{5}$   
 (m)  $8\sqrt{3}$    (n)  $2\sqrt{2}$    (o)  $\sqrt{2}$    (p)  $8\sqrt{2}$    (q)  $6\sqrt{5}$    (r)  $5\sqrt{6}$   
 (s)  $7\sqrt{2}$    (t)  $8\sqrt{3}$    (u)  $4\sqrt{2}$
5. (a)  $5$    (b)  $2$    (c)  $11$    (d)  $a$    (e)  $6$    (f)  $c$   
 (g)  $k$    (h)  $3\sqrt{2}$    (i)  $4$    (j)  $2\sqrt{3}$    (k)  $\sqrt{15}$    (l)  $\sqrt{xy}$   
 (m)  $4$    (n)  $6$    (o)  $10$    (p)  $8$    (q)  $\sqrt{ab}$    (r)  $\sqrt{10x}$   
 (s)  $\sqrt{pq}$    (t)  $\sqrt{6k}$    (u)  $2\sqrt{5}$    (v)  $6\sqrt{2}$    (w)  $5\sqrt{2}$    (x)  $6\sqrt{2}$   
 (y)  $2\sqrt{15}$    (z)  $4\sqrt{2}$
6. (a)  $6$    (b)  $30$    (c)  $6\sqrt{14}$    (d)  $24$    (e)  $3\sqrt{10}$    (f)
- $18\sqrt{2}$

7. (g)  $16\sqrt{6}$  (h)  $15\sqrt{15}$   
 (a) 2 (b)  $\frac{3}{2}$  (c)  $\frac{1}{4}$  (d)  $\frac{1}{3}$  (e) 2 (f)  $\frac{1}{2}$   
 (g)  $\frac{3}{2}$  (h)  $\frac{5}{3}$  (i)  $\frac{1}{2}$  (j)  $\frac{1}{3}$  (k) 6 (l)  $\frac{10}{3}$   
 (m)  $2\sqrt{2}$  (n)  $\frac{1}{2\sqrt{2}}$  (o)  $\sqrt{14}$  (p)  $\frac{1}{\sqrt{5}}$
8. (a)  $\sqrt{2} - 2$  (b)  $3 + \sqrt{3}$  (c)  $5 - \sqrt{5}$  (d)  $5\sqrt{2} + 2$   
 (e)  $3\sqrt{2} + 2\sqrt{3}$  (f)  $4\sqrt{6} + 2\sqrt{3}$  (g)  $3\sqrt{2} - 4\sqrt{6}$  (h)  $5 + 2\sqrt{5}$   
 (i)  $48 - 16\sqrt{3}$  (j)  $4 + 8\sqrt{2}$  (k)  $12 + 12\sqrt{2}$  (l)  $15\sqrt{10}$   
 (m)  $\sqrt{6} + \sqrt{3}$  (n) 6 (o)  $\sqrt{6} + 3\sqrt{2}$  (p)  $3\sqrt{5} - 5$
9. (a)  $2\sqrt{2} - 1$  (b)  $6 - 2\sqrt{5}$  (c)  $16 + 11\sqrt{2}$  (d) 2  
 (e) -1 (f) 1 (g)  $10 - 13\sqrt{2}$  (h)  $10 + 3\sqrt{8}$   
 (i)  $12 + 7\sqrt{6}$  (j)  $11 + 6\sqrt{2}$  (k)  $5 + 2\sqrt{6}$  (l)  $13 - 4\sqrt{3}$   
 (m)  $30 - 4\sqrt{14}$  (n)  $37 - 20\sqrt{3}$  (o) -2 (p)  $8 + 2\sqrt{7}$   
 (q)  $8 + 4\sqrt{3}$  (r) -1
10. a)  $\frac{\sqrt{2}}{2}$  b)  $\frac{\sqrt{3}}{3}$  c)  $\frac{\sqrt{5}}{5}$  (d)  $2\sqrt{3}$  (e)  $2\sqrt{5}$  (f)  $\frac{2\sqrt{3}}{3}$  (g)  $\frac{3\sqrt{5}}{5}$   
 h)  $10\sqrt{2}$  i)  $\sqrt{2}$  (j)  $4\sqrt{3}$  (k)  $\frac{\sqrt{6}}{2}$  (l)  $\frac{4\sqrt{5}}{5}$  (m)  $5\sqrt{2}$  (n)  $5\sqrt{7}$   
 o)  $\frac{\sqrt{5}}{10}$  p)  $\frac{2\sqrt{2}}{5}$  q)  $\frac{\sqrt{2}}{2}$  (r)  $\frac{2\sqrt{6}}{5}$  (s)  $\frac{4\sqrt{2}}{3}$  (t)  $\frac{4\sqrt{5}}{7}$  (u)  $\frac{5\sqrt{10}}{3}$
11. a)  $\frac{\sqrt{6}}{2}$  b)  $\frac{\sqrt{10}}{5}$  c) 2 d)  $\sqrt{6}$  e)  $\frac{1}{2}$  (f)  $\frac{\sqrt{6}}{6}$  (g)  $\sqrt{3}$   
 h)  $\frac{2\sqrt{3}}{3}$  (i)  $\frac{\sqrt{10}}{2}$  (j)  $\frac{\sqrt{22}}{2}$  k)  $\frac{\sqrt{21}}{3}$  (l)  $\frac{\sqrt{65}}{5}$  (m)  $\frac{2}{3}$  (n)  $\frac{\sqrt{6}}{3}$   
 o)  $\frac{\sqrt{15}}{3}$  p)  $\frac{4\sqrt{15}}{15}$  q)  $\frac{\sqrt{3}}{3}$  (r)  $\sqrt{5}$  (s)  $\frac{1}{2}$  (t)  $\frac{\sqrt{10}}{2}$
12. a)  $\frac{\sqrt{2}}{10}$  b)  $2\sqrt{3}$  c)  $\frac{\sqrt{2}}{2}$  d)  $\frac{3\sqrt{5}}{10}$  e)  $\sqrt{2}$  (f)  $\frac{\sqrt{2}}{2}$  (g)  $\frac{5\sqrt{3}}{3}$   
 h)  $\frac{3\sqrt{2}}{10}$  (i)  $\frac{\sqrt{2}}{2}$  (j)  $\frac{\sqrt{2}}{3}$  k)  $\frac{\sqrt{3}}{2}$  (l)  $\frac{2}{3}$