

The Semi-Interquartile Range - A new Measure of Spread

Let us look back at the example on page 181 - the ages of a group on a Sunday school trip.

2, 6, 6, 7, 7, 7, 7, 8, 8, 8, 10, 10, 11, 13, 25.

If we use the only measure of **spread** we have - the **range** - there is a slight problem.

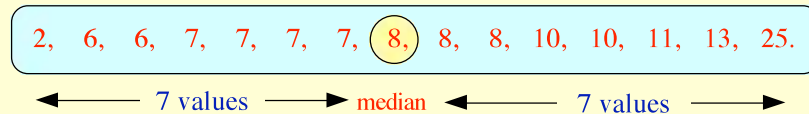
$$\text{Range} = \text{Highest} - \text{Lowest} = 25 - 2 = 23.$$

- can you see that most of the children are aged 6 to 13 ?
- the Sunday school teacher is aged 25 and has her 2-year old son with her.
- the range of **25** gives a false impression of “how widely spread” the actual ages are.
- the **range** only concentrates on the two “end” ages and disregards **all** of the other ages.

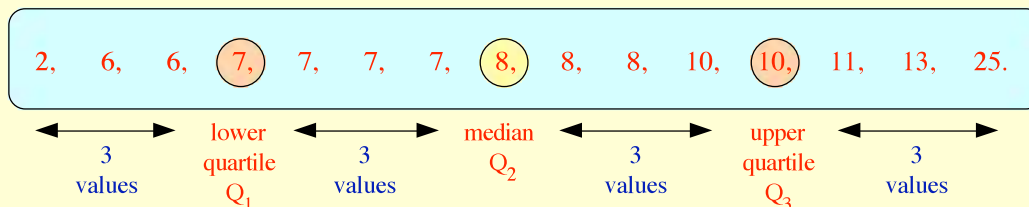
We need a new measure of **SPREAD** which takes into account more of the numbers in the distribution. Such a measure of spread exists - the **semi-interquartile range**, which we will study soon.

The Quartiles of a set of Numbers

The **MEDIAN** is the value that splits a distribution of **ordered** numbers into two equal bits.



The **QUARTILES** are the 3 values that split a distribution of **ordered** numbers into **four** equal bits.



Can you see that, for the above group of ages,

the **lower quartile** (Q_1) = 7, the **middle quartile** (Q_2) = 8, the **upper quartile** (Q_3) = 10 ?

* the **middle quartile** (Q_2) is just another name for the **median** ?

The **quartiles** must split up a distribution of ordered numbers in such a way that there are an equal number of values in each of the 4 “quarters” of the distribution.

Quartiles - continued

Example :- Find the quartiles for the set :- 2, 3, 3, 4, 5, 5, 9, 10, 10, 10, 11, 12, 15, 17, 17, 19, 20

Step 1 There are 17 values in the question.

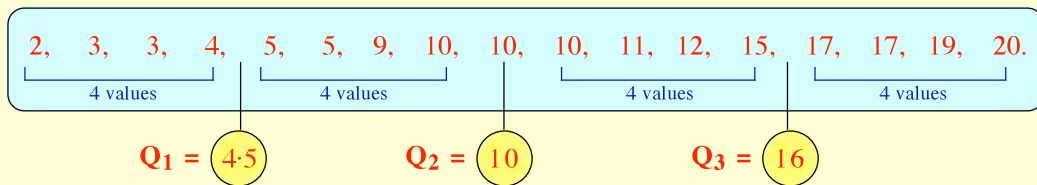
This means that the **median** must be the 9th value. \Rightarrow median = **10**.

2, 3, 3, 4, 5, 5, 9, 10, **10**, 10, 11, 12, 15, 17, 17, 19, 20.

Step 2 This now leaves 8 values in each half of the distribution.

The **middle** of the **LEFT** set is between the 4th and 5th value (up) $\Rightarrow Q_1 = \frac{4+5}{2} = 4.5$.

The **middle** of the **RIGHT** set is between the 4th and 5th value (down) $\Rightarrow Q_3 = \frac{15+17}{2} = 16$.



Exercise 17.4

1. (a) Copy the following 11 numbers. (*you should try to space them fairly widely and fairly evenly*).

8, 8, 9, 10, 10, 11, 11, 11, 12, 13, 14,

(b) Circle the **middle** value - the **median**.

(c) Forgetting this number, how many numbers are there in each of the left and the right halves ?

(d) Find the middle of the left set of numbers - the **lower quartile** - Q_1 .

(e) Find the middle of the right set of numbers - the **upper quartile** - Q_3 .

2. Find the **lower quartile**, the **middle quartile** (the **median**) and the **upper quartile** for each of these :-

(a) 1, 3, 4, 7, 7, 9, 13.

(b) 13, 13, 15, 16, 21, 23, 24, 28, 29.

(c) 3.2, 3.5, 3.6, 3.8, 3.8, 4.0, 4.4, 4.4, 4.7, 5.3, 5.4, 5.9.

(d) 48, 51, 54, 54, 58, 64, 67, 71, 73, 78.

(e) 34, 31, 25, 35, 35, 23, 23, 40, 37, 27, 21, 29, 39. (*order ?*)

3. Mrs Jones weighs the 65 children in Primary 7 and writes them all down in order.

Their weights, (in kilograms), are :- 34, 34, 35, 36, 36, 36, 37, 50, 50, 51.

(a) Of the 65, which child's weight should be given as the **median** weight. (*the 30th, 31st, 32nd..*) ?

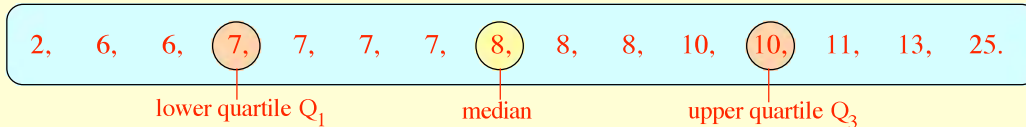
(b) Which of the 65 children's weights will give the **lower quartile** and which will give the **upper** ?

The Semi-Interquartile Range

Let us look once more at the example on page 181 - the ages of a group on a Sunday school trip.

2, 6, 6, 7, 7, 7, 7, 8, 8, 8, 10, 10, 11, 13, 25.

We found the **quartiles** and these are shown below :-



RANGE :- You learned on page 183 that the range was a simple measure of spread.

Range = highest – lowest = 25 – 2 = 23 (but this gave too “big” an answer).

If we now find the **difference** => **upper quartile – lower quartile**, and **halve** this answer, we end up with a new measure of spread, called the **semi-interquartile range**. (S.I.Q.R.)

$$\text{Semi-Interquartile Range} = \frac{\text{Upper Quartile} - \text{Lower Quartile}}{2}$$

$$\text{S.I.Q.R} = \frac{Q_3 - Q_1}{2} = \frac{10 - 7}{2} = 1.5$$

* In many instances, this measure of spread is preferable to the range. It does not simply rely on the two end values, the “highest” and “lowest” – rather, it takes into account more of the numbers in the distribution.

Exercise 17.5

- Calculate the **median** and **lower** and **upper quartiles** for each of the following sets of values. Hence, calculate the **semi-interquartile range** of each.

(a) 13, 13, 15, 19, 23, 23, 24, 26, 27.

(b) 2.4, 2.6, 2.9, 2.9, 3.1, 3.1, 3.3, 3.6, 3.6, 3.8, 4.1, 4.1, 4.5, 4.7, 4.9, 5.0.

(c) 101, 108, 109, 112, 112, 115, 120, 121, 125, 131, 131, 134, 135, 138, 140.

- A group of 25 third year pupils were asked to say how many cousins they had.

3, 1, 4, 2, 3, 4, 5, 2, 2, 4, 5, 1, 0, 6, 8, 2, 4, 4, 6, 2, 3, 1, 0, 9, 6.

- Re-arrange them in order starting with the lowest.
- Calculate the **mean**, **median** and **modal value**.
- Determine the **lower** and **upper quartiles**.
- Calculate the **range** and the **S.I.Q.R**.



- A shoe shop assistant took a note of the sizes of a popular make of trainers that were sold in her shop last week.

1, 4, 4, $4\frac{1}{2}$, 5, 5, 5, $5\frac{1}{2}$, $5\frac{1}{2}$, 6, 6, 6, $6\frac{1}{2}$, $6\frac{1}{2}$, 10.

Calculate the **range** and the **S.I.Q.R**. and say why the S.I.Q.R. would be a better indicator of the true spread of the shoe sizes sold last week.



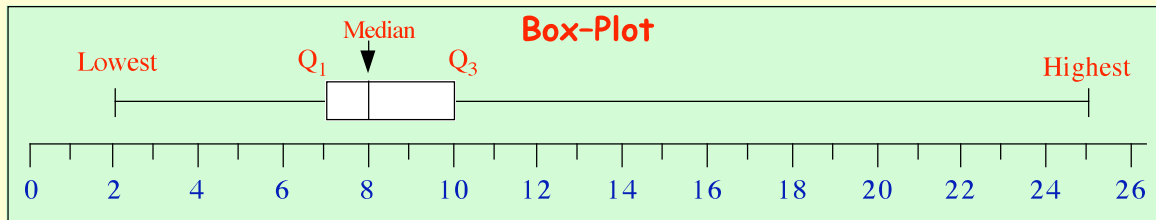
Box-Plots - 5 Point Diagrams

Let us look once more at the example on page 181 - the ages of a group on a Sunday school trip.

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We found that $Q_1 = 7$, $Q_3 = 10$, the **median** = 8, the **lowest** value is 2 and the **highest** is 25.

This can be represented on a **box-plot** (or *5-point summary*) as shown below.



Box-Plots are very useful diagrams, particularly when you wish to compare 2 or more sets of values.

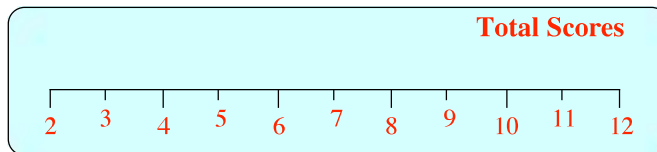
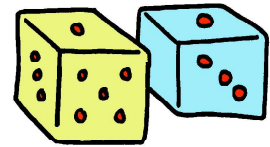
They are also sometimes referred to as **box-whisker** diagrams for obvious reasons.

Exercise 17.6

1. James rolled two die, (*plural of dice*), twelve times, and noted the **total** score each time.

3, 5, 6, 7, 7, 7, 8, 9, 9, 10, 10, 12.

- (a) Calculate the **median** as well as the **upper** and **lower quartiles**.
 (b) **Copy** this scale and draw a neat **box-plot** to represent the above scores.

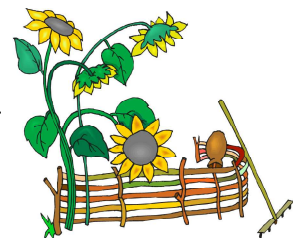


2. A group of pupils took part in a sun-flower growing competition and they all planted their sun-flower seed at the same time.

Eight weeks later, the heights of the plants were measured (*to the nearest 5 cm*).

35, 35, 40, 40, 40, 50, 50, 55, 60, 70, 85, 85, 95, 105.

- (a) Calculate the values of the three quartiles, Q_1 , Q_2 and Q_3 .
 (b) Draw a suitable scale and show the above heights on a neatly drawn labelled box-plot.



- 3.



The weights, (*in kilograms*), of the luggage of the 15 passengers boarding a plane bound for the Orkneys was recorded.

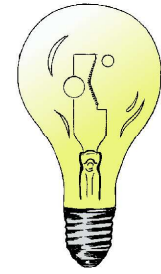
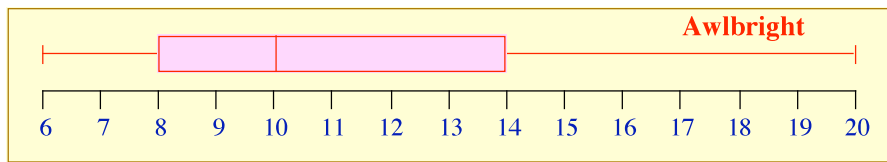
15, 18, 14, 22, 19, 18, 14, 25, 24, 18, 10, 13, 21, 18, 24.

- (a) Re-arrange the weights in order, smallest first and calculate the **median** and the **quartiles**.
 (b) Draw and label a box-plot showing these weights. (*Choose a suitable scale*).

4. Osiris claim that their light-bulbs last longer than Awlbright's bulbs.
A sample of each was tested. **Osiris'** sample is shown below (*in months*).

8, 9, 10, 10, 12, 14, 14, 14, 15, 15, 17, 17, 18.

A box plot was created to represent **Awlbright's** sample and is shown below,



Osiris
We shine
for longer

- (a) What were the **median** and the **quartiles** for the sample of Awlbright's light-bulb lifetime ?
(b) Calculate the **median** and the **quartiles** for the sample of Osiris' light-bulb lifetime.
(c) Make a neat copy of the above box-plot, and **on the same graph**, draw the box-plot above the Awlbright's box plot, showing the Osiris bulbs.
(d) Write a couple of sentences comparing the two samples.
5. A group of men and a group of women, in a local gym, decided to hold a competition. They counted how many pull-ups each person could do in a two minute period.

men	7	9	9	11	13	13	15	18	18	20	25	
women	5	5	6	7	7	10	12	12	14	14	15	17



- (a) Calculate the **medians** and **quartiles** for both the men and the women.
(b) Draw a neat labelled **composite** box-plot diagram to show how the two groups fared.
(c) Write a couple of sentences comparing the men competitors with the women.

6.

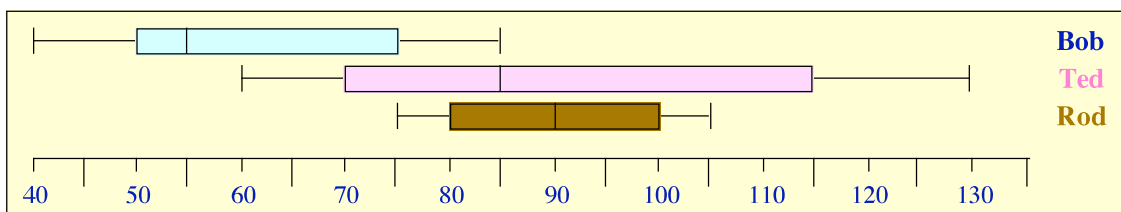


Both "Breezyjet" and "High-on-Air" fly daily from Edinburgh to London Stanstead. The flight is supposed to take 1 hour.

Over the course of a week in June, the flight-times of every Breezyjet and High-on-Air plane from Edinburgh to Stanstead was recorded in minutes.

Easyjet -	55	57	61	63	66	66	67	70	70	72	72	75	75	77	80
High-on-Air -	61	61	61	62	63	65	65	65	66	66	68	68	68		

- (a) Draw a neat labelled composite box-plot diagram to show the above flight-times.
(b) Make a statement comparing both company's flight-times from Edinburgh to London.
7. Three men hit 15 golf balls on a driving range, each using a number 6 iron. The box plot diagram shows the distances (in metres) they hit their golf balls.



Write a few sentences comparing the three men's driving skills, mentioning their **median** scores, the **spread** of their drives, who was likely to be the novice, who was most erratic and who most consistent.