Higher Maths: Nat 5 Revision Work

Exercise 1: Straight Line Revision

1. Calculate the gradient of the line joining each pair of points below:

(a) A(−2, 6) and B(8, 8) **(b)** C(3, −3) and D(4, −1) **(c)** E(5, −9) and F(8, −15)

(d) G(0, 6) and H(5, 11) **(e)** I(−1, −3) and J(7, −9) **(f)** K(−4, 0) and L(−1, 5)

(g) M(2, 2) and N(−3, 4) **(h)** P(5, −1) and Q(−2, 10) **(i)** R(−3, −5) and S(8, −4)

(j) T(4, −6) and U(7, −2) **(k)** V(5, −6) and W(−2, 6) **(l)** X(−1, 7) and Y(−2, 6)

2. Identify the gradient and y – intercept of these lines.

(a) $y = x + 3$ **(b)** $y = -2x - 1$ **(c)** $y = \frac{1}{2}x$ **(d)** $y = -\frac{1}{2}x + 2$

(e) $y = x - 7$ **(f)** $y = -5x + 3$ **(g)** $y = -4x$ **(h)** $x + y = 6$

(i) $2y = x - 4$ **(j)** $3y = x + 12$ **(k)** $4x + 5y = 20$ **(l)** $3x - 2y = 12$

(m) $5y = 3x - 10$ **(n)** $2x + y = 11$ **(o)** $2y = x - 5$ **(p)** $3y - x = 18$

3. Write down the equation of the lines with the given gradients passing through the given points:

(a) gradient 4, through (0, 5) **(b)** gradient −2, through (0, 1)

(c) gradient ¾, through (0, −3) **(d)** gradient 4, through (3, 1)

(e) gradient −5, through (−3, 1) **(f)** gradient ½, through (−5, −2)

(g) gradient $\frac{4}{3}$, through (2, 7) **(h)** gradient $-\frac{3}{4}$, through (−2, −2)

4. Find the equation of the line joining each pair of points below.

(a) (4, 3) & (8, 11) **(b)** (1, 9) & (3, 1) **(c)** (−2, 6) & (8, 8) **(d)** (5, −9) & (8,−15)

(e) (0, 6) & (5, 11) **(f)** (−1,−3) & (7, −9) **(g)** (−4, 0) & (−1, 5) **(h)** (2, 2) & (−3, 4)

(i) (5,−1) & (−2,10) **(j)** (2, 1) & (6, 3) **(k)** (1, 5) & (3, 1) **(l)** (2, 0) & (4, 6)

Exercise 2: Simultaneous Equations

1. Solve the following pairs of simultaneous equations:

(a) $x + y = 4$, $x - y = 2$ **(b)** $x + y = 9$, $x - y = 5$ **(c)** $x + y = 7$, $x - y = 3$ **(d)** $x + y = 1$, $x - y = 3$

(e) $x + y = 3$, $x - y = 9$ **(f)** $x + y = -1$, $x - y = 9$ **(g)** $x + y = -5$, $x - y = -1$ **(h)** $x + y = -14$, $x - y = -8$

2. Solve the following pairs of simultaneous equations:

(a) $5x + 2y = 9$, $2x - 3y = -4$ **(b)** $4x + 5y = 7$, $7x - 3y = 24$ **(c)** $5x + 2y = 14$, $4x - 5y = -2$ **(d)** $3x + y = 16$, $2x + 3y = 13$

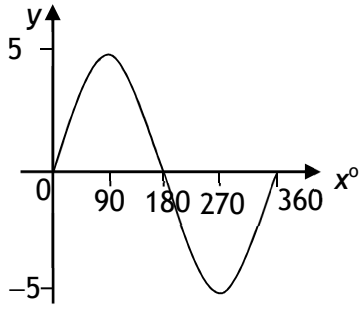
(e) $8x - 3y = 19$, $3x - 2y = 1$ **(f)** $5x + 3y = 19$, $7x - 4y = 43$ **(g)** $2x - 5y = 21$, $3x + 2y = 3$ **(h)** $2x - 3y = 17$, $7x - 4y = 40$

(i) $8x + 2y = 23$, $5x + 6y = 31$ **(j)** $2x + 3y = 7$, $4x + 5y = 12$ **(k)** $7x + 2y = 11$, $6x - 5y = -4$ **(l)** $7x - 5y = 35$, $9x - 4y = 45$

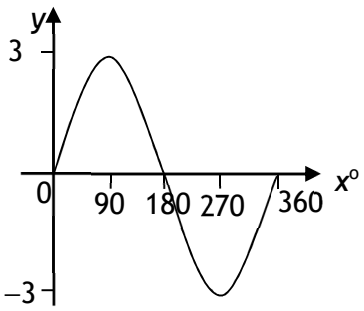
Exercise 3: Trig Graphs and Equations

1. The graphs represent the functions $a \sin x^\circ$ and $a \cos x^\circ$. Write down the equation for each.

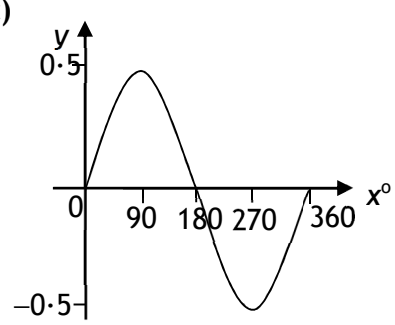
(a)



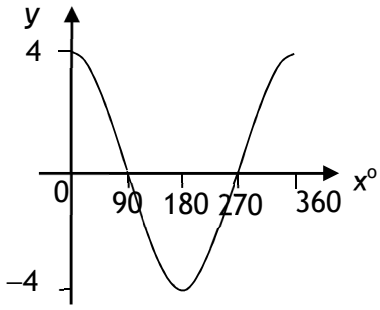
(b)



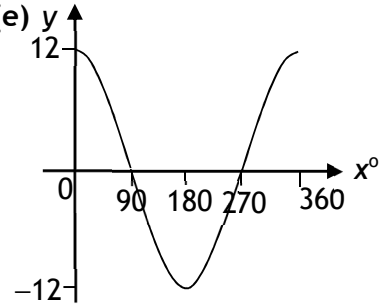
(c)



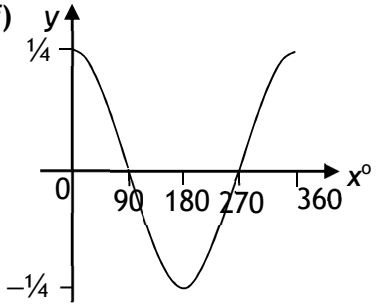
(d)



(e)

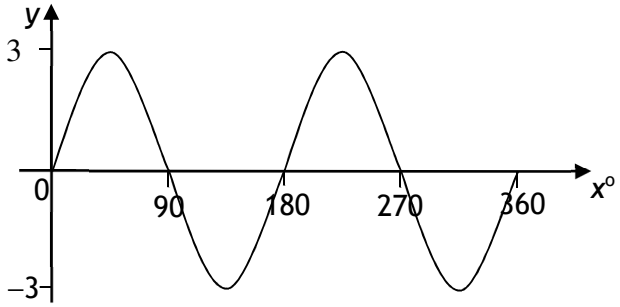


(f)

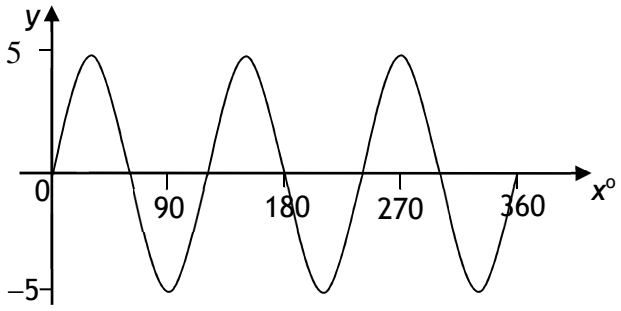


2. The graphs represent trigonometric functions. Write down the equation for each.

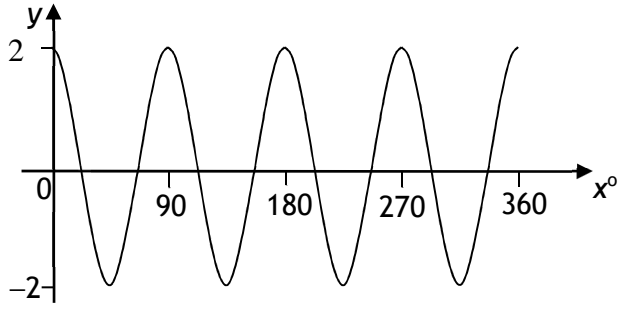
(a)



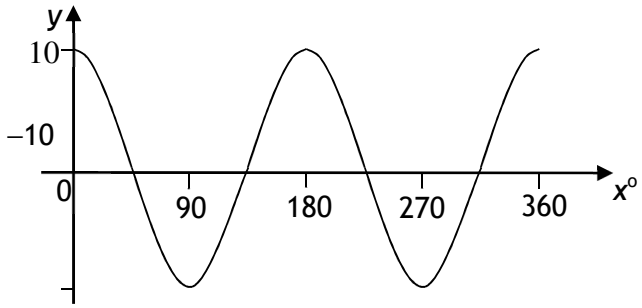
(b)



(c)

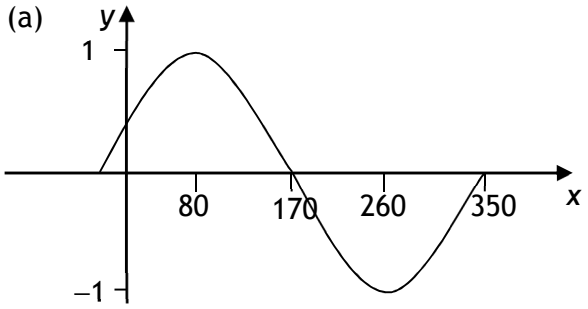


(d)

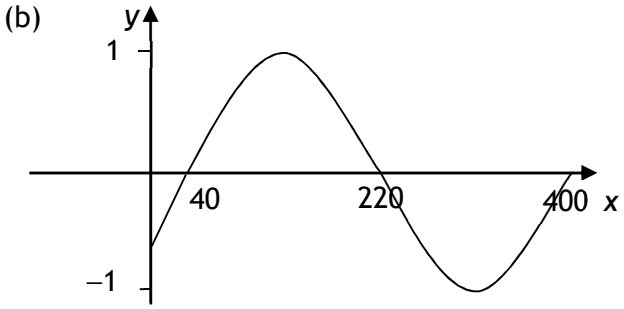


4. Write down the equation for each graph in the form y = sin $(x \pm a)^o$ or cos $(x \pm a)^o$

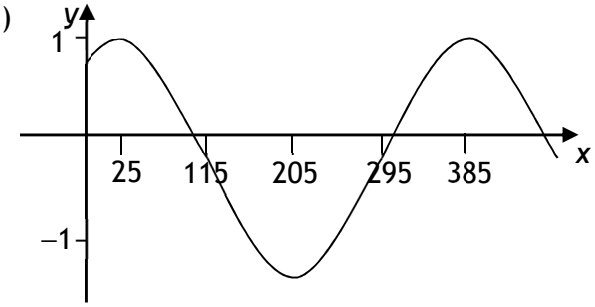
(a)



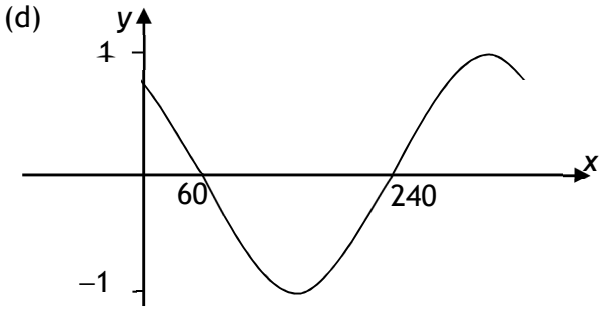
(b)



(c)

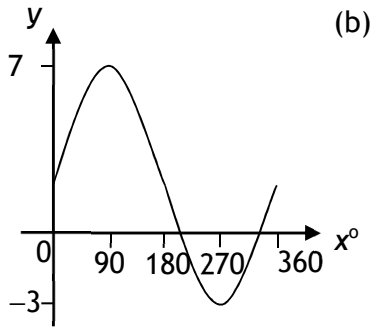


(d)

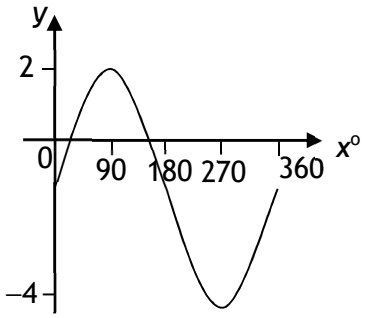


5. State the equation of each graph below.

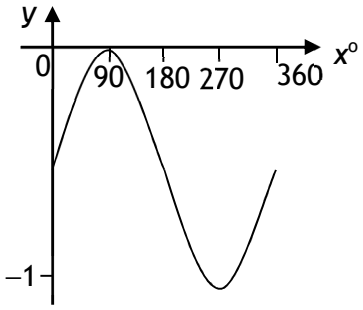
(a)



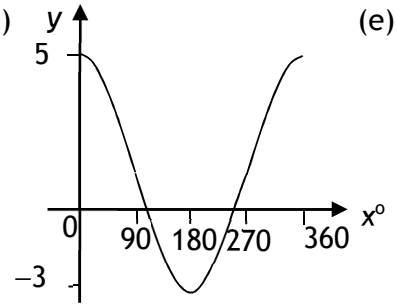
(b)



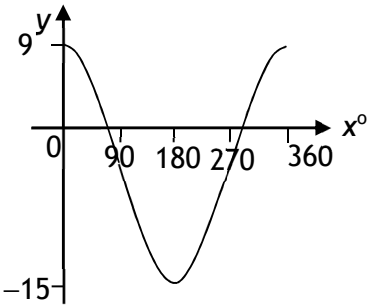
(c)



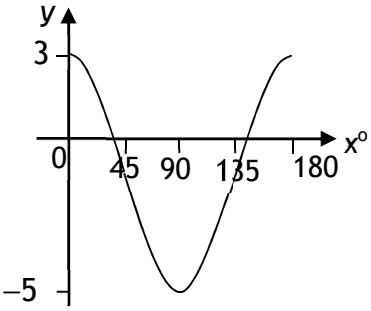
(d)



(e)



(f)



6. Solve the following equations where $0 \leq x \leq 360$

(a)	sin x^o = 0·5	**(b)**	cos x^o = 0·866	**(c)**	tan x^o = 1	**(d)**	cos x^o = − 0·5
(e)	tan x^o = − 0·577	**(f)**	sin x^o = − 0·866	**(g)**	tan x^o = 1·732	**(h)**	sin x^o = 0·707
(i)	cos x^o = 0·707	**(j)**	sin x^o = − 0·707	**(k)**	cos x^o = − 0·866	**(l)**	tan x^o = − 1·732

7. Solve the following equations where $0 \leq x \leq 360$

(a)	2 sin x^o = 1	**(b)**	3 cos x^o = 2	**(c)**	3 tan x^o = 5	**(d)**	2 cos x^o = − 1
(e)	2 tan x^o = − 8	**(f)**	4 sin x^o = − 3	**(g)**	5 tan x^o = 23·5	**(h)**	5 sin x^o = 2
(i)	6 cos x^o = 1	**(j)**	8 sin x^o = − 3	**(k)**	11 cos x^o = − 9	**(l)**	10 tan x^o = − 9

Exercise 4: Brackets and Factorising

1. Expand and simplify:

(a)	$3(3a - 1) + 2a$	**(b)**	$2(5x + 3) - 3x$	**(c)**	$8(b + 2) - 9$	**(d)**	$4(2h - 1) + 7$
(e)	$5(3 - 4x) + 11x$	**(f)**	$3(2c + 1) - 8$	**(g)**	$2(4t + 3) - 10t$	**(h)**	$p(p + q) - 3pq$
(i)	$7(1 - 3c) - 10$	**(j)**	$3 + 2(2x + 5)$	**(k)**	$7a + 3(2a - 3)$	**(l)**	$5 - 2(2x - 7)$

2. Remove the brackets:

(a)	$(x - 1)(x + 5)$	**(b)**	$(a + 3)(a - 7)$	**(c)**	$(t - 5)(t + 4)$	**(d)**	$(y + 8)(y - 4)$
(e)	$(c + 2)(c - 7)$	**(f)**	$(x - 6)(x + 1)$	**(g)**	$(b - 2)(b + 9)$	**(h)**	$(p - 10)(p + 2)$
(i)	$(y - 8)(y + 7)$	**(j)**	$(z + 4)(z - 6)$	**(k)**	$(x + 1)(x - 1)$	**(l)**	$(a + 2)(a - 15)$
(m)	$(c - 3)(c + 3)$	**(n)**	$(p - 7)(p + 1)$	**(o)**	$(b + 10)(b - 5)$		

3. Remove the brackets:

(a)	$(x + 3)^2$	**(b)**	$(w - 2)^2$	**(c)**	$(a - 5)^2$	**(d)**	$(c + 8)^2$	**(e)**	$(y - 4)^2$
(f)	$(a + 6)^2$	**(g)**	$(b + 1)^2$	**(h)**	$(s + 7)^2$	**(i)**	$(b - 9)^2$	**(j)**	$(x - 10)^2$
(k)	$(c - 1)^2$	**(l)**	$(y - 3)^2$	**(m)**	$(2x - 1)^2$	**(n)**	$(5y + 2)^2$	**(o)**	$(4b - 5)^2$

4. Expand and simplify:

(a)	$(x + 2)(x^2 + 3x + 1)$	**(b)**	$(x + 5)(x^2 + 4x + 2)$	**(c)**	$(x + 1)(x^2 + 5x + 4)$
(d)	$(x + 3)(x^2 + x + 5)$	**(e)**	$(x + 8)(x^2 + 2x + 3)$	**(f)**	$(x + 4)(x^2 + 7x + 6)$

5. Expand and simplify:

(a)	$3(x - 4) + (x + 2)^2$	**(b)**	$(2x - 1)(x + 3) + 2x(x - 3)$	**(c)**	$(2x + 3)^2 - 4(x + 1)$
(d)	$-(x + 2)^2 + 4x$	**(e)**	$-3(2x - 1)^2 + 12x^2$	**(f)**	$(x - 3)(x + 2) - (x + 4)^2$
(g)	$3x(x - 4) - (x + 2)(x - 4)$	**(h)**	$(x+2)^2 + (2x - 1)^2 - (x + 3)$	**(i)**	$(2x - 3)^2 - 4(x - 3)(2x+1)$

6. Factorise:

(a)	$x^2 + 3x + 2$	**(b)**	$a^2 + 2a + 1$	**(c)**	$y^2 + 5y + 4$	**(d)**	$x^2 + 8x + 7$
(e)	$b^2 - 6b + 5$	**(f)**	$x^2 - 15x + 14$	**(g)**	$c^2 - 10c + 16$	**(h)**	$x^2 - 7x + 6$
(e)	$q^2 + 2q - 8$	**(f)**	$x^2 - 8x - 20$	**(g)**	$d^2 + 4d - 21$	**(h)**	$c^2 + 9c - 36$
(i)	$p^2 - 5p - 24$	**(j)**	$y^2 - 7y - 8$	**(k)**	$a^2 + 5a - 6$	**(l)**	$x^2 - 5x - 36$

7. Factorise:

(a)	$3x^2 + 7x + 2$	**(b)**	$2a^2 + 5a + 2$	**(c)**	$3c^2 + 8c + 5$	**(d)**	$2p^2 + 11p + 9$
(e)	$5b^2 - 7b + 2$	**(f)**	$6x^2 - 7x + 2$	**(g)**	$4y^2 - 11y + 6$	**(h)**	$7c^2 - 29c + 4$
(i)	$6b^2 + 7b - 20$	**(j)**	$4t^2 - 4t - 3$	**(k)**	$12z^2 + 16z - 3$	**(l)**	$4d^2 - 4d - 15$
(m)	$7s^2 - 27s - 4$	**(n)**	$15x^2 + 16x - 15$	**(o)**	$36v^2 + v - 2$	**(p)**	$3v^2 + 10v + 7$

8. Factorise fully:

(a)	$3x^2 - 3$	**(b)**	$2p^2 + 12p + 10$	**(c)**	$9x^2 - 36$	**(d)**	$5x^2 + 25x + 30$
(e)	$ax^2 + 5ax + 6a$	**(f)**	$3y^2 - 12y - 15$	**(g)**	$15c^2 + 27c + 12$	**(h)**	$16b^2 + 28b + 6$
(i)	$9q^2 + 33q + 18$	**(j)**	$10s^2 - 35s + 15$	**(k)**	$8m^2 - 20m + 12$	**(l)**	$8a^2 - 36a + 36$
(m)	$4t^2 + 2t - 56$	**(n)**	$90d^2 - 60d - 80$	**(o)**	$400x^2 - 4$		

Exercise 5: Quadratic Equations

1. Solve these quadratic equations by factorising first.

(a)	$x^2 + 4x + 3 = 0$	**(b)**	$y^2 + 6y + 5 = 0$	**(c)**	$a^2 + 8a + 7 = 0$
(d)	$m^2 + 5m + 6 = 0$	**(e)**	$c^2 + 6c + 8 = 0$	**(f)**	$z^2 + 7z + 12 = 0$
(g)	$15 - 2x - x^2 = 0$	**(h)**	$b^2 - 8b + 16 = 0$	**(i)**	$x^2 - 7x + 10 = 0$

2. Solve these quadratic equations by factorising first.

(a)	$2x^2 + 7x + 5 = 0$	**(b)**	$2p^2 + 11p + 5 = 0$	**(c)**	$3t^2 + 10t + 3 = 0$
(d)	$3k^2 + 7k + 2 = 0$	**(e)**	$3y^2 + 8y + 5 = 0$	**(f)**	$6 - 7a - 5a^2 = 0$
(g)	$3 - 5w - 2w^2 = 0$	**(h)**	$3d^2 - 5d + 2 = 0$	**(i)**	$5x^2 - 16x + 3 = 0$
(j)	$3m^2 - 14m + 8 = 0$	**(k)**	$7 + 5c - 2c^2 = 0$	**(l)**	$1 - 5y - 6y^2 = 0$
(m)	$3x^2 - 2x = 1$	**(n)**	$4q^2 + 5q = 6$	**(o)**	$4t(t - 1) - 3 = 0$
(p)	$3m^2 + 2m = 5$	**(q)**	$36v^2 = -v + 2$	**(r)**	$7s^2 = 4 + 27s$

3. Solve accurate to 2 decimal places.

(a)	$x^2 + 5x + 5 = 0$	**(b)**	$b^2 + 9b + 2 = 0$	**(c)**	$p^2 + 4p + 1 = 0$
(d)	$c^2 + 4c + 2 = 0$	**(e)**	$y^2 + 7y + 3 = 0$	**(f)**	$a^2 + 8a + 5 = 0$
(g)	$z^2 - 5z + 1 = 0$	**(h)**	$q^2 - 12q + 4 = 0$	**(i)**	$w^2 - 6w + 2 = 0$
(j)	$3x^2 + 8x + 5 = 0$	**(k)**	$2b^2 + 9b + 3 = 0$	**(l)**	$2p^2 + 5p + 1 = 0$

4. For each equation below, find the discriminant and state the nature of the roots.

(a)	$x^2 + 4x + 3 = 0$	**(b)**	$x^2 + 6x + 9 = 0$	**(c)**	$x^2 + 8x + 7 = 0$	**(d)**	$3 - 5x - 2x^2 = 0$
(e)	$2x^2 + 7x + 5 = 0$	**(f)**	$x^2 - 12x + 36 = 0$	**(g)**	$x^2 - 7x + 12 = 0$	**(h)**	$2x^2 + 7x + 9 = 0$
(i)	$5x^2 - 16x + 3 = 0$	**(j)**	$6x^2 - 11x - 2 = 0$	**(k)**	$x^2 - 8x + 9 = 0$	**(l)**	$3x^2 + 2x + 7 = 0$

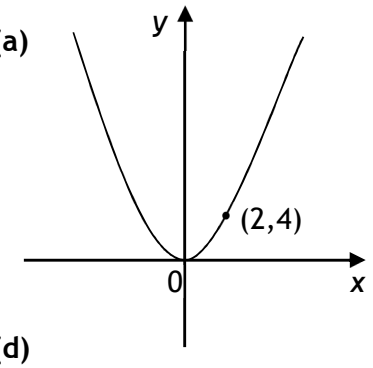
5. Find the value of *a* or *k* so that these quadratic equations have equal roots.

(a)	$x^2 - 4x - a = 0$	**(b)**	$2x^2 + 10x + a = 0$	**(c)**	$ax^2 - 2x + 5 = 0$
(d)	$ax^2 + (4a - 3)x + a = 0$	**(e)**	$3x^2 + 8x + a = 0$	**(f)**	$ax^2 - 7x - 5 = 0$
(g)	$kx^2 - 8x + 4 = 0$	**(h)**	$kx^2 + 6x + 18 = 0$	**(i)**	$x^2 - 2kx + 5 = 0$

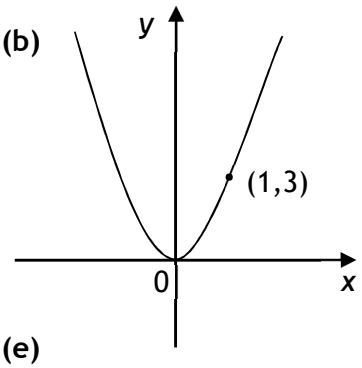
Exercise 6: Quadratic Graphs

1. Write down the equation of the graphs shown below, which have the form $y = kx^2$.

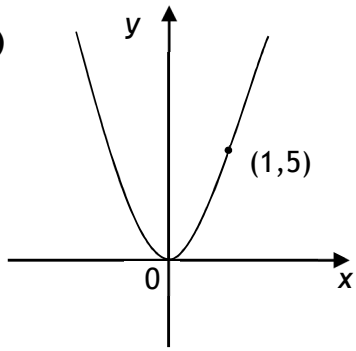
(a)



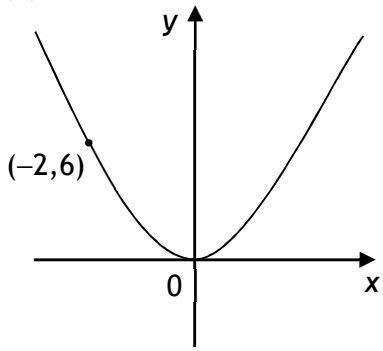
(b)



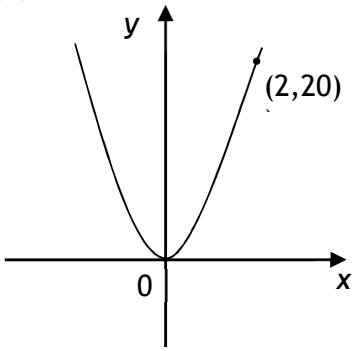
(c)



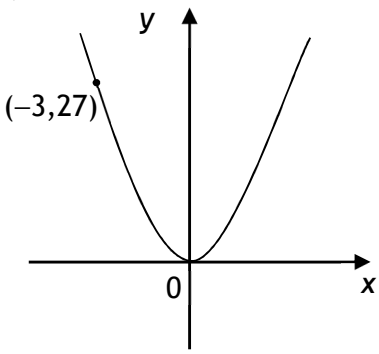
(d)



(e)

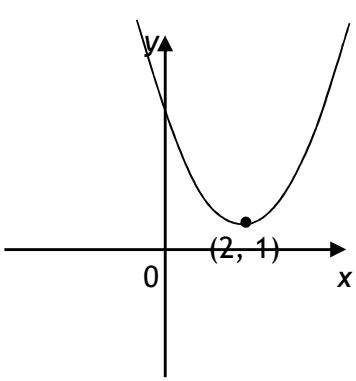


(f)

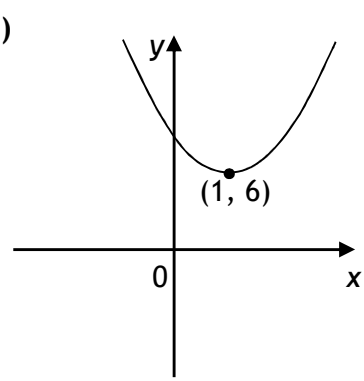


2. Write down the equation of the graphs shown below, which have the form $y = (x + a)^2 + b$.

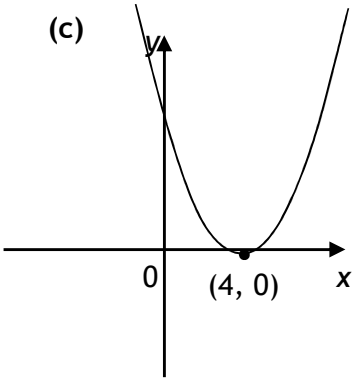
(a)



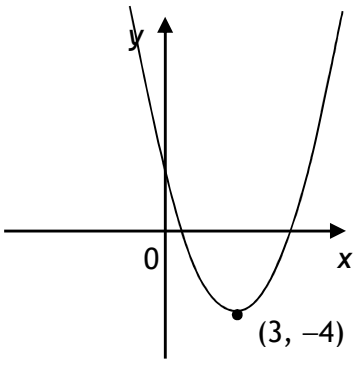
(b)



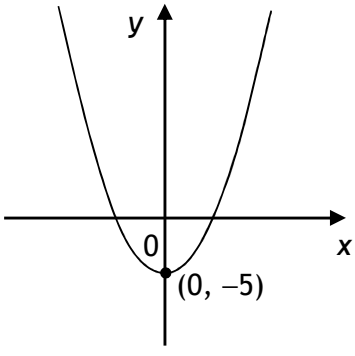
(c)



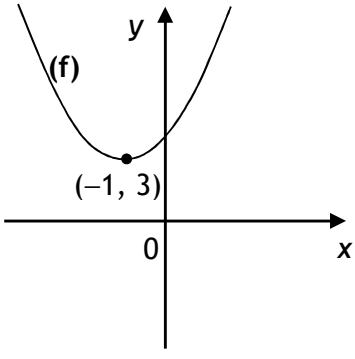
(d)



(e)



(f)



3. For each of the equations below: **(i)** write down the turning point
(ii) state its nature
(iii) write down the equation of the axis of symmetry

(a)	$y = (x - 4)^2 + 1$	**(b)**	$y = (x - 2)^2 + 5$	**(c)**	$y = (x - 1)^2 + 7$	**(d)**	$y = (x - 2)^2 - 3$
(e)	$y = (x - 3)^2 - 4$	**(f)**	$y = (x - 5)^2 - 2$	**(g)**	$y = (x + 4)^2 + 6$	**(h)**	$y = (x + 1)^2 + 5$
(i)	$y = (x + 8)^2 + 1$	**(j)**	$y = (x + 3)^2 - 1$	**(k)**	$y = (x + ½)^2 - ¾$	**(l)**	$y = (x + 0{\cdot}5)^2 - 2{\cdot}5$
(m)	$y = -(x - 1)^2 + 4$	**(n)**	$y = -(x + 6)^2 + 3$	**(o)**	$y = -(x + 7)^2 - 2$	**(p)**	$y = (2 - x)^2 + 12$
(q)	$y = (5 - x)^2 - 1$	**(r)**	$y = (4 - x)^2 + 3{\cdot}75$				

4. Sketch the graphs with the following equations

(a)	$y = (x - 4)^2 + 1$	**(b)**	$y = (x - 2)^2 + 5$	**(c)**	$y = (x - 1)^2 + 7$	**(d)**	$y = (x - 2)^2 - 3$
(e)	$y = (x - 3)^2 - 4$	**(f)**	$y = (x - 5)^2 - 2$	**(g)**	$y = (x + 4)^2 + 6$	**(h)**	$y = (x + 1)^2 + 5$
(i)	$y = (x + 8)^2 + 1$	**(j)**	$y = (x + 3)^2 - 1$	**(k)**	$y = (x + ½)^2 - ¾$	**(l)**	$y = (x + 0{\cdot}5)^2 - 2{\cdot}5$

5. Sketch the graphs with the following equations

(a)	$y = (x - 1)(x - 5)$	**(b)**	$y = (x - 4)(x - 2)$	**(c)**	$y = (x - 3)(x - 7)$	**(d)**	$y = x^2 - 14x + 48$
(e)	$y = x^2 - 7x + 10$	**(f)**	$y = x^2 - 13x + 40$	**(g)**	$y = (x + 2)(x + 3)$	**(h)**	$y = (x + 5)(x + 2)$
(i)	$y = (x + 4)(x + 6)$	**(j)**	$y = x^2 + 7x + 12$	**(k)**	$y = x^2 + 14x + 45$	**(l)**	$y = x^2 + 11x + 24$

6. Write the following in the form $(x + a)^2 + b$ and write down the minimum value of each one.

(a)	$y = x^2 + 4x$	**(b)**	$y = x^2 + 10x$	**(c)**	$y = x^2 + 7x$	**(d)**	$y = x^2 + 9x$
(e)	$y = x^2 - 6x$	**(f)**	$y = x^2 - 8x$	**(g)**	$y = x^2 - 5x$	**(h)**	$y = x^2 - 11x$
(i)	$y = x^2 + 2x + 7$	**(j)**	$y = x^2 + 6x + 2$	**(k)**	$y = x^2 + 8x + 19$	**(l)**	$y = x^2 + 10x + 27$

Exercise 7: Surds and Indices

1. Express each of the following in its simplest form:

(a)	$\sqrt{8}$	**(b)**	$\sqrt{12}$	**(c)**	$\sqrt{50}$	**(d)**	$\sqrt{20}$	**(e)**	$\sqrt{24}$	**(f)**	$\sqrt{108}$
(g)	$\sqrt{60}$	**(h)**	$\sqrt{72}$	**(i)**	$\sqrt{300}$	**(j)**	$\sqrt{27}$	**(k)**	$\sqrt{96}$	**(l)**	$\sqrt{48}$
(m)	$\sqrt{45}$	**(n)**	$\sqrt{98}$	**(o)**	$\sqrt{90}$	**(p)**	$\sqrt{18}$	**(q)**	$\sqrt{28}$	**(r)**	$\sqrt{80}$

2. Express each of the following in its simplest form:

(a)	$\sqrt{12} + \sqrt{27}$	**(b)**	$\sqrt{32} - \sqrt{8}$	**(c)**	$\sqrt{72} - \sqrt{50}$	**(d)**	$\sqrt{2} + \sqrt{98}$
(e)	$\sqrt{80} + \sqrt{20}$	**(f)**	$\sqrt{24} + \sqrt{54}$	**(g)**	$\sqrt{180} - \sqrt{45}$	**(h)**	$\sqrt{1000} - \sqrt{90}$
(i)	$\sqrt{50} - \sqrt{8}$	**(j)**	$\sqrt{3} - \sqrt{12}$	**(k)**	$\sqrt{75} + \sqrt{108} - \sqrt{3}$	**(l)**	$\sqrt{5} + \sqrt{20} + \sqrt{80}$

3. Simplify:

(a)	$\sqrt{5} \times \sqrt{5}$	**(b)**	$\sqrt{2} \times \sqrt{2}$	**(c)**	$\sqrt{11} \times \sqrt{11}$	**(d)**	$\sqrt{a} \times \sqrt{a}$	**(e)**	$\sqrt{6} \times \sqrt{6}$
(f)	$\sqrt{c} \times \sqrt{c}$	**(g)**	$\sqrt{k} \times \sqrt{k}$	**(h)**	$\sqrt{3} \times \sqrt{6}$	**(i)**	$\sqrt{8} \times \sqrt{2}$	**(j)**	$\sqrt{6} \times \sqrt{2}$
(k)	$\sqrt{3} \times \sqrt{5}$	**(l)**	$\sqrt{x} \times \sqrt{y}$	**(m)**	$\sqrt{2} \times \sqrt{8}$	**(n)**	$\sqrt{12} \times \sqrt{3}$	**(o)**	$\sqrt{5} \times \sqrt{20}$

4. Simplify:

(a) $\frac{\sqrt{8}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{27}}{\sqrt{12}}$ **(c)** $\frac{\sqrt{2}}{\sqrt{32}}$ **(d)** $\frac{\sqrt{3}}{\sqrt{27}}$ **(e)** $\frac{\sqrt{20}}{\sqrt{5}}$ **(f)** $\frac{\sqrt{12}}{\sqrt{48}}$

(g) $\frac{\sqrt{54}}{\sqrt{24}}$ **(h)** $\frac{\sqrt{175}}{\sqrt{63}}$ **(i)** $\frac{\sqrt{18}}{\sqrt{72}}$ **(j)** $\frac{\sqrt{6}}{\sqrt{54}}$ **(k)** $\frac{\sqrt{288}}{\sqrt{8}}$ **(l)** $\frac{\sqrt{1000}}{\sqrt{90}}$

(m) $\frac{\sqrt{48}}{\sqrt{6}}$ **(n)** $\frac{\sqrt{3}}{\sqrt{24}}$ **(o)** $\frac{\sqrt{98}}{\sqrt{7}}$ **(p)** $\frac{\sqrt{50}}{\sqrt{250}}$

5. Expand and simplify where possible:

(a) $(\sqrt{2} + 3)(\sqrt{2} - 1)$ **(b)** $(\sqrt{5} + 1)(2\sqrt{5} - 4)$ **(c)** $(2\sqrt{2} + 3)(\sqrt{2} + 4)$ **(d)** $(\sqrt{3} + 1)(\sqrt{3} - 1)$

(e) $(2 + \sqrt{5})(2 - \sqrt{5})$ **(f)** $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$ **(g)** $(\sqrt{2} - 4)(3\sqrt{2} - 1)$ **(h)** $(\sqrt{8} + 2)(\sqrt{8} + 1)$

(i) $(\sqrt{2} + 3)^2$ **(j)** $(2\sqrt{3}+\sqrt{2})(\sqrt{3} + 3\sqrt{2})$ **(k)** $(\sqrt{2} + \sqrt{3})^2$ **(l)** $(2\sqrt{3} - 1)^2$

6. Express each of the following with a *rational denominator* and simplify where possible:

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{3}}$ **(c)** $\frac{1}{\sqrt{5}}$ **(d)** $\frac{6}{\sqrt{3}}$ **(e)** $\frac{10}{\sqrt{5}}$ **(f)** $\frac{2}{\sqrt{3}}$

(g) $\frac{3}{\sqrt{5}}$ **(h)** $\frac{20}{\sqrt{2}}$ **(i)** $\frac{2}{\sqrt{2}}$ **(j)** $\frac{12}{\sqrt{3}}$ **(k)** $\frac{3}{\sqrt{6}}$ **(l)** $\frac{4}{\sqrt{5}}$

(m) $\frac{10}{\sqrt{2}}$ **(n)** $\frac{35}{\sqrt{7}}$ **(o)** $\frac{1}{2\sqrt{5}}$ **(p)** $\frac{4}{5\sqrt{2}}$ **(q)** $\frac{3}{3\sqrt{2}}$ **(r)** $\frac{12}{5\sqrt{6}}$

7. Write each of the following in its simplest index form.

(a) $3^4 \times 3^2$ **(b)** 2×2^3 **(c)** $10^5 \times 10^2$ **(d)** $8^3 \times 8^5$ **(e)** $7^6 \times 7$

(f) $5^4 \times 5^4$ **(g)** $9^6 \times 9^2$ **(h)** $6^8 \times 6^5$ **(i)** $x^3 \times x^5$ **(j)** $c^2 \times c^9$

(k) $a^2 \times a^{12}$ **(l)** $y^5 \times y^5$ **(m)** $b^{10} \times b^{30}$ **(n)** $p \times p^9$ **(o)** $d^2 \times d^4$

(p) $q^{11} \times q^9$ **(q)** $t^3 \times t^7$ **(r)** $f^4 \times f^3$ **(s)** $k \times k^{12}$ **(t)** $z^{50} \times z^{50}$

(u) $x^{30} \times x^{50}$ **(v)** $y^{19} \times y$ **(w)** $a^{25} \times a^{65}$ **(x)** $b^1 \times b^0$

8. Write each of the following in its simplest index form.

(a) $2^8 \div 2^3$ **(b)** $5^4 \div 5^2$ **(c)** $12^9 \div 12^6$ **(d)** $7^{11} \div 7^4$ **(e)** $20^5 \div 20$

(f) $8^8 \div 8^4$ **(g)** $3^{18} \div 3^3$ **(h)** $4^{15} \div 4^{13}$ **(i)** $x^7 \div x^2$ **(j)** $a^9 \div a^5$

(k) $y^{20} \div y^{10}$ **(l)** $b^4 \div b^1$ **(m)** $p^{12} \div p^{11}$ **(n)** $c^7 \div c^7$ **(o)** $q^8 \div q^2$

(p) $d^4 \div d$ **(q)** $\frac{x^9}{x^3}$ **(r)** $\frac{a^8}{a^2}$ **(s)** $\frac{m^{14}}{m}$ **(t)** $\frac{s^7}{s^7}$

(u) $\frac{d^{20}}{d^{12}}$ **(v)** $\frac{y^{100}}{y^{10}}$ **(w)** $\frac{t^{100}}{t}$ **(x)** $\frac{w^{10}}{w^0}$

9. Write each of the following in its simplest index form.

(a) $(3^2)^4$ **(b)** $(8^2)^2$ **(c)** $(10^3)^2$ **(d)** $(2^2)^5$ **(e)** $(4^5)^3$ **(f)** $(1^7)^2$

(g) $(12^3)^3$ **(h)** $(5^5)^5$ **(i)** $(x^4)^2$ **(j)** $(y^8)^5$ **(k)** $(a^3)^7$ **(l)** $(m^4)^4$

(m) $(b^3)^6$ **(n)** $(p^5)^3$ **(o)** $(k^5)^{20}$ **(p)** $(z^6)^0$

10. Write the following without brackets.

(a) $(2b)^2$ **(b)** $(7a)^3$ **(c)** $(3x)^4$ **(d)** $(2y)^5$ **(e)** $(ab)^4$ **(f)** $(xy)^7$

(g) $(wz)^5$ **(h)** $(st)^3$ **(i)** $(pq^2)^3$ **(j)** $(x^4y)^2$ **(k)** $(a^2b^3)^5$ **(l)** $(6a^5)^2$

(m) $(10x^2)^3$ **(n)** $(2c^4)^5$ **(o)** $(3ab^2)^3$ **(p)** $(4m^2k)^2$

11. Simplify these expressions.

(a) $2a^3 \times 5a^5$ **(b)** $7x \times 9x^8$ **(c)** $12p^7 \div 4p^4$ **(d)** $50b^{12} \div 10b^6$

(e) $3y \times (2y^2)^3$ **(f)** $(4q^3)^2 \times 5q^4$ **(g)** $(4c^3)^3 \div 8c^2$ **(h)** $72z^{12} \div (3z^4)^2$

(i) $k^2(k^3 + k^5)$ **(j)** $m^5(m^2 - m^3)$ **(k)** $2x^4(x^3 + 3x^2)$ **(l)** $5a^5(2a^2 - 3a^3)$

(m) $\dfrac{x^5 \times x^4}{x^6}$ **(n)** $\dfrac{(m^5)^4}{m^6}$ **(o)** $\dfrac{5c^3 \times 4c^7}{2c^6}$ **(p)** $\dfrac{(3q^3)^2 \times 4q^4}{6q^7}$

(q) $\dfrac{(3xy^5)^3}{9x^2y}$ **(r)** $\dfrac{(2a^2b^5)^6}{(4ab)^2}$ **(s)** $\dfrac{(4p^4)^3}{2p^3 \times 8p^6}$ **(t)** $\dfrac{(2ab^3)^5}{3a^2b \times 4ab^2}$

12. Rewrite the following with positive indices.

(a) 3^{-2} **(b)** 5^{-4} **(c)** 2^{-6} **(d)** 10^{-3} **(e)** 4^{-5} **(f)** 200^{-7}

(g) a^{-5} **(h)** x^{-2} **(i)** p^{-7} **(j)** y^{-10} **(k)** $2b^{-3}$ **(l)** $10\,q^{-x}$

(m) $\dfrac{1}{x^{-3}}$ **(n)** $\dfrac{1}{w^{-5}}$ **(o)** $\dfrac{3}{a^{-2}}$ **(p)** $\dfrac{10}{c^{-8}}$ **(q)** $\dfrac{2}{3t^{-1}}$ **(r)** $\dfrac{5}{4y^{-3}}$

13. Rewrite the following with negative indices.

(a) $\dfrac{1}{3^2}$ **(b)** $\dfrac{1}{6^9}$ **(c)** $\dfrac{1}{5^4}$ **(d)** $\dfrac{1}{2^7}$ **(e)** $\dfrac{1}{10^3}$ **(f)** $\dfrac{1}{4^4}$

(g) $\dfrac{1}{x^3}$ **(h)** $\dfrac{1}{a^5}$ **(i)** $\dfrac{1}{p^4}$ **(j)** $\dfrac{1}{y^{10}}$ **(k)** $\dfrac{1}{q^6}$ **(l)** $\dfrac{1}{c^8}$

14. Write the following in surd form.

(a) $x^{\frac{1}{2}}$ **(b)** $y^{\frac{1}{3}}$ **(c)** $a^{\frac{1}{4}}$ **(d)** $y^{\frac{2}{3}}$ **(e)** $b^{\frac{3}{4}}$ **(f)** $x^{\frac{5}{3}}$ **(g)** $c^{\frac{3}{5}}$

(h) $c^{-\frac{1}{3}}$ **(i)** $z^{-\frac{1}{2}}$ **(j)** $m^{-\frac{2}{3}}$ **(k)** $k^{-\frac{3}{5}}$ **(l)** $p^{-\frac{4}{3}}$ **(m)** $x^{-\frac{5}{3}}$ **(n)** $w^{-\frac{4}{5}}$

15. Write the following in index form.

(a) $\sqrt{x}$ **(b)** $\sqrt[3]{a}$ **(c)** $\sqrt{y^3}$ **(d)** $\sqrt[3]{z^2}$ **(e)** $\sqrt[3]{c^2}$ **(f)** $\sqrt[4]{x^3}$ **(g)** $\sqrt[3]{p^5}$

(h) $\frac{1}{\sqrt{a}}$ **(i)** $\frac{1}{\sqrt[3]{z}}$ **(j)** $\frac{1}{\sqrt[3]{x^4}}$ **(k)** $\frac{1}{\sqrt{a^5}}$ **(l)** $\frac{1}{\sqrt[3]{b^2}}$ **(m)** $\frac{1}{\sqrt[5]{m^3}}$ **(n)** $\frac{1}{\sqrt[3]{c^5}}$

16. Find the value of

(a) $16^{\frac{1}{4}}$ **(b)** $8^{\frac{1}{3}}$ **(c)** $36^{\frac{1}{2}}$ **(d)** $27^{\frac{2}{3}}$ **(e)** $64^{\frac{1}{3}}$ **(f)** $1000^{\frac{1}{3}}$

(g) $25^{\frac{1}{2}}$ **(h)** $81^{\frac{3}{4}}$ **(i)** $125^{\frac{2}{3}}$ **(j)** $64^{\frac{1}{2}}$ **(k)** $216^{\frac{1}{3}}$ **(l)** $16^{-\frac{1}{4}}$

17. Simplify each of the following by ……..
(i) changing root signs to fractional powers;
(ii) moving *x*'s onto the numerators;
(iii) expanding brackets …… where necessary.

(a) $x^{\frac{1}{2}}(x^4+1)$ **(b)** $x^{-\frac{1}{2}}(x^{\frac{3}{2}}-x^2)$ **(c)** $\frac{1}{x^2}(x^{\frac{1}{2}}+x)$ **(d)** $\frac{2}{x^{-3}}(x^2+\frac{1}{x})$

(e) $\frac{1}{\sqrt{x}}(x^2-\sqrt{x})$ **(f)** $\left(x^2+\frac{1}{x}\right)^2$ **(g)** $\frac{1}{x}(\sqrt{x}+x)$ **(h)** $\left(x+\frac{1}{\sqrt{x}}\right)^2$

(i) $x^{-2}\left(\frac{1}{x}-\sqrt[3]{x}\right)$ **(j)** $\frac{x^2+3}{x}$ **(k)** $\frac{\sqrt{x}-x}{x^2}$ **(l)** $\frac{(2x+1)^2}{x^{\frac{3}{2}}}$

Exercise 8: Vectors

1. **(i)** Draw diagrams on squared to illustrate $\boldsymbol{a}$ + $\boldsymbol{b}$ for each the following pairs of vectors.
(ii) State the components of the resultant vector and calculate its magnitude.

(a) $\boldsymbol{a}=\begin{pmatrix}3\\5\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}3\\-4\end{pmatrix}$ **(b)** $\boldsymbol{a}=\begin{pmatrix}4\\7\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-9\\3\end{pmatrix}$ **(c)** $\boldsymbol{a}=\begin{pmatrix}-4\\-2\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}6\\-5\end{pmatrix}$

(d) $\boldsymbol{a}=\begin{pmatrix}0\\-5\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-3\\3\end{pmatrix}$ **(e)** $\boldsymbol{a}=\begin{pmatrix}-6\\-4\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-5\\6\end{pmatrix}$ **(f)** $\boldsymbol{a}=\begin{pmatrix}4\\0\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}0\\-3\end{pmatrix}$

(g) $\boldsymbol{a}=\begin{pmatrix}0\\5\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}3\\0\end{pmatrix}$ **(h)** $\boldsymbol{a}=\begin{pmatrix}-3\\4\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}2\\-4\end{pmatrix}$

2. **(i)** Draw diagrams on squared to illustrate $\boldsymbol{a}$ – $\boldsymbol{b}$ for each the following pairs of vectors.
(ii) State the components of the resultant vector and calculate its magnitude correct to one decimal place.

(a) $\boldsymbol{a}=\begin{pmatrix}9\\7\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}8\\4\end{pmatrix}$ **(b)** $\boldsymbol{a}=\begin{pmatrix}-4\\-7\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-1\\4\end{pmatrix}$ **(c)** $\boldsymbol{a}=\begin{pmatrix}5\\8\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-2\\-1\end{pmatrix}$

(d) $\boldsymbol{a}=\begin{pmatrix}2\\4\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}2\\1\end{pmatrix}$ **(e)** $\boldsymbol{a}=\begin{pmatrix}-2\\-4\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-2\\-6\end{pmatrix}$ **(f)** $\boldsymbol{a}=\begin{pmatrix}4\\-3\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-4\\0\end{pmatrix}$

(g) $\boldsymbol{a}=\begin{pmatrix}0\\7\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-1\\0\end{pmatrix}$ **(h)** $\boldsymbol{a}=\begin{pmatrix}0\\-6\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}-4\\5\end{pmatrix}$ **(i)** $\boldsymbol{a}=\begin{pmatrix}3\\-1\end{pmatrix}$; $\boldsymbol{b}=\begin{pmatrix}0\\3\end{pmatrix}$

3. Calculate the magnitude of each of these vectors giving answers to one decimal place:

(a) $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ **(b)** $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix}$ **(c)** $\boldsymbol{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ **(d)** $\boldsymbol{t} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$

(e) $\boldsymbol{u} = \begin{pmatrix} 6 \\ -1 \\ -4 \end{pmatrix}$ **(f)** $\boldsymbol{q} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **(g)** $\boldsymbol{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ **(h)** $\boldsymbol{b} = \begin{pmatrix} 5 \\ -12 \\ 0 \end{pmatrix}$

4. Calculate the magnitude of these vectors , leaving you answer a surd in its in simplest form.

(a) $\boldsymbol{u} = \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$ **(b)** $AB = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ **(c)** $\boldsymbol{t}$ where point T has coordinates ($\sqrt{3}$, $\sqrt{5}$, $2\sqrt{2}$)

5. $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are 3 vectors with components $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ -1 \end{pmatrix}$ respectively.

(i) Find the components of the following:

(a) $2\boldsymbol{u} + 3\boldsymbol{v}$ **(b)** $3\boldsymbol{u} - 6\boldsymbol{v}$ **(c)** $3\boldsymbol{w} + 2\boldsymbol{v}$ **(d)** $4\boldsymbol{u} - 2\boldsymbol{w}$
(e) $-3\boldsymbol{u} - 4\boldsymbol{v}$ **(f)** $3\boldsymbol{w} - 4\boldsymbol{u}$ **(g)** $3\boldsymbol{u} - 6\boldsymbol{v} + 2\boldsymbol{w}$ **(h)** $2\boldsymbol{u} + 3\boldsymbol{v} - 4\boldsymbol{w}$

(ii) Calculate the magnitude of each resultant vector above giving answers to 1 decimal place.