Exercise 15: Solving simultaneous equations algebraically

1. Solve each pair of equations below using the method of substitution.

   (a) $y = x$ and $3x - y = 10$ (b) $y = x$ and $5x - y = 4$

   (c) $y = 2x$ and $5x + y = 14$ (d) $y = 2x$ and $2x + 3y = 24$

   (e) $y = 3x + 1$ and $y = x + 7$ (f) $y = 5x - 4$ and $y = 2x + 11$

2. Solve the following pairs of simultaneous equations:

   (a) $x + y = 4$ (b) $x + y = 9$ (c) $x + y = 7$ (d) $x + y = 1$
   $\quad\;\; x - y = 2$ $x - y = 5$ $x - y = 3$ $x - y = 3$

   (e) $x + y = 3$ (f) $x + y = -1$ (g) $x + y = -5$ (h) $x + y = -14$
   $\quad\;\; x - y = 9$ $x - y = 9$ $x - y = -1$ $x - y = -8$

3. Solve the following pairs of simultaneous equations:

   (a) $2x + y = 15$ (b) $3x + 2y = 32$ (c) $5x + 3y = 26$ (d) $3x + y = 9$
   $\quad\;\; x - y = 6$ $x - 2y = 8$ $2x - 3y = 2$ $x + y = 5$

   (e) $4x + y = 11$ (f) $7x + 2y = 36$ (g) $2x - 5y = -21$ (h) $3x + 8y = 23$
   $\quad\;\; 2x + y = 5$ $2x + 2y = 16$ $3x + 10y = 56$ $x - 4y = 1$

   (i) $3x + 4y = 10$ (j) $5x - 2y = 16$ (k) $7x + 3y = -13$ (l) $3x - 5y = 8$
   $\quad\;\; 6x + 5y = 17$ $3x + 4y = 20$ $3x + y = -5$ $x - 7y = 8$

4. Solve the following pairs of simultaneous equations:

   (a) $5x + 2y = 9$ (b) $4x + 5y = 7$ (c) $5x + 2y = 14$ (d) $3x + y = 16$
   $\quad\;\; 2x - 3y = -4$ $7x - 3y = 24$ $4x - 5y = -2$ $2x + 3y = 13$

   (e) $8x - 3y = 19$ (f) $5x + 3y = 19$ (g) $2x - 5y = 21$ (h) $2x - 3y = 17$
   $\quad\;\; 3x - 2y = 1$ $7x - 4y = 43$ $3x + 2y = 3$ $7x - 4y = 40$

   (i) $8x + 2y = 23$ (j) $2x + 3y = 7$ (k) $7x + 2y = 11$ (l) $7x - 5y = 35$
   $\quad\;\; 5x + 6y = 31$ $4x + 5y = 12$ $6x - 5y = -4$ $9x - 4y = 45$

5. Solve the following pairs of simultaneous equations:

   (a) $2x + 4y = 24$ (b) $4a - 3b = 18$ (c) $2e + 7f = 26$ (d) $5x + y = -2$
   $\quad\;\; 7x - 2y = 4$ $2a + 6b = -6$ $8e - 5f = 38$ $3x + 2y = 3$

   (e) $2x - 3y = 10$ (f) $4p + 3q = 1$ (g) $2g + 3h = 1$ (h) $-2x + 3y = 6$
   $\quad\;\; 3x - 6y = 18$ $8p + 5q = -1$ $5g - 2h = -26$ $9x - 7y = -1$

| **(i)**	$2u + 4v = -16$ $11u - 7v = -1$	**(j)**	$2x - 8y = 0$ $5x - 5y = 15$	**(k)**	$3p + 2q = -11$ $4p + 3q = -14$	**(l)**	$10a - 3b = 46$ $6a - 8b = 40$
(m)	$x + 3y = 17$ $3x - 2y = -4$	**(n)**	$a - 3b = 6$ $3a + b = 8$	**(o)**	$2e + f = 1$ $5e - 2f = -20$	**(p)**	$5x + 3y = 7$ $4x + y = 0$
(q)	$2x - 5y = -14$ $x - 2y = -5$	**(r)**	$2p + 3q = 6$ $4p + q = -8$	**(s)**	$2g + h = 11$ $7g - 8h = 96$	**(t)**	$3x - 2y = 25$ $x + 5y = -3$
(u)	$u - 4v = 10$ $9u - 2v = 22$	**(v)**	$2x = 3y + 5$ $x + 5y = 9$	**(w)**	$3p - 2q + 7 = 0$ $4p + q = -2$	**(x)**	$4a + b - 30 = 0$ $6a = 38 - 5b$

Exercise 16: Simultaenous equations in context

1. Find two numbers whose sum is 56 and whose difference is 16.

2. Four chocolate bars and six packets of crisps together cost £3.40.

Ten chocolate bars and three packets of crisps cost £4.90.

Form simultaneous equations and solve them to find the cost
each packet of crisps and each bar of chocolate.
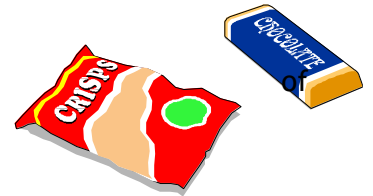
3. Four sandwiches and 3 hot-dogs cost £7.50.

Two sandwiches and 4 hot-dogs cost £6.

Form simultaneous equations and solve them

to find the cost of each sandwich and hot-dog.
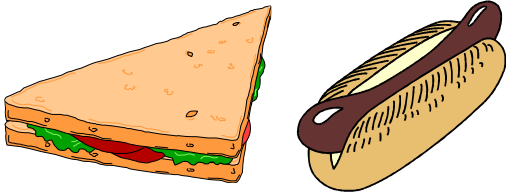
4. At *Smith's Stationers*, the cost of a ruler and a pencil together is 57p. The ruler costs 23p more
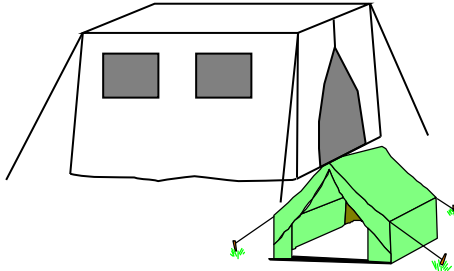than the pencil.

Find the cost of each.

5. A photographer produces 2 sizes of print, Standard and Jumbo.

A customer who orders 24 standard and 5 jumbo prints pays £7.79

Another customer pays £8.60 for 20 standard and 8 jumbo prints.

How much would I have to pay for 1 standard and 1 jumbo print ?
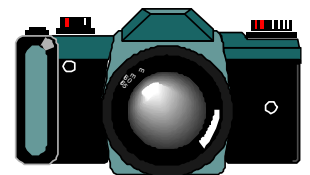
6. There are 2 types of ticket on sale for a football match – Side Stand and Centre Stand.

You pay £71.75 for 4 Side and 3 Centre tickets.

Your friend pays £75.25 for 2 Side and 5 Centre tickets.

What is the price for each type of ticket?
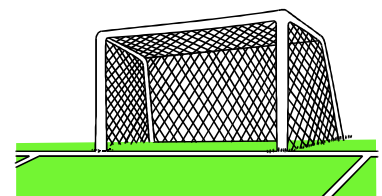
7. Two small wine glasses and five large wine glasses together contain 915 ml.

One small glass and three large glasses together hold 530 ml.

How much does each glass hold?

8. On a camping holiday a group of 30 students take 3 frame tents and 2 ridge tents.

Another group of 25 students take 2 frame tents and 3 ridge tents.

How many people does each type of tent hold ?

9. A magazine pays different rates for *Star Letters* and *Readers' Letters*.

In June the magazine editor paid out £195 for 3 Star Letters and 8 Readers' Letters.

In July £215 was paid out for 2 Star Letters and 11 Readers' Letters.

How much does the magazine pay for each type of letter?

10. Brian is a potter and is making 2 different sizes of vase.

Five small vases and four large ones require 17 kg of clay.

Three small vases and two large vases take 9·4 kg of clay.

How much clay is needed for each size of vase?

11. Look at the two rectangles opposite.

The smaller one has a perimeter of 60cm.

The larger one has a perimeter of twice the smaller.
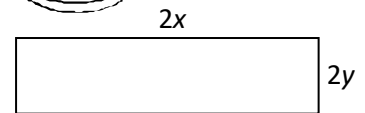
   (a) Form two equations and solve them simultaneously to find the values of *x* and *y*.

   (b) Hence calculate the area of the smaller rectangle.

$2x$

$2y$

$3x$

$8y$
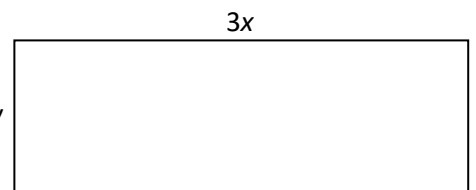
12. A van is carrying eight identical boxes and five identical parcels.

   (a) If 3 boxes and 2 parcels weigh a total of 22kg and 4 boxes and 3 parcels weigh 30kg, find the weight of an individual box and a single parcel.

   (b) What is the total weight carried by the van?
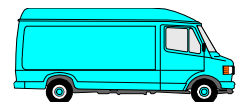
13. 3 pounds of butter and 4 pints of milk costs £3.84.

5 pounds of butter and 7 pints of milk costs £6.48.

Find the cost of a pound of butter and a single pint of milk.

14. In a certain factory, the basic rate of pay is £4.50 per hour, with overtime at £6.40.

Paul's total wage for a certain week was £215.80.

If he worked a total of 45 hours in all, how many hours did he work at the basis rate?

15. At a concert 500 tickets were sold. Cheap tickets cost £5 whereas more expensive ones cost £9.

If the total receipts were £3 220, how many cheap tickets were sold?

16. John saves money by putting every 50p and every 20p coin he receives in a box. After a while he discovers that he has 54 coins amounting to £17.10.

How many of each coin does he have?

Exercise 17: Further simultaneous equations in context

1. A small printing company sends out letters to customers every day.

On Monday they sent out 20 first class letters and 15 second class letters and the charge for postage was £19·50.

On Tuesday they sent out 18 first class letters and 25 second class letters and the charge was £23.30.

How much will it cost on Wednesday to send 10 first class letters and 30 second class?

2. A concert hall sells two types of tickets, stall tickets and balcony tickets. When **all** seats are sold the concert hall holds a total of 640 people.

(a) Let s be the number of stall tickets and b the number of balcony tickets.

From the information above write down an equation connecting s and b.

(b) On a particular night a concert is sold out (all seats are taken) with stall tickets priced at £8.50 and balcony tickets at £12.20. The total takings at the box office for that night was £6143.

From this information write down a second equation connecting s and b.

(c) Hence find how many stall and balcony seats are in this concert hall.

3. In a fast food restaurant Ian buys 3 burgers and 4 portions of French fries and it costs £5.64.

Sarah buys 2 burgers and 3 portions of French fries and it costs £4.01.

Jack had a voucher to receive one burger and one portion of fries for free.

How much would it cost Jack for 5 burgers and 3 portions of French fries?

4. A hotel owner is buying some new duvets for his hotel.

One week he bought 7 double duvets and 12 single duvets which cost £168.

The next week he bought 4 double duvets and 9 singles for £111.

The hotel owner was given a 14% discount on his next order for 5 double duvets and 5 single duvets.

How much did he pay for this third order?

5. Clare has baked 60 scones to sell at the school fayre. Some are fruit scones (f) and some are treacle scones (t).

(a) Write down an equation using f and t to illustrate this information.

She sells the fruit scones for 25p and the treacle scones for 20p each.

She sells all the scones for a total of £13.25.

(b) Write down another equation using f and t to illustrate this information.

(c) Hence, find **algebraically** the number of treacle scones Clare sold.

6. At the funfair coloured tokens are awarded as prizes in some of the games. These tokens can be saved up and exchanged for larger items.

3 green tokens and 4 red tokens have a total value of 26 points.

5 green tokens and 2 red tokens have a total value of 20 points.

Dave has 10 green tokens and 10 red tokens.

Does he have enough points to exchange for a large soft toy with a points value

7. In a week Peter downloads 5 tracks and 4 films and pays £21.23.

In the same week Frank downloads 7 tracks and 3 films and pays £18.49.

Calculate how much Richard would pay if he downloaded 3 tracks and 2 films.

8. In the Garden centre there are 2 types of plants on special offer.

| This week's specials! Rose bushes and Poppy plants |

Carly bought 3 Rose bushes and 2 Poppy plants which cost £15.23

Steph paid £26.71 for 4 Poppy plants and 5 Rose bushes.

How much would Sally pay for a Rose bush and 3 Poppy plants?

9. Peter is buying new furniture for his flat.

He can buy two sofas and one chair for £1145, or one sofa and three chairs for £1310.

Find the cost of one sofa and the cost of one chair.

10. Eric orders goods from a mail-order company.

5 books and 2 CDs cost £40·80.

2 books and 3 CDs cost £37·78.

Each order includes £2·95 post and packing regardless of the size of the order.

How much would it cost Eric to have 3 books and 1CD and have them delivered?

11. Shereen goes shopping in the summer sales.

The store has an advert in the window.

Shereen buys 2 tops and 3 skirts and
pays £33·90. Her friend Nadia buys 3 tops and 4 skirts and
£46·70.

Another friend Kay buys 2 tops and 2 skirts. How much does she pay?

> All tops one price!
>
> SALE
>
> All skirts one price!

Exercise 15, page 27

1.	**(a)**	$x = 5$; $y = 5$	**(b)**	$x = 1$; $y = 1$	**(c)**	$x = 2$; $y = 4$	**(d)**	$x = 3$; $y = 6$		
	(e)	$x = 3$; $y = 10$	**(f)**	$x = 5$; $y = 21$						
2.	**(a)**	$x = 3$; $y = 1$	**(b)**	$x = 7$; $y = 2$	**(c)**	$x = 5$; $y = 2$	**(d)**	$x = 2$; $y = -1$		
	(e)	$x = 6$; $y = -3$	**(f)**	$x = 4$; $y = -5$	**(g)**	$x = -3$; $y = -2$	**(h)**	$x = -11$; $y = -3$		
	(i)	$x = -8$; $y = -10$								
3.	**(a)**	$x = 7$; $y = 1$	**(b)**	$x = 10$; $y = 1$	**(c)**	$x = 4$; $y = 2$	**(d)**	$x = 2$; $y = 3$		
	(e)	$x = 3$; $y = -1$	**(f)**	$x = 4$; $y = 4$	**(g)**	$x = 2$; $y = 5$	**(h)**	$x = 5$; $y = 1$		
	(i)	$x = 2$; $y = 1$	**(j)**	$x = 4$; $y = 2$	**(k)**	$x = -1$; $y = -2$	**(l)**	$x = 1$; $y = -1$		
4.	**(a)**	$x = 1$; $y = 2$	**(b)**	$x = 3$; $y = -1$	**(c)**	$x = 2$; $y = 2$	**(d)**	$x = 5$; $y = 1$		
	(e)	$x = 5$; $y = 7$	**(f)**	$x = 5$; $y = -2$	**(g)**	$x = 3$; $y = -3$	**(h)**	$x = 4$; $y = -3$		
	(i)	$x = 2$; $y = 3·5$	**(j)**	$x = 0·5$; $y = 2$	**(k)**	$x = 1$; $y = 2$	**(l)**	$x = 5$; $y = 0$		
5.	**(a)**	$x = 2$; $y = 5$	**(b)**	$a = 3$; $b = 2$	**(c)**	$e = 6$; $f = 2$	**(d)**	$x = -1$; $y = 3$		
	(e)	$x = 2$; $y = -2$	**(f)**	$p = -2$; $q = 3$	**(g)**	$g = -4$; $h = 3$	**(h)**	$x = 3$; $y = 4$		
	(i)	$u = -2$; $v = -3$	**(j)**	$x = 4$; $y = 1$	**(k)**	$p = -5$; $q = 2$	**(l)**	$a = 4$; $b = -2$		
	(m)	$x = 2$; $y = 5$	**(n)**	$a = 3$; $b = -1$	**(o)**	$e = -2$; $f = 5$	**(p)**	$x = -1$; $y = 4$		
	(q)	$x = 3$; $y = 4$	**(r)**	$p = -3$; $q = 4$	**(s)**	$g = 8$; $h = -5$	**(t)**	$x = 7$; $y = -2$		
	(u)	$u = 2$; $v = -2$	**(v)**	$x = 4$; $y = 1$	**(w)**	$p = -1$; $q = 2$	**(x)**	$a = 8$; $b = -2$		

Exercise 16, page 28

1. 36 and 20 **2.** Chocolate costs 40p and crisps cost 30p
3. Sandwich costs £1.20 and hotdog costs 90p. **4.** Ruler costs 49p and pencil costs 26p.
5. Standard print is 21p and Jumbo costs 55p. **6.** Centre costs £11.25 and Side costs £9.50
7. Large glass holds 145ml and small holds 95ml. **8.** Frame tent holds 8 and ridge tent holds 3.
9. Reader's letter pays £15 and Star letter pays £25. **10.** Small takes 1·8kg and the large takes 2kg.
11. (a) $4x + 4y = 60$; $6x + 16y = 120$; $x = 12$ and $y = 3$ **(b)** 144cm^2
12. (a) Box weighs 6kg and parcel weighs 2kg. **(b)** 58kg
13. Milk is 24p and butter is 96p. **14.** 38 hours basic and 7 hours overtime
15. 320 cheaper tickets were sold **16.** 33 × 20p coins and 21 × 50p coins

Exercise 17, page 30

1. £21
2. (a) $s + b = 640$ **(b)** $8·5s + 12·2b = 6143$ **(c)** 450 stalls tickets and 190 balcony tickets
3. £5.02 **4.** £81.70
5. (a) $f + t = 60$ **(b)** $25f + 20t = 1325$ **(c)** Clare sold 35 treacle scones
6. 75 points are needed – tokens give only 70 points so not enough.
7. £11.01. **8.** £9.72 **9.** Sofa costs £425 and chair costs £295.
10. £23.87 **11.** £25.60