National 5 Applications of Mathematics Revision Notes

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Use this booklet to practise working independently like you will have to in an exam.

- Get in the habit of turning to this booklet to refresh your memory.
- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** (back page) to look it up.

As you get closer to the exam, you should be aiming to use this booklet less and less.

This booklet is for:
- Students doing the National 5 Lifeskills Mathematics course.
- Students studying one or more of the National 5 Lifeskills Mathematics units: **Numeracy**, **Geometry and Measures** or **Managing Finance and Statistics**.

This booklet contains:
- The most important facts you need to memorise for National 5 Lifeskills Mathematics.
- Examples that take you through the most common **routine** questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:
- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a practice test question.
- To memorise key facts when revising for the exam.

The key to revising for a maths exam is to do questions, not to read notes. **As well as using this booklet, you should also:**
- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or other exercises suggested by your teacher.
- Work through practice tests.
- Ask your teacher when you come across a question you cannot answer.
- Use resources online (a link that can be scanned with a Smartphone is on the last page).

Contents

Formula Sheet

The following formulae are mentioned in these notes and are collected on this page for ease of reference.

Formulae that are given on the formula sheet in the exam (or in unit assessments)

Topic	Formula(e)	Page Reference
Pythagoras' Theorem	$a^2 + b^2 = c^2$	See page 50
Gradient	$\text{Gradient} = \dfrac{\text{Vertical height}}{\text{Horizontal distance}}$	See page 54
Circumference of a Circle	$C = \pi d$	See page 57
Area of a Circle	$A = \pi r^2$	See page 57
Volume of a prism	$V = Ah$	See page 61
Volume of a cylinder	$V = \pi r^2 h$	See page 61
Volume of a cone	$V = \dfrac{1}{3}\pi r^2 h$	See page 62
Volume of a sphere	$V = \dfrac{4}{3}\pi r^3$	See page 63
Standard deviation	$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$ or $\sqrt{\dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$	See page 90

Formulae that are not given in the exam (or in unit assessments)

Topic	Formula(e)	Page Reference
Percentage increase and decrease	$\dfrac{\text{increase (or decrease)}}{\text{original amount}} \times 100$	See page 16
Area of a rectangle	$A = LB$	See page 20
Area of a square	$A = L^2$	See page 20
Area of a triangle	$A = \dfrac{BH}{2}$	See page 20
Volume of a cuboid	$V = LBH$	See page 21
Speed, Distance, Time	$S = \dfrac{D}{T}$ $T = \dfrac{D}{S}$ $D = ST$	See page 21
Net Pay	Net Pay = Gross Pay – Total Deductions	See page 69
InterQuartile Range (IQR)	IQR = upper quartile – lower quartile	See page 89
Semi InterQuartile Range (SIQR)	$SIQR = \dfrac{\text{upper quartile} - \text{lower quartile}}{2}$	See page 89

How these Notes are Structured

The National 5 Applications of Mathematics course comprises three units:
- Numeracy.
- Geometry and Measures.
- Managing Finance and Statistics.

These notes are organised with one chapter per unit. There is also a brief introductory chapter covering exam technique, which provides candidates with some key tips that will reappear throughout the examples in the rest of the notes.

Types of Example

All assessment questions in National 5 Applications of Mathematics are supposed to be set in a real- life context. This requires candidates to choose the correct strategy for each real-life situation.

However when a learner is practicing a new skill a context can sometimes be distracting, and in the early days it can help to practice a mathematical skill in isolation before encountering it in a context or as part of a longer question.

For this reason there are two types of example used in these notes: **Basic Skills Examples** and **Assessment Style Examples**. The style and content of each are outlined below.

BASIC SKILLS EXAMPLE

A Basic Skills Example will focus on the mathematical steps required to undertake one particular skill in isolation. They will assess a skill that might be required for this course, but the whole question might be below the level of course assessments.

They may be set in a real-life context, but only if the context doesn't distract from the mathematical skill. They will not contain any problem solving elements.

Basic skills Examples will be enclosed in a bold blue frame such as this one.

Assessment Style Example

Examples of the type of question that may be found in exams or unit assessments are written in this style.

These questions will always be set in a real-life context, and may contain problem solving elements or other complications such as rounding, making/explaining decisions or links with other parts of the course.

Most topics will also have a box in red-brown entitled '**What should an exam question look like?**' These boxes give an idea of what features can be expected from an exam or unit assessment question.

Exam and Unit Assessment Technique

This page outlines some of the key things that should be remembered when sitting exams and unit assessments in order to avoid losing marks.

Units

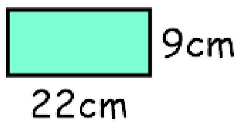
To be fully correct, answers should always contain the correct units. As a general rule, you will lose the mark for the final answer if you do not include the correct units.

There are some occasions where your teacher will still be allowed to give you the mark, even without the correct units, but you do not need to know what these are: the only way to guarantee that you do not lose a mark is to ensure you always include units!

It is especially important to use the correct units in questions relating to area and volume:
- If the question asks you to calculate a **volume**, the units are 'cubic'– e.g. m^3, cm^3, mm^3.
- If the question asks you to calculate an **area**, the units are 'squared'– e.g. m^2, cm^2, mm^2.
- If the question asks you to calculate a length (including a **perimeter**, a **circumference** or an **arc length**), the units are 'normal' units – e.g. m, cm, mm.

Assessment Style Example (this example is below National 5 standard, but is included to focus on the units)

Calculate the area of this rectangle (1 mark)



Example Solutions

Solution One

$$\begin{aligned} A &= LB \\ &= 22 \times 9 \\ &= 198 \end{aligned}$$

You would not get the mark for this answer as the answer contains no units

Solution Two

$$\begin{aligned} A &= LB \\ &= 22 \times 9 \\ &= 198\text{cm} \end{aligned}$$

You would not get the mark for this answer as the answer contains incorrect units.

This is an area question, so squared units are needed for the answer

You would also have lost the mark if you had used cm^3, or if you had writen m^2 instead of cm^2

Solution Three

$$\begin{aligned} A &= LB \\ &= 22 \times 9 \\ &= 198\text{cm}^2 \end{aligned}$$

You would get the mark for this answer as the answer contains the correct units

Rounding

If a question requires you to round your answer, you have to write down your unrounded answer before you then do the rounding. This is because there are often two marks related to the rounding:

- **The final mark** (for the rounding).
- **The previous mark** (for the actual answer).

A lot of people get frustrated by this. But whether you like it or not, if you do not write down your unrounded answer first, you will risk losing both marks **even if you have the correct answer**.

This rule applies in questions where you are explicitly told to round (e.g. 'give your answers correct to 2 significant figures'), or questions where you have to realise for yourself that you have to round. There are two common examples of this:

- **Money questions**, where you have to know to give decimal answers to 2 decimal places. (e.g. you cannot have an answer of £2·587, you have to round it to £2·59.)
- **Real-life situations where decimal answers make no sense**, where you have to know to round up (or down) to the nearest whole number. The example below is an example of this type.

Assessment Style Example (this example is below National 5 standard, but is included to focus on the rounding)

One bus holds 56 people. How many buses must be ordered to take 1294 football fans from Inverness to Glasgow?

Example Solutions

In this question, you have to realise that it is impossible to have a decimal number of buses, so you have to round to the nearest whole number. You also have to realise that you always have to round up to the nearest whole number.

Solution That Would Get Full Marks	Solution That Would Lose Marks
$1294 \div 56 = 23 \cdot 107...$	$1294 \div 56$
24 buses are needed	24 buses are needed

Notice that you do not have to write down every decimal place from your calculator. It is enough to write a few 'extra' decimal places and then to write dots to indicate that they keep on going.

Further examples of assessment style questions requiring answers to be rounded can be found in:

- Appreciation: **Assessment Style Example 2** on page 18 (significant figures).
- Pythagoras' Theorem: **Assessment Style Example 1** and **Assessment Style Example 2** starting on page 51 (significant figures).
- Composite Area: **Assessment Style Example 2** on page 60 (rounding up to the nearest whole number).
- Cones: **Assessment Style Example 1** on page 62 (this question requires rounding down to the nearest whole number).
- Volume of a solid: **Assessment Style Example 2** on page 64 (significant figures).
- Volumes: **Assessment Style Example 3** on page 65 (this question requires rounding down to the nearest whole number).
- Currency Conversions: **Assessment Style Example 2** on page 75 (significant figures).

Fractions

If your answer contains a fraction (e.g. a probability), you should <u>always</u> give your fraction in its simplest form. For example:

1. An answer of $\frac{6}{8}$ should be simplified to $\frac{3}{4}$.
2. An answer of $\frac{30}{25}$ should be simplified to $\frac{6}{5}$.

Types of Question

Each of the outcomes of the *Geometry and Measures* and *Managing Finance and Statistics* units are split into three Assessment Standards, which cover three 'types' of skill that learners should be able to demonstrate when responding to questions. These skills will also be tested in the final exam paper.

- **Analysing a situation and identifying a valid strategy.** In everyday language, this assessment standard most commonly requires you to choose a method to answer a question; usually in a question that is slightly unfamiliar to you (a 'non-routine' question).
- **Using appropriate mathematical processes and/or calculations to determine a solution**. These are the 'process' marks awarded for getting calculations correct.
- **Justifying a solution in relation to the context.** In everyday language, this assessment standard most commonly requires you to either *communicate* your answer after performing a calculation; for instance by writing a sentence or to ensure the answer is in the appropriate form. More detail is given in the next section.

Making and Explaining Decisions

Some questions will ask you to make a decision, which may then be followed by asking you to **'give a reason for your answer'** or **'explain your answer'** or **'justify your answer'**.

A valid reason at National 5 should involve **comparing <u>two</u> numbers**.

<u>Assessment Style Example</u> (this example is below National 5 standard, but is included to focus on the reason)

Jack has £17. Ben has £8. They need £30 to buy a computer game. Do they have enough? Give a reason for your answer

Solution

You <u>would not</u> get a mark for the following explanations:

- No. *(no reason at all)*
- No, because they do not have £30. *(just repeating the number from the question)*
- No, because they only have £25. *(not comparing <u>two</u> numbers)*

You <u>would</u> get a mark for the following explanations:

- No, because they have £25 and they need £30. *(two numbers compared)*
- No, because they need £5 more. *(Writing down the difference between the numbers counts as comparing two numbers)*

Further examples of assessment style questions requiring decisions and explanations can be found in:

- Gradient, **Assessment Style Example 1** on page 55.
- Perimeter: **Assessment Style Example 1** on page 58.
- Pythagoras' Theorem: **Assessment Style Example 1** on page 51.
- Savings: **Assessment Style Example 1** on page 77.
- Savings: **Assessment Style Example 2** on page 78.
- Tolerance, **Assessment Style Example** on page 34. *This example is slightly different because it requires three numbers as there is both an upper and a lower limit.*

In statistics questions, the style of explanation required for a comparison is often different, and this is covered in the **Comparing Statistics** section on page 93.

Numeracy

Units and Notation

The course notes state that you are expected to be able to work with the following units. You should also know the key facts connecting them.

Context	Units	Key Facts
Money	Pounds (£) and pence Dollars ($) and cents Euros (€) and cents	£1 = 100p $1 = 100 cents €1 = 100 cents
	There are also many other units for money for other currencies which are used in the Foreign Exchange topic.	
Time	Months Weeks Days Hours Minutes Seconds	1 (normal) year = 365 days 1 (leap) year = 366 days 1 year = 12 months 1 year = 52 weeks 1 day = 24 hours 1 hour = 60 minutes 1 minute = 60 seconds
Length	Millimetres (mm) Centimetres (cm) Metres (m) Kilometres (km) Miles	1cm = 10mm 1m = 100cm 1km = 1000m 1m = 1000mm
Weight	Kilograms (kg) Grams (g) Milligrams (mg) Tonnes (t)	1 g = 1000 mg 1 kg = 1000 g 1 tonne = 1000 kg
Volume	Millilitres and Litres Millilitres and cm³	1 litre = 1000ml 1 ml = 1 cm³
Temperature	Degrees Celsius (°C) Degrees Fahrenheit (°F)	

Important
As explained on page 5, it is essential to give the correct units in your answers. In many questions, you will lose the final mark if you have the 'correct' answer without units.

The course syllabus states that you are expected to be familiar with all of these symbols:

Symbol	Meaning
=	Equal
+ – × ÷	Add, take away, multiply, divide
/	An alternative way of showing a divide sum – i.e. 15 ÷ 3 could also be written 15/3 or $\frac{15}{3}$
<	Less than (≤ means less than *or equal to*)
>	Greater/bigger than (≤ means greater than *or equal to*)
( )	Sums in brackets have to be done first
%	Percentage (see page 16)
:	Ratio symbol (see page 23)
·	Decimal point

Calculations

Calculations without a Calculator

Add and Subtract

You are expected to be able to add and subtract decimal numbers (with up to 2 decimal places). Some people are happy to do these calculations without requiring working, but if you need to write working, it is best to add extra 'trailing' zeroes after the decimal point at the end of the number so that all numbers have the same number of decimal places. Do not forget to include the decimal point in your answer. It should be directly below all of the decimal points in the original question.

BASIC SKILL EXAMPLE 1: Decimal addition and subtraction

(a) Add: 34·7 + 17·39
(b) Subtract: 30 – 14·26

Solution

Add trailing zeroes to all numbers so that every number has two decimal places.

(a) becomes 34·70 + 17·39 and (b) becomes 30·00 – 14·26.

The solutions, with all carrying and borrowing shown, are shown below:

(a)
$$\begin{array}{r} 34\cdot70 \\ +17\cdot39 \\ \hline 52\cdot09 \end{array}$$

(b)
$$\begin{array}{r} 30\cdot00 \\ -14\cdot26 \\ \hline 15\cdot74 \end{array}$$

Basic Multiply and Divide

You are expected to be able to multiply and divide decimal numbers (with up to 2 decimal places) by a single digit using your knowledge of multiplication tables. There is no need to write trailing zeroes in multiplication sums.

In division sums, you only need to add trailing zeroes if you still have a remainder at the end. Keep on adding zeroes until you no longer have a remainder (or until you decide to round the number instead). Do not forget to include the decimal point in your answer. It should be directly below the decimal point in the original question.

BASIC SKILL EXAMPLE 2: Decimal multiplication and division by a single digit

(a) Multiply: 15·29 × 6
(b) Divide: 7·3 ÷ 5

Solution

The solutions, with all carrying and borrowing shown, are shown below. In the division question, an extra zero had to be added to allow us to complete the calculation

(a)
$$\begin{array}{r} 15\cdot29 \\ \times\quad 6 \\ \hline 91\cdot74 \end{array}$$

(b)
$$\begin{array}{r} 1\cdot46 \\ 5\overline{)7\cdot30} \end{array}$$

You are also expected to multiply and divide decimal numbers (with up to 2 decimal places) by multiples of 10, 100 or 1000 (e.g. multiplying by 30 or 6000; dividing by 200 or 8000).

The key to this method is splitting into two steps:

- Step A involves multiplying or dividing by a single digit using the method above.
- Step B involves multiplying or dividing by 10, 100 or 1000 (by moving all the digits to the left or right).

The order for Steps A and B does not matter. The following table illustrates how this can be done:

Calculation	**Step A**	**Step B**
Multiply by 20	Multiply by 2	Multiply by 10
Divide by 20	Divide by 2	Divide by 10
Multiply by 300	Multiply by 3	Multiply by 100
Divide by 5000	Divide by 5	Divide by 1000

BASIC SKILL EXAMPLE 3: Multiplying by a multiple of 10, 100 or 1000

Multiply without a calculator: 3·14 × 3000

Solution

First do 3·14 × 3, and then multiply the answer by 1000, (or 3·14 × 1000, and then × 3).

3·14 × 3 = 9·42 *(working with carrying shown on right)*

$$\begin{array}{r} 3{\cdot}14 \\ \times \ {}_1 3 \\ \hline 9{\cdot}42 \end{array}$$

9·42 × 1000 = <u>9420</u>

BASIC SKILL EXAMPLE 4: Dividing by a multiple of 10, 100 or 1000

Divide without a calculator: 124·65 ÷ 50

Solution

First do 124·65 ÷ 5, and then divide the answer by 10 (or 124·65 ÷ 10, and then ÷ 5).

124·65 ÷ 5 = 24·93 *(working with carrying shown on right).*

$$\begin{array}{r} 24\cdot93 \\ 5\overline{)12{}^{2}4\cdot{}^{4}6{}^{1}5} \end{array}$$

24·93 ÷ 10 = <u>2·493</u>

<u>More Complex Multiply and Divide</u>

Whilst it is not stated in the Course and Unit Support Notes, for the final exam you could be expected to multiply or divide by more complicated numbers – such as multiplying or dividing two two-digit numbers; or a two-digit and a three digit number.

For multiplication, there are two main methods that can be used. It is possible that you will have been taught only one of these, or perhaps you will have been taught both and will have chosen the one that you prefer. These can be described as **long multiplication** (where the sum is written out in a similar way to the single-digit multiplication sum, but with additional lines) or the **box method**.

In the box method, each number is split up according to its digits (e.g. 58 is split into 50 and 8; 238 is split into 200, 30 and 8) and then a mini 'tables square' is produced. All the answers in the table square are then added together.

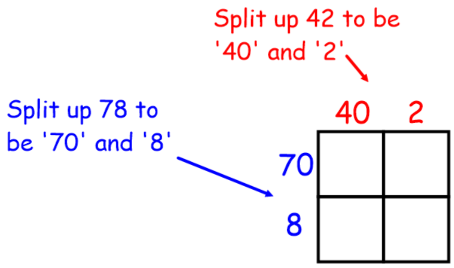
BASIC SKILL EXAMPLE 5: Multiplying two two-digit numbers

Multiply 78 × 42

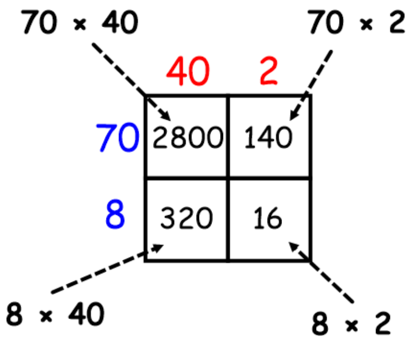
Solutions

Method A (Box Method)

Step 1 – construct a multiplication square with two numbers along the side and two numbers along the top.



Step 2 – multiply the numbers in each row and column to obtain one number in each of the four smaller squares.

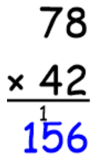


Step 3 – add the four numbers to obtain the final answer.

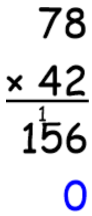
2800 + 140 + 320 + 16 = 3276

Method B (Long Multiplication)

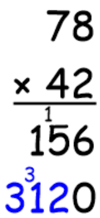
Step 1 – start doing a usual multiplication sum and do 78 × 2 using the usual method.



Step 2 – the next line will be for 78 × 4. However because the sum should really be 78 × 40, we write one zero in the units column.



Step 3 – complete the sum 78 × 4 usual the usual method.



Step 4 – add the two answers to obtain the final answer.

156 + 3120 = 3276.

These methods can be extended to any multiplication sum, including multiplication of three- (or more)-digit numbers, or multiplication of decimals.

Assessment Style Example 1: Multiplying a three-digit number and a two-digit number

It costs £56 to cover one square metre of pathway with concrete. How much will it cost to cover a path measuring 247m^2?

Solution

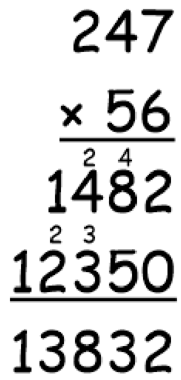
The calculation is 247 × 56. We can use either method outlined above. These methods are illustrated on the next page.

Using **method A** (box method) the working is as follows:

	200	40	7
50	10000	2000	350
6	1200	240	42

10000 + 2000 + 350
+ 1200 + 240 + 42
= 13832

Using **method B** (long multiplication) the working is as follows:



The final answer is <u>£13832</u>

For division, there are also two main methods that can be used. Again, it is possible that you will have been only taught one of these, or maybe that you will have been taught both and will have chosen the one that you prefer. These can be described as **long division** (where the sum is written out in a similar way to the single-digit division sum, but with added complexity) or by writing the division sum as a **fraction** and simplifying the fraction until you obtain an easier division.

BASIC SKILL EXAMPLE 6: Division by a two-digit number

Divide 920 ÷ 32

<u>Solutions</u>

Method A (Fractions)

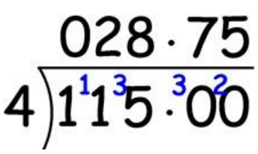
<u>Step 1</u> – write the division sum as a fraction.

$$\frac{920}{32}$$

<u>Step 2</u> – simplify the fraction by dividing the top and bottom by the same number.

$$\frac{920^{\div 2}}{32_{\div 2}} = \frac{460^{\div 4}}{16_{\div 4}} = \frac{115}{4}$$

<u>Step 3</u> – complete the (easier) divide sum using the usual method outlined on page 10. The sum we are left with is 115 ÷ 4.

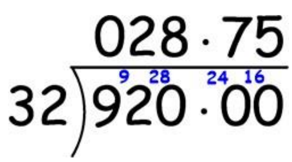


Method B (Long Division)

To use long division for this question, you need to know your 32 times table (writing it down before you start would be advisable!). If you can't do this, practice Method A instead.

We use the usual method outlined on page 10.

Using long division, the remainders will be much larger than we are used to dealing with. If this is a problem, practice method A instead,



The answer is <u>28·75</u>

Assessment Style Example 2: Division by a three-digit number

28 000 shares are bought for £238 000. How much does one share cost?

Solution

The calculation is 238000 ÷ 28000. We can use either method outlined above. Method A will be used here because it is quicker:

$$238000 \div 28000 = \frac{238000^{\div 1000}}{28000_{\div 1000}} = \frac{238^{\div 7}}{28_{\div 7}} = \frac{34^{\div 2}}{4_{\div 2}} = \frac{17}{2}$$

We can now complete the calculation by doing 17 ÷ 2. The answer is £8·50.

Fractions

To calculate a fraction, you divide by the number on the bottom (the denominator) and multiply by the number on the top (the numerator)

BASIC SKILL EXAMPLE 7: Fractions

A washing machine costs £385. $\frac{3}{5}$ of the cost is for the materials. What is the cost of the materials?

Solution

$$\frac{3}{5} \text{ of } 385 = 385 \div 5 \times 3 = 77 \times 3 = \underline{\underline{£231}}$$

Percentages

You should know the following equivalences:

Percentage	Fraction	Percentage	Fraction
50%	$\frac{1}{2}$	10%	$\frac{1}{10}$
25%	$\frac{1}{4}$	1%	$\frac{1}{100}$
75%	$\frac{3}{4}$	$33\frac{1}{3}\%$	$\frac{1}{3}$
20%	$\frac{1}{5}$	$66\frac{2}{3}\%$	$\frac{2}{3}$

Other percentages can be worked out without a calculator by finding 1% or 10% first
For example to find 30%, find 10% first, then multiply the answer by 3.
To find 4%, find 1% first then multiply the answer by 4.

BASIC SKILL EXAMPLE 8: Percentages without a calculator

What is 40% of £120? **What is 7% of 3000kg?**

Solution

10% of £120 = £12
So 40% of £120 = 12 × 4 = **£48**

1% of 3000kg = 30kg
so 7% of 3000kg = 30 × 7 = **210kg**

More complex percentages can be found by combining other percentages.

For example:

- To find 32%, you could work out 10% and 1% and then do **10% × 3 + 1% × 2** (or 10% + 10% + 10% + 1% + 1%).
- To find 95% you could do 10% × 9 + 1% × 5, or 100% **-** (half of 10%). There are also many other possible methods.
- To find 7·5%, you could find 5% (half of 10%) and 2·5% (half of 5%) and then do 5% + 2·5%.

Assessment Style Example 2 – more complex percentages without a calculator

Alan wins £42000 on the lottery. He gives 22½% of his winnings to his daughter, Kerrie. How much money does Kerrie receive?

Solution

10% of £42000 = £4200
5% of £42000 = half of 10% = £2100
2½% of £42000 = half of 5% = £1050

22½% of £42000 = 10% × 2 + 2½%
= 4200 × 2 + 1050
= £9450

Rounding to Significant Figures

You are expected to be able to round an answer to a number of significant figures.

BASIC SKILL EXAMPLE 1: rounding to significant figures

Round the number 446•586 to:

(i) **1 significant figure**
(ii) **2 significant figures**
(iii) **3 significant figures**
(iv) **4 significant figures**

Solutions

446·586	→	Rounded to 1 significant figure is:	**400** (not 400·0)
	→	Rounded to 2 significant figures is:	**450**
	→	Rounded to 3 significant figures is:	**447**
	→	Rounded to 4 significant figures is:	**446·6** (not 446·600)

BASIC SKILL EXAMPLE 2: rounding numbers less than 1 to significant figures

Round the number 0•00567 to:

(i) **1 significant figure**
(ii) **2 significant figures**

Solutions

0·00567	→	Rounded to 1 significant figure is:	**0·006** (not 0·0060)
	→	Rounded to 2 significant figures is:	**0·0057**

As explained on page 5, it is essential to always write your unrounded answer before rounding.

Further examples of assessment style questions requiring answers to be rounded to significant figures can be found in:

- Appreciation: **Assessment Style Example 2** on page 18.
- Pythagoras' Theorem: **Assessment Style Example 1** and **Assessment Style Example 2** starting on page 51.
- Volume of a solid: **Assessment Style Example** on page 64.
- Currency Conversions: **Assessment Style Example 2** on page 75.

Percentages and Fractions

To find the percentage, there are three steps:

1. Write as a fraction.
2. Change to a decimal by **dividing**.
3. Change to a percentage by **multiplying by 100**.

A quick way of remembering this is to do smaller ÷ bigger × 100 (or top ÷ bottom × 100)

BASIC SKILL EXAMPLE 1: Finding the Percentage

Out of 1250 pupils, 475 get to school by bus. What percentage is this?

Solution

As a fraction, this is $\frac{475}{1250}$.

To change this to a percentage divide and then multiply by 100:

$475 \div 1250 \times 100 = \underline{\underline{38\%}}$

Without a calculator, the calculation can be found using equivalent fractions. Multiply and divide the top and bottom by the same number to obtain the number 100 on the bottom of the fraction. The number on the top is then the percentage.

BASIC SKILL EXAMPLE 2: Finding the Percentage (non-calculator)

Darren baked 20 cakes. 13 of these cakes are carrot cakes. What percentage of cakes are carrot cakes?

Solution

The fraction of carrot cakes is $\frac{13}{20}$. We need to change this into a percentage. To obtain the number 100 on the bottom of the fraction we need to multiply both top and bottom by 5.

$\frac{13^{\times 5}}{20_{\times 5}} = \frac{65}{100}$, so the percentage is $\underline{65\%}$.

More difficult questions ask you to find the percentage increase or decrease. In these questions, you always have to work out the percentage of the **original** amount.

Formula: <u>not given</u> on the formula sheet in National 5 Lifeskills Mathematics assessments

$$\text{Percentage increase/decrease} = \frac{\text{change}}{\text{original amount}} \times 100$$

BASIC SKILL EXAMPLE 3: Finding the Percentage Increase or Decrease

The temperature in an oven was 180°C. It went up to 207°C.
What was the percentage increase in temperature?

Solution

Step one – what is the increase? 207 – 180 = 27°C.

Step two – write as a fraction of the original amount:

Original amount was 180°C, so as a fraction this is $\frac{27}{180}$.

Step three – divide and multiply by 100 to change to a percentage:
27 ÷ 180 × 100 = 15%

For National 5 Numeracy assessment questions, it is likely that the numbers for a percentage question will not be stated in the question. Instead there might be a table, graph or scale to read to determine the numbers.

Assessment Style Example 1

The table on the right shows the numbers of student vets at four Scottish Universities.

What percentage of the vet students are women?

University	Men	Women
Edinburgh	110	100
Glasgow	214	223
Dundee	120	197
St Andrew's	132	121

Solution

Total number of women at all four Universities = 100 + 223 + 197 + 121 = 641.
Total number of students = 100 + 223 + 197 + 121 + 110 + 214 + 120 + 132 = 1217.

The fraction of women is $\frac{641}{1217}$. We convert this to a percentage using the usual method:
641 ÷ 1217 × 100 = 52·67… = 52·7% (1 d.p.)

In National 5 exam percentage questions, you could be asked to increase or decrease an amount by a percentage – this will usually be either **compound interest**, or **appreciation** or **depreciation**.

For every question, there is a longer way and a quicker way to do it. Use the one you are happiest with. In the examples below, the quicker method will be preferred.

Percentage	Longer method	Quicker method
3% increase	Multiply by **0·03**, then add answer on	*[100% + 3% = 103%]* Multiply by **1·03**
3% decrease	Multiply by **0·03**, then take answer away	*[100% – 3% = 97%]* Multiply by **0·97**
2·4% increase	Multiply by **0·024**, then add answer on	*[100% + 2·4% = 102·4%]* Multiply by **1·024**
15% decrease	Multiply by **0·15**, then take answer away	*[100% – 15% = 85%]* Multiply by **0·85**
4·5% decrease	Multiply by **0·045**, then take answer away	*[100% – 4·5% = 95·5%]* Multiply by **0·955**

Definition: Appreciation is an increase in value; **Depreciation** is a decrease in value.

When appreciation or depreciation is repeated, using powers can make for a quicker method.

BASIC SKILL EXAMPLE 4: Appreciation and Depreciation

Peterhead has a population of 30 000. Its population depreciates by 15% per year. What is its population after two years?

Solution

Depreciation means decrease, so we will be taking away.
100% – 15% = 85%, so we use **0·85** in the quicker method.
The question is for two years so need to repeat 2 times (a power of 2).

Longer method	Quicker method
Year 1: $0 \cdot 15 \times 30000 = 4500$ $30000 - 4500 = 25500$	
Year 2: $0 \cdot 15 \times 25500 = 3825$ $25500 - 3825 = 21675$	$30000 \times 0 \cdot 85^2 = 21675$
Answer: 21 675	**Answer:** 21 675

You might be asked to work out the percentage increase or decrease first, rather than being told what the percentage is.

Assessment Style Example 2

A house cost £240 000 when first bought. One year later its value has appreciated to £250 800.

a) Find the rate of appreciation.

b) If the house continues to appreciate at this rate, what will its value be after a further 4 years? Round your answer to 3 significant figures.

Solution

a) The increase is 250 800 – 240 000 = £10 800.

Using the formula, the percentage increase is given by:

$$\frac{10\,800}{240\,000} \times 100 = \underline{\underline{4 \cdot 5\%}}$$

b) Using the quicker method:

Appreciation means increase, so we will be adding.
100% + 4·5% = 104·5%, so we use **1·045** in the quicker method.
The question is for four years so need to repeat 4 times (a power of 4).
(The question says a further four years – so we start with £250800 not £240000).

$250800 \times 1 \cdot 045^4 = 299083 \cdot 665$ (When the question requires rounding, you must state the unrounded answer first)

Answer: after a further 4 years, the house will be worth £299 000 (3 s.f.)

It does not matter if we do not have a start value. We can just do the multiplication calculation with the multipliers alone.

Assessment Style Example 3 – no start value

A vintage car depreciated in value by 5% and then appreciated in value by 12%. How much had its value changed overall?

Solution

The multiplier for a 5% depreciation is 0·95 (because 100 – 5 = 95).
The multiplier for a 12% appreciation is 1·12 (because 100 + 12 = 112).

$0\cdot95 \times 1\cdot12 = 1\cdot064$, which is the multiplier for a 6·4% appreciation.

Answer: A 5% depreciation followed by a 12% appreciation is the same as a 6·4% appreciation.

Tip: Always use multipliers and powers in any National 5 percentages exam question.

In an exam, you might be expected to work with **fractions**, including topheavy fractions (e.g. $\frac{7}{2}$) or those which may be expressed as a mixed number (e.g. $3\frac{1}{2}$).

You need to be able to change a fraction from a topheavy fraction or vice versa. To do this we can think about the link between fractions and dividing.

$\frac{a}{b} = a \div b = p\frac{q}{b}$, where p is the quotient and q is the remainder when doing $a \div b$.

BASIC SKILL EXAMPLE 5: Changing topheavy fractions to mixed numbers

Change the topheavy fractions to mixed numbers: (a) $\frac{13}{3}$ (b) $\frac{46}{7}$

Solutions

(a) 13 ÷ 3 = **4** r **1**, so $\frac{13}{3} = 4\frac{1}{3}$

(b) 46 ÷ 7 = **6** r **4**, so $\frac{46}{7} = 6\frac{4}{7}$

To go the other way, we use the fact that $a\frac{b}{c} = \frac{ac+b}{c}$

BASIC SKILL EXAMPLE 6: Changing mixed numbers to topheavy fractions

Change the mixed numbers to topheavy fractions: (a) $6\frac{2}{5}$ (b) $10\frac{4}{9}$

Solutions

(a) 5× 6 + 2 = **32**, so $6\frac{2}{5} = \frac{32}{5}$

(b) 9× 10 +4 = **94**, so $10\frac{4}{9} = \frac{94}{9}$

You can only add and subtract fractions when the denominators are the same. When they are not the same, we have to change the fractions into another fraction that *is* the same.

A quick method for doing this, and the one used in these notes, is known as the **'kiss and smile'** method because of the shape formed when you draw lines between the terms you are combining.

BASIC SKILL EXAMPLE 7: Adding or subtracting fractions

Take away: $\frac{7}{8} - \frac{2}{3}$

Solution

Step one (smile) $\frac{7}{8} - \frac{2}{3}$

Step two (kiss) $\frac{7}{8} \times \frac{2}{3}$

Step three take away (or add) the top line

$$\frac{7}{8} - \frac{2}{3} = \frac{}{24} = \frac{21-16}{24} = \frac{5}{24}$$

Adding works in exactly the same way as taking away. The only difference is that step three involves an add sum rather than a take away sum.

BASIC SKILL EXAMPLE 8: Comparing fractions

Which fraction is bigger: $\frac{4}{7}$ **or** $\frac{3}{8}$**?**

Solution

We change both fractions to have the same number on the bottom. 56 is a multiple of both 7 and 8, so we can choose 56.

$\frac{4^{\times 8}}{7_{\times 8}} = \frac{32}{56}$ $\quad$ $\frac{3^{\times 7}}{8_{\times 7}} = \frac{21}{56}$, therefore $\frac{4}{7}$ is larger because $\frac{32}{56} > \frac{21}{56}$.

Area, Perimeter and Volume

The topics of Area, Perimeter and Volume are covered thoroughly in the *Geometry and Measures* unit starting on page 57. However you should also remember how to calculate the area, perimeter and volume of basic shapes such as rectangles, triangles and cuboids. The relevant section of the notes from National 4 Lifeskills Maths are included here as a reminder,

It is especially important to use the correct units in questions relating to area and volume:

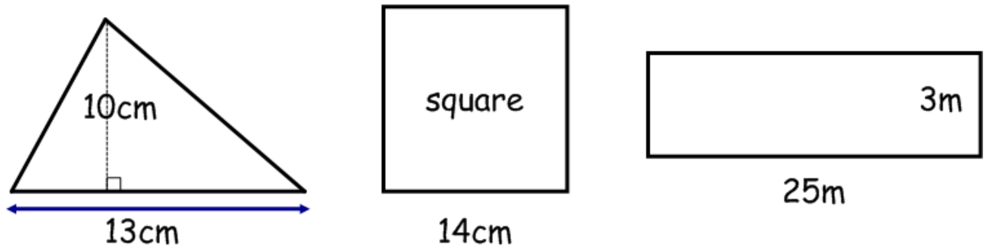
- If the question asks you to calculate a **volume**, the units are 'cubic'– e.g. m^3, cm^3, mm^3.
- If the question asks you to calculate an **area**, the units are 'squared'– e.g. m^2, cm^2, mm^2.
- If the question asks you to calculate a length (including a **perimeter**, a **circumference** or an **arc length**), the units are 'normal' units – e.g. m, cm, mm.

Formulae: not given on the formula sheet in National 5 Lifeskills Mathematics assessments

Area of a rectangle:	$A = \text{Length} \times \text{Breadth}$, or $A = LB$
Area of a square:	$A = L^2$
Area of a triangle:	$A = \frac{1}{2}\text{Base} \times \text{Height}$, or $A = \frac{BH}{2}$

BASIC SKILL EXAMPLE 1: Area of triangle, square and rectangle

Find the areas of these shapes



Solutions

$$A = \frac{BH}{2}$$

$$A = \frac{13 \times 10}{2} = \frac{130}{2} = \underline{65\text{cm}^2}$$

$$A = L^2$$

$$A = 14^2 = \underline{196\text{cm}^2}$$

$$A = LB$$

$$A = 25 \times 3 = \underline{75\text{m}^2}$$

Formula: <u>not given</u> on the formula sheet in National 5 Lifeskills Mathematics assessments
Volume of a cuboid:
$V = \text{Length} \times \text{Breadth} \times \text{Height}$, or $V = LBH$

BASIC SKILL EXAMPLE 2: Volume of a cuboid

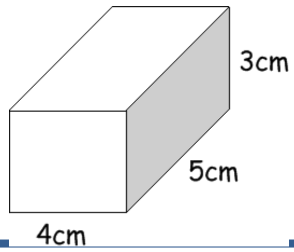
Calculate the volume of this cuboid

Solution

$$V = LBH$$

$$= 5 \times 4 \times 3$$

$$= \underline{60\text{cm}^3}$$



Speed, Distance, Time

Formulae: <u>not given</u> on the formula sheet in National 5 Lifeskills Mathematics assessments

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}} \qquad \text{Distance} = \text{Speed} \times \text{Time}$$

If you have been taught it, you might also like to use the Speed, Distance and Time triangle

<u>Units:</u> the units for speed, distance and time are interlinked:

- If distance is in **km** and time is in **hours** (h), speed is measured in **km/hour**.
- If distance is in **metres** (m) and time is in **seconds** (s), speed is measured in **m/s**.
- If the speed is in **cm/min**, then the distance is in **cm** and the time in **minutes**.

Sometimes speed is referred to as **average speed** or **mean speed** to reflect the fact that it can vary during a journey. This makes no difference to how you answer a question.

BASIC SKILL EXAMPLE 1: calculate speed

I drive 90km in 2 hours and 15 minutes. Calculate my average speed.

Solution

We are working out speed, so we use the formula $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$.

2 hours and 15 minutes is not 2·15 hours. It is 2 ¼ hours = 2·25 hours.

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}}\\ &= \frac{90}{2\cdot 25}\\ &= \underline{40\text{km/h}}\end{aligned}$$

BASIC SKILL EXAMPLE 2: calculate distance

A bird flies 3½ hours at an average speed of 42km/h. How far does it fly?

Solution

We are working out distance, so we use the formula $\text{Distance} = \text{Speed} \times \text{Time}$

3 ½ hours is not 3·30 hours. It is 3·5 hours.

$$\begin{aligned}\text{Distance} &= \text{Speed} \times \text{Time}\\ &= 42 \times 3\cdot 5\\ &= \underline{147\text{km}}\end{aligned}$$

BASIC SKILL EXAMPLE 3: calculate time taken

A driver travels 129 miles at an average speed of 30mph. How long does it take her? Give your answer in hours and minutes.

Solution

We are working out time, so we use the formula $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$.

$$\begin{aligned}\text{Time} &= \frac{\text{Distance}}{\text{Speed}}\\ &= \frac{129}{30} = 4.3\end{aligned}$$

4·3 hours is not 4 hours 3 minutes. 0·3 × 60 = 18, so 4·3 hours = **<u>4 hours 18 minutes</u>**.

Further examples of assessment style questions requiring knowledge of speed, distance and time can be found in:

- Converting Measurements: **Assessment Style Example** on Page 33.
- Time Zones: **Assessment Style Example** on Page 48.

Reading Scales

An essential mathematical skill is to read a number from a measuring scale.
Scales usually have **major divisions** and **minor divisions**. The major divisions are the main ones: in the diagram on the right, these are shown by the bolder, longer lines. The minor divisions are the smaller sub-divisions between the major ones.

Usually (but not always) the major divisions will be numbered and the minor divisions will be unnumbered. At National 5 level, you are expected to read a scale to the nearest "marked, minor, unnumbered" division.

In order to do this, it is essential to work out what numbers the minor divisions are 'going up in'. If you are struggling to do this, the following may help:

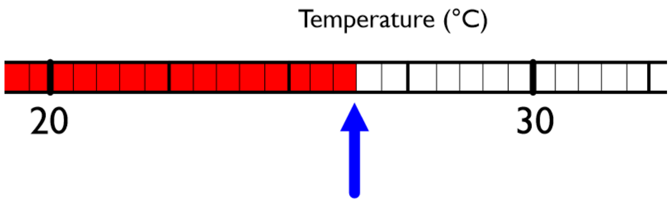
Each minor division on a measuring scale will 'go up in' the amount given by the calculation:
Difference between marked numbers ÷ Number of minor divisions between marked numbers

The next example goes into this question in a lot of detail. Many people do not need this level of detail and can just 'do it'. However the question is explained in detail to offer a method to anybody who struggles with this type of question.

BASIC SKILL EXAMPLE: Reading a Scale

The temperature in a medical store room is measured using a thermometer.

The diagram on the right shows the thermometer.
What is the temperature in the store room?



Solution
The two numbers marked are 20°C and 30°C. This is a difference of 10°C.
There are 20 minor division between 20°C and 30°C.

Using the formula above, we are 'going up in':
Difference between marked numbers ÷ Number of minor divisions between marked numbers
= 10°C ÷ 20 = 0·5°C

Counting up (13 minor divisions) in 0·5°C from 20°C gives us a measurement of **26·5°C**
Note: The diagram in this question is also used in Assessment Style Example on page 34.

Ratio

As well as fractions and percentages, another way to describe the proportions in which quantities are split up is with **ratio**. Ratios consist of numbers separated by a colon symbol e.g., 2:3, 4:1, 3:2:4.
For example, it might be said that a particular shade of purple paint is made by mixing red paint and blue paint in the ratio 4:5. This means that for every 4 litres (or spoonfuls, tins, gallons) of red paint, you must add 5 litres (or spoonfuls, tins, gallons…) of blue paint to get the correct shade of purple.

BASIC SKILL EXAMPLE 1: sharing in a ratio

Dave and Jim share £7200 in the ratio 7:2. How much money does each get?

Solution

Dave gets 7 shares of the money, Jim gets 2 shares.
This means that there are **9 shares** in total.

Dave gets 7 out of 9 shares:

$\frac{7}{9}$ of £7200

$= 7200 \div 9 \times 7$

$=$ £5600

Jim gets 2 out of 9 shares:

$\frac{2}{9}$ of £7200

$= 7200 \div 9 \times 2$

$=$ £1600

Therefore Dave gets £5600 and Jim gets £1600.

Ratios can be simplified. Simplifying a ratio is very similar to simplifying a fraction: divide through all the numbers by a common factor.

BASIC SKILL EXAMPLE 2: simplifying a ratio

Simplify the ratios:

(a) **15:10**

(b) **4:2:12**

Solution

(a) The highest common factor of 15 and 10 is 5, so we divide by 5.

$_{\div 5}\,15:10\,_{\div 5}$

$=$ <u>3: 2</u>

(b) The highest common factor of 4, 2 and 12 is 2, so we divide by 2.

$_{\div 2}\,4:2:12\,_{\div 2}$

$=$ <u>2:1: 6</u>

<u>Assessment Style Example</u>

The table on the right shows the numbers of boys and girls in each year of a Primary School. What is the ratio of girls to boys in its simplest form?

	Boys	Girls
P1	25	25
P2	30	25
P3	28	34
P4	15	17
P5	22	24
P6	40	20
P7	30	25

Solution

The total number of boys
$= 25 + 30 + 28 + 15 + 22 + 40 + 30 = 190.$

The total number of girls
$= 25 + 25 + 34 + 17 + 24 + 20 + 25 = 170.$

We are asked for the ratio of girls to boys (not boys to girls), so the girls' number must come first and the boys' number second. The ratio is 170:190.

We can simplify this by dividing through both numbers by 10. The simplified ratio is **<u>17:19</u>**.

When we know a ratio and we know any one related quantity, we can work out what the other quantities must be to match. We can do this using the principle of equivalent ratios: multiplying (or dividing) through by a common scale factor.

BASIC SKILL EXAMPLE 3: equivalent ratios

A cake is made using flour, butter and sugar in the ratio 4:5:2.
If 785 grams of butter are used, how much flour and sugar should be used?

Solution

To keep track of the numbers, it can be useful to lay the question out as a table, although this is not essential. In the first row of the table, we insert the numbers from the ratio (4, 5 and 2). In the final row of the table, we insert any numbers we have been told in the question (in this case 785g).

flour	butter	sugar
4	5	2
	785g	

We need to multiply through all numbers by the same number, so we need to work out what number was used in the middle column, where 5 ×_____= 785g.

785 ÷ 5 = 157, so we multiply all numbers by 157.

flour	butter	sugar
$_{\times 157}$ 4	5	2 $_{\times 157}$
628g	785g	314g

The answer is that <u>628g flour</u> and <u>314g sugar</u> should be used.

Direct and Indirect Proportion

A **direct proportion** question is one where two sets of numbers change at the same rate. The method for one of these questions is usually to find the cost for *one* first.

BASIC SKILL EXAMPLE 1a: Direct proportion

Jo is an electrician. She charges customers £27 for every 15 minutes she has to work.
How much will a customer have to pay Jo for a job that lasts 3 hours?

Solution

<u>Step One</u> – How much does it cost for <u>one</u> hour?

15 minutes cost £27. There are four ‘lots’ of 15 minutes in an hour,
So for one hour it costs 27 × 4 = £108

<u>Step Two</u> – How much does it cost for three hours?

Multiply: 108 × 3 = **<u>£324</u>**.

Numbers in direct proportion will always have the same ratio between them, so we can also use a ratio method to obtain the answer.

An alternative solution involves a table, as used in the ratio section on page 24.

BASIC SKILL EXAMPLE 1b: Alternative solution

(Same question as Basic Skill Example 1a above)

Alternative Solution

First set up a table containing the numbers we are given (£27 and 15 minutes) in the top row; and the number we are given (3 hours) in the bottom row. We can't mix units (15 minutes and 3 hours), so we write 0·25 hours instead of 15 minutes.

time (hours)	cost (£)
$0 \cdot 25$	27
3	

$3 \div 0{\cdot}25 = 12$, so we multiply through by 12. *(There are other possible methods, such as ×4 first, then ×3)*

time (hours)	cost (£)
$_{\times 12}\, 0 \cdot 25$	$27_{\times 12}$
3	324

The final answer is **£324**.

An **indirect proportion** question is one in which when one set of numbers *increases*, the other set of numbers *decreases* in proportion.

When two quantities are in indirect proportion, their **product** is always the same.

BASIC SKILL EXAMPLE 2: Indirect proportion

The time taken to build a house is inversely proportional to the number of builders employed.

If it takes 4 days to build a house when 6 builders are employed, how long would it take to build a house when 8 builders are employed?

Solution

When two variables are in indirect proportion, their product is always the same.

For the example given, the product is $4 \times 6 = 24$.

Therefore for the other example given, days × 8 = 24.

We can get the time by doing $\frac{24}{8} = 3$, so the job would take 3 days.

Solving an Equation

Solving an equation is not a required skill for the National 5 Lifeskills Mathematics syllabus.

However it is included in these notes because it is a skill that may come in useful in 'problem solving' questions (mostly those that require you to 'go backwards' to find out an original number when you know the answer). An example is given in the next section.

There are two commonly taught methods. The first is *change side and do the opposite*.

BASIC SKILL EXAMPLE 1a: Solving an equation using 'change side do the opposite'

Solve algebraically the equation $8a + 4 = 34$

Solution

Step one – move the '+4' over to the other side to become '–4'.

Step two – divide by the 8.

Step three – write down the answer.

Step four (check your answer) – double check that 8 × 3·75 + 4 does equal 34.

$$\begin{aligned} 8a + 4 &= 34 \\ 8a &= 34 - 4 \\ 8a &= 30 \\ a &= \frac{30}{8} \\ a &= 3{\cdot}75 \end{aligned}$$

The second commonly taught method is 'balancing' where you do the same operation to both sides of an equation.

BASIC SKILL EXAMPLE 1b: Solving an equation using Balancing

Solve algebraically the equation $8a + 4 = 34$

Solution

Step one – take 4 away from each side.
Step two – divide both sides by 8.
Step three – write down the answer.

Step four (check your answer) – double check that 8 × 3·75 + 4 does equal 34.

$$\begin{aligned} 8a + 4_{-4} &= 34_{-4} \\ 8a_{\div 8} &= 30_{\div 8} \\ a &= 3{\cdot}75 \end{aligned}$$

Using a Formula

For National 5, you need to be able to use a formula in a real-life context both with or without a calculator.

BASIC SKILL EXAMPLE 1: Using a formula

The distance, *s* metres, travelled by an accelerating object is given by the formula $s = ut + \frac{1}{2}at^2$, where *s* means distance (in metres), *t* means time (in seconds), *a* means acceleration (in m/s²) and *u* means the original speed (m/s).

Calculate the distance travelled when the original speed is 40m/s, the time is 7 seconds and the acceleration is 4m/s².

Solution

First we have to match up the information given in the final sentence with the letters in the formula. The first paragraph explains what each letter stands for:

- 'Calculate the distance travelled' means 'Find *s*'
- 'The original speed is 40m/s' means '$u = 40$'.
- 'The time is 7 seconds' means '$t = 7$'.
- 'The acceleration is 4 m/s²' means '$a = 4$'.

(continued on next page)

(Basic Skills Example 1 continued…)

So the question is saying 'Find s when $u = 40$, $t = 7$ and $a = 4$'.

$$s = ut + \frac{1}{2}at^2$$
$$= 40 \times 7 + \frac{1}{2} \times 4 \times 7^2$$
$$= 280 + 2 \times 49$$
$$= 280 + 98$$
$$= \underline{378\text{m}} \quad \text{(remembering to put units in our answer)}$$

You need to be able to use a formula normally, but also to go backwards to work out what one of the original numbers must have been when you know the final answer. To do this we can use the solving an equation skill outlined on page 26.

BASIC SKILL EXAMPLE 2: Going backwards with a formula

Using the same formula as the last example $\left(s = ut + \frac{1}{2}at^2\right)$, calculate u when $s = 294$, $t = 6$ and $a = 3$.

Solution

Step One – put all the numbers in and do any sums you can to simplify

$$s = ut + \frac{1}{2}at^2$$
$$294 = u \times 6 + \frac{1}{2} \times 3 \times 6^2$$
$$294 = 6u + 54$$

Step Two – solve the equation you are left with:

$$6u + 54 = 294$$
$$6u = 294 - 54$$
$$6u = 240$$
$$u = \frac{240}{6} = \underline{40\text{m/s}}$$

Probability and Expected Frequency

Probability is a measure of how likely an event is:

- If the event is **impossible**, the probability is 0.
- If the event is **certain** to happen, the probability is 1.
- If the event is between certain and uncertain, the probability is given as a fraction or a percentage or a decimal.

BASIC SKILL EXAMPLE 1: Probability
of First Year pupils.

A girl is picked at random from this sample, what is the probability that they are wearing a tie?

	Wearing a tie	*Not wearing a tie*
Boys	40	22
Girls	29	9

Solution
We are told that a girl is picked, so we do not take boys into account.
In total there are 29 + 9 = 38 girls.
29 of these were girls wearing a tie.

Answer: the probability is $\frac{29}{38}$. (As a decimal this is 29 ÷ 38 = 0·763 (to 3 d.p.), as a percentage this is 76·3% (to 1d.p.).)

To compare probabilities, decimals (or percentages) are the easiest to use. However probabilities as a fraction could still be compared without a calculator using the method outlined in the example on page 20.

BASIC SKILL EXAMPLE 2: Comparing Probabilities
In Jackson High School there are 900 pupils and 522 are girls.
In Sienna High School there are 200 pupils and 104 are girls.

A pupil is picked at random from each school to represent the school on a TV show. In which school is the probability of picking a girl higher? Explain your answer.

Solution
The probability of picking a girl in Jackson High School is $\frac{522}{900}$ = 522 ÷ 900 = 0 · 58 (or 58%).

The probability of picking a girl in Sienna High School is $\frac{104}{200}$ = 104 ÷ 200 = 0 · 52 (or 52%).

Decision: the probability is greater in Jackson High School because 0·58 > 0·52.

Understanding Graphs

In the *Managing Finance and Statistics* unit (starting on page 83) you will be expected to calculate statistics and draw your own graphs. In the *Numeracy* unit you will be expected to read information from graphs that have already been drawn.

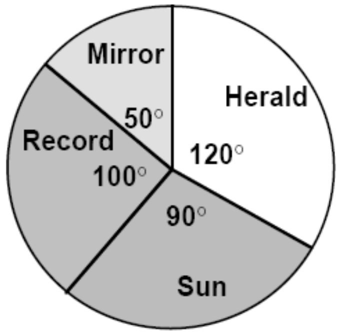
It is not possible to cover every type of graph in these notes, but some of the key methods are covered in this section. Elsewhere in these notes are Scatter Graphs (covered starting on page 83) and Box Plots (covered starting on page 86).

Pie Charts

When interpreting a pie chart, we can work out the fraction of any particular slice if we know its angle. The fraction will always be $\frac{\text{angle}}{360}$.

BASIC SKILL EXAMPLE 1: Interpreting a pie chart

1800 people were asked what newspaper they bought. The pie chart on the right shows the results. How many people bought *The Mirror*?



Solution

The slice for *The Mirror* is 50°, so the fraction of people who chose *The Mirror* is $\frac{50}{360}$.

There were 1800 people in total, so we calculate $\frac{50}{360}$ of 1800:

1800 ÷ 360 × 50 = 250, so **250** people said they bought *The Mirror*.

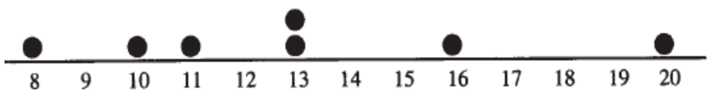
For the Expressions and Formulae unit, you also need to know how to draw a pie chart, which is covered on page 93.

Dot Plots

A dot plot is a very easy way of showing a list of numbers in a diagram. It is a lot like a bar graph or a frequency table, in which every dot represents one individual number.

BASIC SKILL EXAMPLE 2: Dot Plot

The dot plot on the right shows the age of Betty's grandchildren.



(a) How many grandchildren does Betty have?
(b) How old is her second youngest grandchild?

Solution

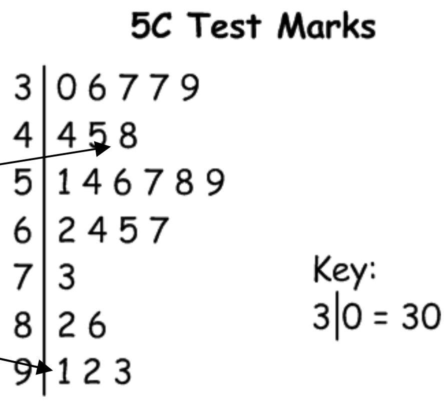
The dot plot shows that the ages of Betty's grandchildren are: 8, 10, 11, 13, 13, 16 and 20.
(a) Betty has 7 grandchildren.
(b) Betty's second youngest grandchild is aged 10.

Stem and Leaf Diagrams

A stem and leaf diagram is a way of arranging a list of numbers in a structured order whilst still being able to tell what the original numbers were.

For example the numbers on the left below can be written as the stem-and-leaf diagram on the right.

93	44	37	57	82	92
39	54	65	67	30	48
86	91	58	64	37	62
59	73	45	56	51	36



It is also possible to compare two stem-and-leaf diagrams at once by showing them in the same diagram called a 'back-to-back' stem-and-leaf diagram, such as the one in the next example.

BASIC SKILL EXAMPLE 3: Reading a Stem and Leaf Diagram

The back-to-back stem and leaf diagram below shows the number of people who went to the cinema to see two different films every night for ten nights.

Comedy Film		Action Film
8	4	1 7 8
4 0	5	2 3 5 6
1 1 1	6	0 4
7 6 5 3	7	5

8 | 4 = 48 people   4 | 1 = 41 people

a) **What was the largest number of people who went to see the action film?**
b) **What was the smallest number of people who went to see the comedy film?**

Solution

The number of people who went to see the comedy film are:
48, 50, 54, 61, 61, 61, 73, 75, 76, 77

The numbers of people who went to see the action film are:
41, 47, 48, 52, 53, 55, 56, 60, 64, 75

Therefore the answers to the question are as follows:

a) 75 people.

b) 48 people.

Geometry and Measures Unit

Measurement

Converting Measurements, including Time

You will be expected to convert between different units of measurement. To do this you should be familiar with all the key facts in the table below. Many of these should be basic to you.

Length
1 kilometre (km) = **1000** metres (m)
1 metre (m) = **1000** millimetres (mm)
1 metre (m) = **100** centimetres (cm)
1 centimetre (cm) = **10** millimetres (mm)

Volume
1 litre (l) = **1000** millilitres (ml)
1 litre (l) = **1000** cubic centimetres (cm^3)
1 millilitre (ml) = **1** cubic centimetre (cm^3)

Weight
1 tonne (t) = **1000** kilograms (kg)
1 kilogram (kg) = **1000** grams (g)
1 gram (g) = **1000** milligrams (mg)

Time
1 normal year = **365** days
1 leap year = **366** days
1 year = **12** months
1 hour = **60** minutes
1 minute = **60** seconds

BASIC SKILL EXAMPLE 1: Converting measurements (tonnes and kilograms)
Change 125700kg into tonnes.

Solution
1 tonne = 1000kg, so we divide by 1000. 125700 ÷ 1000 = 125·7 tonnes

BASIC SKILL EXAMPLE 2: Converting measurements (hours and minutes)
Change 8·35 hours into hours and minutes.

Solution
8·35 hours = 8 hours + 0·35 hours = 8 hours____minutes.
We need to change 0·35 hours into minutes.

1 hour = 60 minutes, so we multiply by 60 to change hours into minutes.

0·35 × 60 = 21 minutes.
So 8·35 hours = 8 hours 21 minutes.

BASIC SKILL EXAMPLE 3: Converting measurements (hours and minutes)
Change 3 hours 47 minutes into hours.

Solution
We need to change 47 minutes into hours.

1 hour = 60 minutes, so we divide by 60 to change minutes into hours.

47 ÷ 60 = 0·78333… (keep at least 3 decimal places, preferably more)

47 minutes = 0·783 hours, so 3 hours 47 minutes = 3·783 hours (rounded to 3 d.p.).

Assessment Style Example

A racing driver travels 1·6km in a time of 1 minute 22·5 seconds. What is his speed in metres per second?

Solution

To give our speed in metres per second we must convert the distance into metres and the time into seconds.

1km = 1000m. Therefore 1·6km = 1·6 × 1000 = 1600m.
1 minute = 60 seconds. Therefore 1 minute 22·5s = 1× 60 + 22·5 = 82·5s.

$$\begin{aligned}\text{Speed} &= \frac{\text{Distance}}{\text{Time}}\\ &= \frac{1600}{82\cdot5}\\ &= 19\cdot3939....\\ &= \underline{\underline{19\cdot4\text{ m/s}}}\text{ (1 d.p.)}\end{aligned}$$

Tolerance

In real-life situations, when measuring, it is not always possible to be completely exact. We have to accept that errors may be made and that the final answer may not be completely accurate. However some errors are acceptable and others are not, this depends on the situation.

- If you were building a bookcase, making it 1mm too wide is probably OK as it will probably still fit in the room.
- If you are making a key, making it 1mm too wide is probably not OK as the key probably will not fit in the lock.

The degree of accuracy that we are prepared to accept is called the **tolerance**. The tolerance is usually written using the '±' sign. A tolerance of ±1p means 'you are allowed to be up to 1p away from the real measurement'. A tolerance of ±5mm means 'you are allowed to be up to 5mm away from the real measurement'.

The tolerance allows us to work out a **minimum** and a **maximum** value. Any values between (and, usually, including) these values are acceptable. Any value that is lower than the minimum or higher than the maximum is unacceptable.

BASIC SKILL EXAMPLE: Tolerance

The body temperatures temperature of a person in good health is 36·8°C ± 0·4°C. Write down the minimum and maximum body temperatures of a person in good health.

Solution

Minimum temperature is 36·8 – 0·4 = 36·4°C
Maximum temperature s 36·8 + 0·4 = 37·2°C

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, tolerance questions should always:

- Require you to work out the maximum and minimum values.
- Require you to compare at least one value to test if it is within the given range.
- Be set in a real-life context.

You **might** also have to:

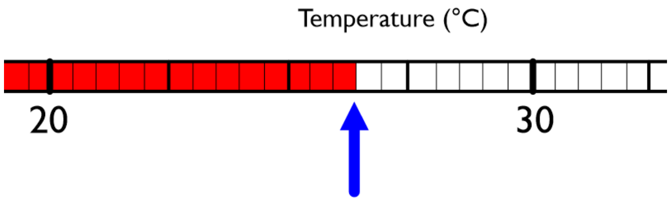
- Write an explanation for a decision (see **Assessment Style Example** on page 34).

(There are many ways questions may be adapted and so this list can never cover everything).

When comparing a measurement it is important to remember that it should be compared to both the maximum and the minimum before we can decide if it acceptable.

Assessment Style Example

A medicine store room at a hospital must be kept at a temperature of 26·2°C ± 0·4°C or the medicines will not work.



The diagram on the right shows a thermometer displaying the current temperature in the store room. Will the medicine work? Explain your answer.

Solution

The diagram is the same diagram used in the example on page 23. Using the method on that page, the measurement is 26·5°C.

The maximum temperature for the medicines to work is 26·2°C + 0·4°C = 26·6°C.
The minimum temperature for the medicines to work is 26·2°C – 0·4°C = 25·8°C.

The *decision* is that the medicines will work.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

- Yes, because the temperature is 26·5°C which is in between 25·8°C and 26·6°C. *(comparing to both limits at once)*
- Yes, because the temperature is 26·5°C which is more than 25·8°C and less than 26·6°C. *(comparing to both limits separately)*
- Yes, because the temperature is 0·1°C below the maximum and 0·3°C above the minimum. *(stating the difference between the limits).*

You would **not** get a mark for these explanations:

- Yes because the temperature is 26·5°C which is less than 26·6°C *(no mention of the minimum)*
- Yes because the temperature is 26·5°C which is more than 25·8°C *(no mention of the maximum)*
- Yes because the temperature is between 25·8°C and 26·6°C *(just repeating the limits, no mention of the measurement)*

Scale Drawing

You are expected to be able to construct a scale drawing. A scale drawing is (usually) a reduction of the original diagram with all lengths kept in proportion.

A scale drawing used in this course would usually have a scale of the form 1cm =_____cm (or 1cm =_________km, 1cm =______metres etc.). This could also be expressed as a ratio in the form 1:_. The number in the scale is known as the **scale factor**.

- A scale of 1cm = 30cm could be expressed as 1:30. The scale factor is **30**.
- A scale of 1:200 could also be expressed as 1cm = 200cm (or 1cm = 2m). The scale factor is **200**.

When a scale is expressed as a ratio, the first number refers to the **measurement on the diagram**, and the second number refers to the **measurement in real-life**. So if a diagram has been enlarged with a scale of 5:2 this means that 'every 5cm on the page represents 2cm in real-life'.

If you are asked to use a scale drawing it is essential that you have a ruler (and possibly a protractor). You will probably be expected to draw all lengths to a tolerance of ±2mm. Angles have to be exactly the same size – they do not change in a scale drawing, but you will probably be allowed to draw them to a tolerance of ±2°C.

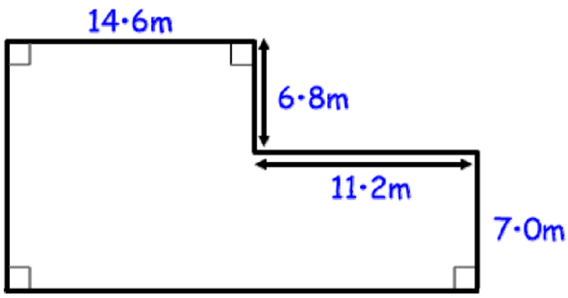
Once a scale drawing has been produced it could also be used to work out another real-life length that was not included in the original diagram. You can use the scale factor to calculate lengths:

- To find out a real-life length, you **multiply** by the scale factor.
- To find out how long a line should be on the page, you **divide** by the scale factor.

BASIC SKILL EXAMPLE 1: constructing a scale drawing when the scale is given

The diagram on the right shows a rough sketch of a function room used for weddings.

Using a scale of 1:200, make a scale drawing of the room.



Solution

It will be useful to convert all lengths from metres to centimetres first. We do this by multiplying each length by 100.

The marked lengths then become 1460cm, 680cm, 1120cm and 700cm.

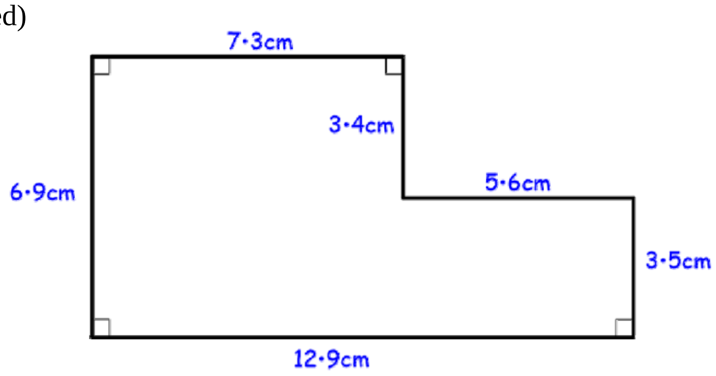
The scale factor is 200. We divide each length by 200 to obtain the lengths in the scale drawing.

1460 ÷ 200 = 7·3cm
680 ÷ 200 = 3·4cm
1120 ÷ 200 = 5·6cm
700 ÷ 200 = 3·5cm
The long side on the left should be 3·5 + 3·4 = 6·9cm
The long side along the bottom should be 7·3 + 5·6 = 12·9cm

(continued on next page)

(Basic Skills Example 1 continued)

The completed diagram should look like the one on the right. All lengths must be drawn to a tolerance of ±2mm and all right angles must be measured accurately. The actual diagram printed on this page will not be the exact size.



What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, scale drawing questions should always:

- Ask you to make an enlargement or reduction of a given sketch.
- Be set in a real-life context.

You **<u>might</u>** also have to:

- Choose your own scale (more likely in unit assessments than the final exam) (see **Assessment Style Example** on page 36 below).
- Use your diagram to calculate another real-life distance using the scale factor (see **Assessment Style Example** on page 36 below).

(There are many ways questions may be adapted and so this list can never cover everything).

At National 5 level you will be expected to choose a suitable scale. However the idea of a 'suitable' scale is a vague one: you can choose <u>any</u> scale that you like so long as your diagram will fit onto your page. There will always be lots of possible scales you could choose.

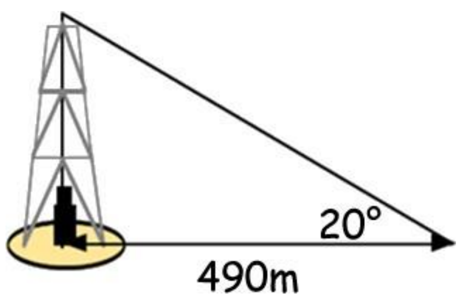
If you are not confident with choosing a scale, the following tips might be useful:

1. Write your scale in the form 1cm =_________.
2. The units on the right-hand side should match the units used for the real-life distances in the question (e.g. if the real life distances in the question are 980**km** and 860**km**, then your scale will be 1cm =_______**km**).
3. Choose any number you like to be the scale factor, but if in doubt, choose:
   - The number 10, 100 or 1000.
   - Or a number that divides 'nicely' into the numbers in the question (e.g. if the numbers in the question are 15km and 18km, you could use the number 3).
4. Start drawing the diagram, beginning with the longest length.
   - If the diagram is going to be too big to fit on the page, choose a <u>bigger</u> number to be the scale factor instead.
   - If the diagram is going to be too small, choose a <u>smaller</u> number instead.

<u>Assessment Style Example</u>

The diagram on the right is a sketch of an oil rig in a desert.

Using a suitable scale, make a scale drawing of the triangle in the diagram and use it to find the real-life height of the oil rig.



Solution

Task One – choose the scale

We need to choose a scale in the form 1cm =_______m.

If we use the scale 1cm = 10m, the longest edge would be 49cm long which is too long to fit on the page.

We could choose to use a scale of **1cm = 20m**, so the scale factor is 20. However this is a random choice – other scales are possible.

Task Two – calculate the lengths needed for the drawing (the angles stay the same).

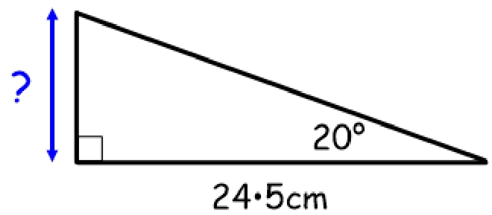
> **Fact:** To find out how long a line should be on the page,
> **divide** the real-life length by the scale factor.

There is only one real-life length (490m).

490 ÷ 20 = 24·5cm.

Task Three – draw your diagram accurately.

We draw a triangle on our page with a horizontal length of 24·5cm (with an allowed tolerance of ±2mm), and angles of 90° and 20° (with an allowed tolerance of ±2°). This is shown in the diagram on the right, however the actual diagram printed on this page will not be the exact size.



Task Four – use your diagram to calculate the real-life lengths.

> **Fact:** To find out a real-life length,
> **multiply** the length on the page by the scale factor.

We now measure the height of the oil rig on the page (indicated by a question mark in the diagram). If the diagram has been drawn perfectly, you should get a measurement of 8·9cm.

To find a real-life distance, we multiply the length on the page by the scale factor. The real-life distance is then 8·9 × 20 = **178 metres**.

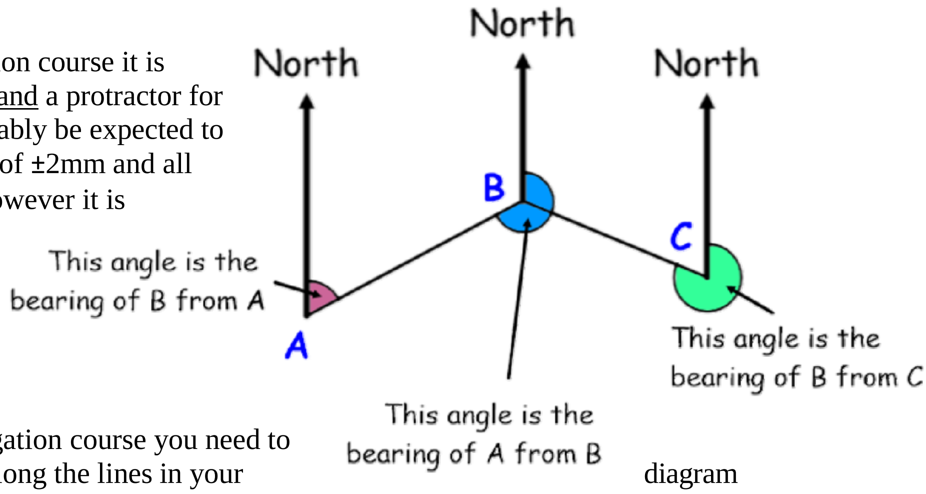
Bearings and Navigation

For unit assessments, you need to be able to plot a **navigation** course showing a journey when given distances and three-figure bearings. A three-figure bearing is a way to describe a direction more accurately than a compass. The bearing is the angle measured clockwise from North (with North being 000°).

You also have to use three figures, so we write 085° instead of 85°, and we write 002° instead of 2°.

When describing bearings, it is important to go in the correct direction. The bearing of A **from** B refers to the angle that is centred at B and pointing towards A.

When asked to draw a navigation course it is essential that you have a ruler <u>and</u> a protractor for these questions. You will probably be expected to draw all lengths to a tolerance of ±2mm and all angles to a tolerance of ±2°, however it is possible that the SQA might get stricter and ask for a tolerance of ±1mm and ±1° in the future, so be as accurate as you can.



When drawing your own navigation course you need to use the scale to work out how long the lines in your diagram must be.

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, navigation and bearings questions should always:

- Ask you to draw a navigation course when told at least two sets of distances and bearings.
- Be set in a real-life situation.

You **<u>might</u>** also have to:

- Choose your own scale (more likely in unit assessments than the final exam) (see **Assessment Style Example** on page 36 above).
- Use your diagram to calculate another real-life distance using the scale factor (see **Assessment Style Example** on page 38 below).
- Work out one or more of the real-life lengths first using Pythagoras (see page 50), or Speed-Distance-Time (see pages 21 and 32) techniques.

(There are many ways questions may be adapted and so this list can never cover everything).

<u>Assessment Style Example</u>

Some soldiers are marching across the countryside. From the start, they march:
2400m on a bearing of 040° to reach a lake;
then
800m on a bearing of 200° to reach a hut.

Using the scale 1cm = 200m, make a scale drawing of the soldiers' route and use your scale drawing to calculate the soldiers' distance from their starting point.

Solution

<u>Task One</u> – choose the scale

On this occasion the scale (1cm = 200m) was chosen for us, so we don't need to do this.

<u>Task Two</u> – calculate the lengths needed for the drawing (the angles do not change).

Fact: To find out how long a line should be on the page, **divide** the real-life length by the scale factor.

We divide the real-life lengths by the scale factor (200).

<u>1st leg of journey:</u>

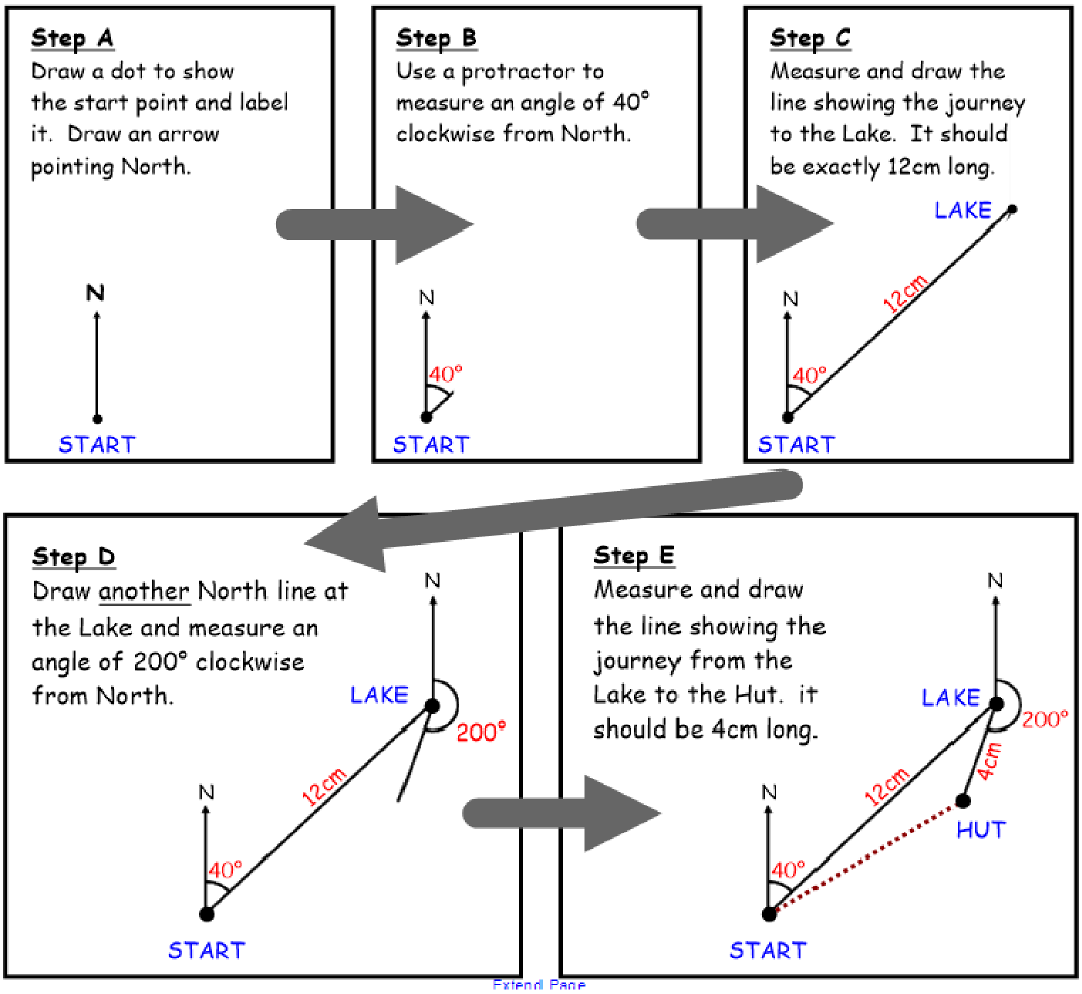
$2400 \div 200 = 12$, *so we will draw a line* ***12cm*** *long on a bearing of* ***040°*** *(40° clockwise from North).*

<u>2nd leg of journey:</u>

$800 \div 200 = 4$, *so we will draw a line* ***4cm*** *long on a bearing of* ***200°*** *(200° clockwise from North).*

<u>Task Three</u> – draw your diagram accurately.

The steps involved in drawing the route are outlined here. The finished diagram should look something like the **<u>Step E</u>** picture. Always annotate (label) your diagram thoroughly with all lengths, angles and place names.



<u>Step Four</u> – use your diagram to calculate the real-life lengths.

Fact: To find out a real-life length, **multiply** the length on the page by the scale factor.

We measure the distance on the page from the 'start' point to the 'end' point, (indicated by a dotted line in the diagram above). If the diagram has been drawn perfectly, the dotted line should measure 8·35cm. The real life distance is then $8{\cdot}35 \times 200 =$ **<u>1670m.</u>**

Container packing

You need to be able to work out how to pack smaller three-dimensional objects inside larger containers. When doing so, we have to bear a number of factors in mind:

- It is *essential* that none of the edges of the smaller objects end up being too big for the larger container.
- It is OK to have extra space left over. However we want as little unused space as possible as unused space could result in wasted money to a business.
- Some objects may have to be stacked a particular way up so that they do not break.

To find out how many objects fit in, we need to do a division sum with the lengths of the objects and the length of the container. It is not possible to have a fraction of an object so if the answer is a fraction we have to round down (never up) to the nearest whole number.

BASIC SKILL EXAMPLE 1: how many objects can you fit in?

A tin of beans has diameter 8·5cm.

A supermarket shelf measures 120cm by 48cm. What is the largest number of tins that can be fitted in one layer on the shelf?

Solution

For the 120cm edge: $120 \div 8{\cdot}5 = 14{\cdot}111$, so 14 cans can be fitted along that edge.
For the 48cm edge: $48 \div 8{\cdot}5 = 5{\cdot}647$, so 5 cans can be fitted along that edge.

In total, $14 \times 5 = 70$ cans are able to be fitted on the shelf.

What should an exam question look like?

In assessments for National 5 Lifeskills, container packing questions should always:

- Involve packing smaller items into larger containers. The larger containers will all be the same size. The smaller items may be all the same size, or may vary.
- Ask you to find the best way of packing, so that you can fit the maximum possible number of smaller items into each larger container.
- Be set in a real-life context.

You **might** also have to:

- Identify different ways that the smaller containers could be turned around and determine how this affects the maximum (see **Basic Skill Example 2** on page 41, **Assessment Style Example 1** on page 42 and Assessment Style Example 2 on page 43 below).
- Convert between units of measurement (see Assessment Style Example 2 on page 43).
- Work out the best order to pack *differently* sized items (see **Assessment Style Example 3** on 44).

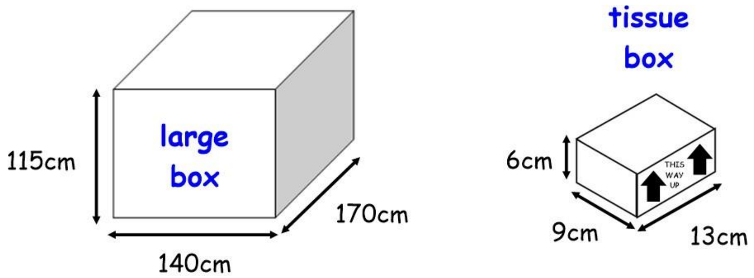
(There are many ways questions may be adapted and so this list can never cover everything).

When stacking smaller boxes inside a larger container, it matters which way around you turn the smaller containers.

- If the smaller container is a **cuboid** and **has to** be placed a particular way up (e.g. to prevent breakage) there are two possible ways that it can be turned.
- If the smaller container is a **cuboid** and **does not have to** be placed a particular way up then there are six possible ways that it can be turned.
- If the smaller container is a **cylinder** and **does** not have to be placed a particular way up then there are three possible ways that it can be turned.

BASIC SKILL EXAMPLE 2: container packing when smaller items are all the same size and shape

Tissue boxes measuring 13cm by 9cm by 6cm are being packed into a larger box measuring 115cm by 140cm by 170cm as shown in the diagram.



The boxes must be packed upright so the tissues stay in a neat pile. What is the maximum number of tissue boxes that can be packed into the larger box?

Solution

Step One – identify the different ways that the smaller boxes can be stacked.

The tissue boxes must be kept upright. This means that the two vertical heights (6cm on the tissue box, and 115cm on the larger box) must be lined up.

We can make a table showing the two ways that the tissue boxes can be stacked. The numbers in the top row are the dimensions of the larger box. The numbers in the other rows are the dimensions of the smaller box showing how they line up with the larger one. *The only number that can go in the 115cm column is 6cm, because they have to line up. The other numbers (9cm and 13cm) can go either way around in the other two columns.*

	115cm	**140cm**	**170cm**
Method 1	6cm	9cm	13cm
Method 2	6cm	13cm	9cm

Step Two – work out how many boxes will fit in along each edge.

We now need to do a division sum for each pair of numbers. Each answer must be rounded **down** (not up) to the nearest whole number. The decimal parts of the answers are shown in brackets and are ignored.

	115cm	**140cm**	**170cm**
Method 1	115 ÷ 6 = **19**(·16…)	140 ÷ 9 = **15**(·55…)	170 ÷ 13 = **13**(·07…)
Method 2	115 ÷ 6 = **19**(·16…)	140 ÷ 13 = **10**(·76…)	170 ÷ 9 = **18**(·88…)

Step Three – for each arrangement, multiply the three results to find the total number of smaller boxes that can be fitted in.

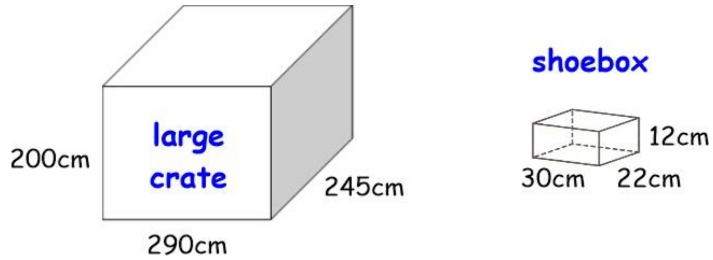
Method 1: 19 × 15 × 13 = 3705 tissue boxes.
Method 2: 19 × 10 × 18 = 3420 tissue boxes.

Step Four – conclusion.

The maximum number of tissue boxes that can be fitted into the larger box is 3705 because the other arrangement gives 3420, which is less than 3705.

Assessment Style Example 1

Shoe boxes measuring 30cm by 22cm by 12cm are being packed into a large crate measuring 200cm by 245cm by 290cm.



It does not matter which way around the shoe boxes are packed. Find the maximum number of shoeboxes that can be packed into the large crate, and show which way around the shoe boxes must be packed to allow this to happen.

Solution

Step One – identify the different ways that the smaller boxes can be stacked.

On this occasion, the boxes do not need to be stacked upright, so we can make a table showing the six ways that the tissue boxes can be stacked.

	290cm	**245cm**	**200cm**
Method 1	30cm	22cm	12cm
Method 2	30cm	12cm	22cm
Method 3	22cm	30cm	12cm
Method 4	22cm	12cm	30cm
Method 5	12cm	30cm	22cm
Method 6	12cm	22cm	30cm

Step Two – work out how many boxes will fit in along each edge.

*Each answer must be rounded **down** (not up) to the nearest whole number. The decimal parts of the answers are shown in brackets and are ignored.*

	290cm	**245cm**	**200cm**
Method 1	290 ÷ 30 = **9**(·66…)	245 ÷ 22 = **11**(·13…)	200 ÷ 12 = **16**(·66…)
Method 2	290 ÷ 30 = **9**(·66…)	245 ÷ 12 = **20**(·41…)	200 ÷ 22 = **9**(·09…)
Method 3	290 ÷ 22 = **13**(·18…)	245 ÷ 30 = **8**(·16…)	200 ÷ 12 = **16**(·66…)
Method 4	290 ÷ 22 = **13**(·18…)	245 ÷ 12 = **20**(·41…)	200 ÷ 30 = **6**(·66…)
Method 5	290 ÷ 12 = **24**(·16…)	245 ÷ 30 = **8**(·16…)	200 ÷ 22 = **9**(·09…)
Method 6	290 ÷ 12 = **24**(·16…)	245 ÷ 22 = **11**(·13…)	200 ÷ 30 = **6**(·66…)

Step Three – for each arrangement, multiply the three results to find the total number of smaller boxes that can be fitted in.

- Method 1: 9 × 11 × 16 = 1584 shoeboxes.
- Method 2: 9 × 20 × 9 = 1620 shoeboxes.
- Method 3: 13 × 8 × 16 = 1664 shoeboxes.
- Method 4: 13 × 20 × 6 = 1560 shoeboxes.
- Method 5: 24 × 8 × 9 = 1728 shoeboxes. (*** the maximum)
- Method 6: 24 × 11 × 6 = 1584 shoeboxes.

Step Four – conclusion.

The maximum number of shoeboxes that can be fitted into the large crate is 1728.
This happens when:

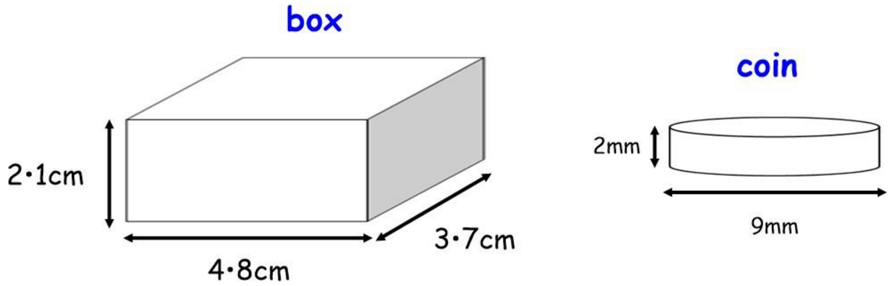
- The 12cm edge of the shoebox is lined up against the 290cm side of the crate.
- The 30cm edge of the shoebox is lined up against the 245cm side of the crate.
- The 22cm edge of the shoebox is lined up against the 200cm side of the crate.

When the smaller shape is a cylinder, you may only be given two dimensions. However the cylinder is actually a 3-dimensional shape, so we need to use three numbers: it just happens that two of the dimensions are the same, as the diameter is both the length and breadth of the cylinder.

Assessment Style Example 2

Chocolate coins are in the shape of a cylinder with diameter 9mm and height 2mm. They must be packed in piles inside a cuboid box measuring 2·1cm by 4·8cm by 3·7cm.

It does not matter which way around the coins are packed. Find the maximum number that can be packed into one box.



Solution

The diameter of the cylinder is 9mm. The three dimensions of the cylinder are therefore 2mm, 9mm and 9mm (again).

The dimensions of the cuboid are in centimetres, and the dimensions of the cylinder are in millimetres. We cannot mix units within a question, so we convert one set of units. The dimensions of the cuboid in millimetres are 21mm, 48mm and 37mm.

Step One – identify the different ways that the smaller items can be stacked.
On this occasion, the coins do not need to be stacked upright, so we can make a table showing the three ways that the coins can be stacked.

	21mm	**48mm**	**37mm**
Method 1	2mm	9mm	9mm
Method 2	9mm	2mm	9mm
Method 3	9mm	9mm	2mm

Step Two – work out how many boxes will fit in along each edge.
We now need to do a division sum for each pair of numbers. Each answer must be rounded **down** (not up) to the nearest whole number. The decimal parts of the answers are shown in brackets and are ignored.

	21mm	**48mm**	**37mm**
Method 1	21 ÷ 2 = **11**(·5)	48 ÷ 9 = **5**(·33…)	37 ÷ 9 = **4**(·11…)
Method 2	21 ÷ 9 = **2**(·33…)	48 ÷ 2 = **24**	37 ÷ 9 = **4**(·11…)
Method 3	21 ÷ 9 = **2**(·33…)	48 ÷ 9 = **5**(·33…)	37 ÷ 2 = **18**(·5)

Step Three – for each arrangement, multiply the three results to find the total number of smaller boxes that can be fitted in.

- Method 1: 11 × 5 × 4 = 220 coins (**** the maximum).
- Method 2: 2 × 24 × 4 = 192 coins.
- Method 3: 2 ×5 × 18 = 180 coins.

Step Four – conclusion.
The maximum number of coins that can be fitted into each box is 220.

It is possible that the smaller items will be of different sizes. In these questions, you will be asked to find the best way of packing the items in order to fit all the smaller items in. You want to minimise wasted space, and you will need to take account of any special instructions given in the question.

Many people will just 'experiment' until they find a method that works, and this is fine, but the following tips may be useful:

- It is **essential** that you read any instructions in the questions clearly. For example you may be told that certain items must be put in the same container; or kept separate from other items (see example below).
- Try to start by filling containers completely, rather than leaving small gaps.
- It can be useful to deal with the biggest items first; as they have the least flexibility as to where they can go. You can leave smaller items until later as they may be able to 'fill in the leftover gaps' in some of the containers.
- It can be useful to keep track of how much space is left in each container.

Assessment Style Example 3

A farm shop is packing fruit and vegetables into five crates.
Each crate can hold a maximum of 20kg.
The apples and bananas must be packed in a different crate to all of the other items.
Find a way of packing the following items into the crates in order to minimise the number of crates needed:

- **12kg Potatoes**
- **4kg Carrots**
- **8kg Broccoli**
- **15kg Apples**
- **9kg Swede**
- **6kg Cabbage**
- **7kg Turnips**
- **5kg Rhubarb**
- **13kg Lettuce**
- **2kg Leeks**
- **3kg Bananas**
- **11kg Parsnips**

Solution

Draw a diagram to represent the crates. It is also useful to keep track of how much space is left inside each crate:

Crate 1	**Crate 2**	**Crate 3**	**Crate 4**	**Crate 5**
20kg left	20kg left	20kg left	20kg left	20kg left

Start by following the instructions in the question. Apples and Bananas must be packed separately to the rest, so put them in a crate on their own. There is space left over, but we won't be able to fill this because the question tells us that we can't pack anything else in to that particular container.

Crate 1	**Crate 2**	**Crate 3**	**Crate 4**	**Crate 5**
15kg Apples 3kg Bananas				
2kg left (can't use this)	20kg left	20kg left	20kg left	20kg left

There are no restrictions on the other items, so try and fill some of the crates completely by finding combinations of items that add to make 20kg (such as the 12kg potatoes and 8kg broccoli). It may help to choose the biggest item first (e.g. the 13kg Lettuce), and then to try and spot some smaller items that can add on to it to make 20kg (i.e. the 7kg turnips). Cross off the items as you go along so that you don't miss any, or put an item in twice. One possible arrangement is as follows (there are others):

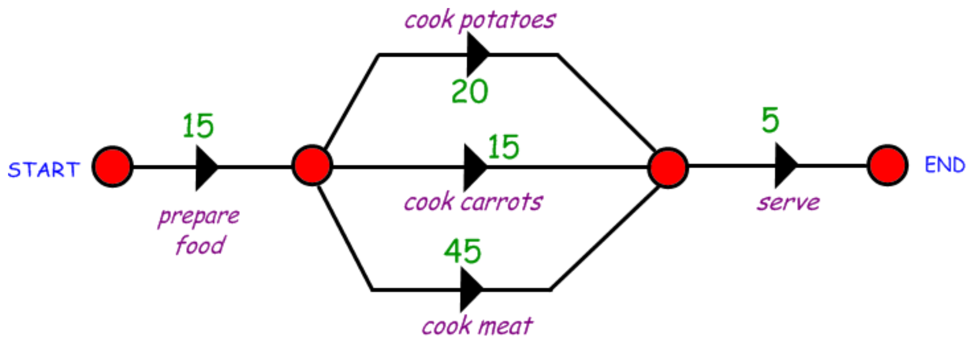
Crate 1	**Crate 2**	**Crate 3**	**Crate 4**	**Crate 5**
15kg Apples 3kg Bananas	13kg Lettuce 7kg Turnips	12kg Potatoes 8kg Broccoli	11kg Parsnips 9kg Swede	
2kg left (can’t use this)	FULL	FULL	FULL	20kg left

The remaining items should then fit in the final crate(s). The following is one possible final solution:

Crate 1	**Crate 2**	**Crate 3**	**Crate 4**	**Crate 5**
15kg Apples 3kg Bananas	12kg Potatoes 8kg Broccoli	11kg Parsnips 9kg Swede	13kg Lettuce 7kg Turnips	4kg Carrots 6kg Cabbage 5kg Rhubarb 2kg Leeks
2kg left (can’t use this)	FULL	FULL	FULL	3kg left

Time: Task Planning

An activity network diagram is one way of showing all the difference smaller tasks that go together to make up a larger task. For example below is a network diagram showing the process required to cook an evening meal. The numbers on each arrow indicate the **time** (in minutes) required for each smaller task.



Definition: the **critical path** is the <u>longest</u> path in the network. It is called the critical path because it tells us the shortest possible time in which this task can be completed.

The critical path is “critical”, because if any job on its path is delayed, the entire job will be delayed. However other jobs not on the critical path *could* possibly take a little longer without the whole overall task being delayed.

BASIC SKILL EXAMPLE 1: critical path

For the activity network above representing the evening meal, determine the critical path and the overall shortest time required to cook the meal.

<u>Solution</u>

The longest path is:

Prepare food + Cook meat + Serve = 15 + 45 + 5 = 65 minutes.

Therefore this is the critical path, and the shortest possible time for the task is **65 minutes**.

Definition: a **prerequisite task** is a task that must be completed before others can be started.

For example, in the diagram above for cooking the evening meal, the task "serve" has prerequisites of "cook potatoes", "cook carrots" and "cook meat". All three of these jobs have "prepare food" as a prerequisite.

There are two ways of indicating prerequisite tasks:

1. Using an **activity network**, as in the diagram above.
2. In a **precedence table**.

A precedence table for the evening meal example could look like the table on the right. To minimise space, each of the tasks has been assigned a letter.

Task		Prerequisite
A	Prepare Food	-
B	Cook potatoes	A
C	Cook carrots	A
D	Cook meat	A
E	Serve	B, C, D

BASIC SKILL EXAMPLE 2: construct a precedence table

The following tasks are involved in 'baking a cake'
They are not in the correct order for doing the job.

- **Go home from shop**
- **Turn on oven**
- **Remove cake from oven**
- **Make cake mixture**
- **Ice cake**
- **Make icing**
- **Bake cake mixture in oven**
- **Buy ingredients in shop**

Put the tasks in order and construct a precedence table to show any prerequisites

Solution
A precedence table is shown on the right.

Note: 'Make icing' has been put in position F, which is probably the most likely place. However it is possible that it could also be put in position C, D or E.

Task		Prerequisite
A	Buy ingredients	-
B	Go home from shop	A
C	Turn on oven	B
D	Make cake mixture	B
E	Bake cake mixture	C, D
F	Make icing	B
G	Remove from oven	E
H	Ice cake	F, G

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, task planning questions should always:

- Involve a precedence table.
- Be set in a real-life context.

You **might** also have to:

- Calculate the minimum time for the task (see **Assessment Style Example** on page 46).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example (2014 SQA exam question, slightly adapted)

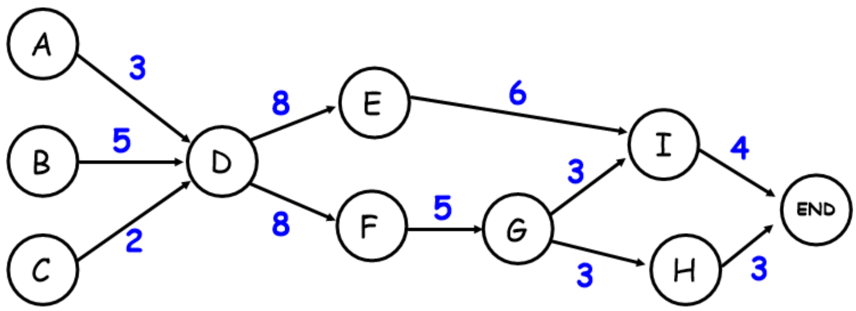
The Clark family are having a new kitchen fitted by a company called Kitease. Kitease provide a team of workers to install the kitchen. The precedence table shows the list of tasks and the time required for each.

Task	Detail	Preceding Task	Time (hours)
A	Begin electrics	None	3
B	Build cupboards	None	5
C	Begin plumbing	None	2
D	Plaster walls	A, B, C	8
E	Fit wall cupboards	D	6
F	Fit floor cupboards	D	5
G	Fit worktops	F	3
H	Finish plumbing	G	3
I	Finish electrics	E, G	4

(a) Construct an activity network.
(b) What is the minimum possible time that in which this kitchen can be installed?

Solution

(a) Taking each task in turn we can construct an activity network as follows. It is essential to check that there is one arrow going from every letter in the ‘preceding task’ column to the corresponding letter in the ‘Task’ column.



(b) The minimum time is shown by the longest path through the network.

The longest path is B → D →F → G → I → END.

The time taken is 5 + 8 + 5 + 3 + 4 = 25 hours, so the minimum time for the job is 25 hours.

Time: Time Zones

Definition: A **time zone** is an area of the world in which all the people use the same time.

The sun rises and sets at different times around the world. For that reason different countries choose to have different times. The time in another country may be a few hours ahead or behind the time we use in the UK.

Definitions
- The Time Zone we use in the UK is called **Greenwich Mean Time** (GMT).
- Another name for GMT is **Co-ordinated Universal Time** (UTC) because standard time across the world is based on the time measured in Greenwich.
- When we refer to **local time** we are always referring to the time in the specific place being referred to.

A country whose time is 2 hours ahead of the UK could be described as GMT+2 or UTC+2.

A country whose time is 11 hours behind the UK could be described as GMT–11 or UTC–11.

When working with time zones, if we go past midnight when adding or subtracting times then the date changes as well as the time.

BASIC SKILL EXAMPLE 1: Time Zones

The time in Beijing is 8 hours ahead of GMT (GMT+8).
If the time in the UK is 8:30pm on the 1st March, what is the date and time in Beijing?

Solution

We add 8 hours on to UK time: the time in Beijing is 4:30am on the 2nd March. The date has changed because we went past midnight when adding on the 8 hours.

BASIC SKILL EXAMPLE 2: Working across more than one time zone

The time in Moscow is GMT+3. The time in Las Vegas is GMT–8.
If the time in Moscow is 1725 on 25th December, what is the date and time in Las Vegas?

Solution

The time difference between Moscow and Las Vegas is 3 – (–8) = 11 hours, with Las Vegas being 11 hours behind Moscow.

We subtract 11 hours from the Moscow time: the time in Las Vegas is 0625. We have not gone past midnight when subtracting the time, so the date has not changed: it is also the 25th December.

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, time questions should always:
- Require working across time zones.
- Be set in a real-life context.

You **might** also have to:
- Do a calculation involving speed, distance and time (see **Assessment Style Example** on page 48).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

A plane leaves London at 0820 and flies from London to Rio de Janeiro. The time in Rio de Janeiro is 3 hours behind GMT.

The distance flown by the plan is 5875 miles and the average speed of the plane is 550 miles per hour.

What is the local time in Rio de Janeiro when the plane lands?

Solution

For this journey the distance is 5875 miles and the speed is 550mph and we need to calculate the time taken. The formula for time taken is $T = \frac{D}{S}$.

$$T = \frac{D}{S} = \frac{5875}{550}$$
$$= 10 \cdot 6818\ldots\ldots \text{hours}$$

We need to change 10·6818… hours into hours and minutes.

$0 \cdot 6818 \times 60 = 40 \cdot 908 = 41$ minutes.
Therefore $10 \cdot 6818$ hours = 10 hours 41 minutes.

The plane leaves at 0820 and lands 10 hours 41 minutes later. Therefore the plane lands at 1901 *London time*.

We now change this to Rio de Janeiro time by subtracting three hours. The plane lands at 1601 local (Rio de Janeiro) time.

Assessment Style Example 2

The time in Athens, Greece is 7 hours ahead of the time in New York, USA. Jen lives in New York. Her partner Jo is working in Athens for two months.

Jen is at home in New York and able to make calls between 1800 and 2245 (New York time).
Jo is in her hotel room in Athens and able to receive calls between 1730 and 0115 (Athens time).

Between which times (New York time) is Jen able to call Jo?

Solution

We need to identify the times when Jen is able to make calls and Jo is able to receive them. To do this we must convert all the times in the question into New York times.

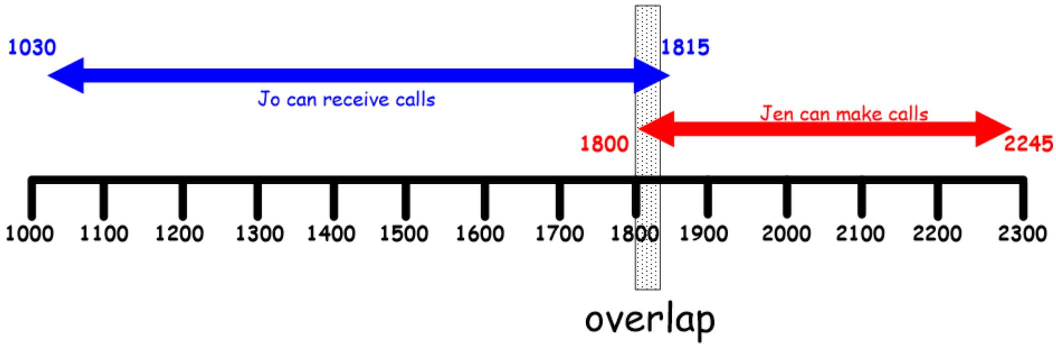
Athens is 7 hours ahead of New York, so we must take 7 hours off all Athens times to change them into New York time:

- 1730 Athens time = 1030 New York time.
- 0115 Athens time = 1815 New York time.

Jen can make calls between 1800 and 2245 (New York time).
Jo can receive calls between 1030 and 1815 (New York time).

The only times between which Jen and Jo are both available are 1800 to 1815, so Jen should call between these times.

If you find it hard to work out which times are the important ones, try drawing a diagram like the one below and looking for the overlap.



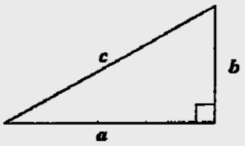
Geometry

Pythagoras' Theorem

At National 4 level you will have learnt that when you know the length of any two sides of a right angle triangle you can use Pythagoras' Theorem (usually just known as **Pythagoras**) to find the length of the third side without measuring.

Formula: given on the formula sheet in National 5 Lifeskills Mathematics assessments

Theorem of Pythagoras:



$a^2 + b^2 = c^2$

There are three steps to any Pythagoras question:

Step One – square the length of the two given sides.

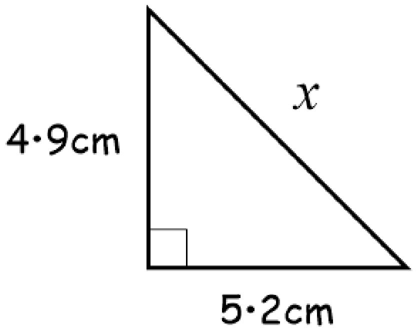
Step Two – either add or take away:

- If you are finding the length of the longest side (the hypotenuse), you **add** the squared numbers.
- If you are finding the length of a shorter side, you **take away** the squared numbers.

Step Three – square root.

BASIC SKILL EXAMPLE 1: Pythagoras for the hypotenuse

Calculate the length of *x* in this triangle.



Solution

We are finding the length of *x*.

x is the hypotenuse, so we **add**:

$x^2 = 4\cdot9^2 + 5\cdot2^2$

$x^2 = 51\cdot05$

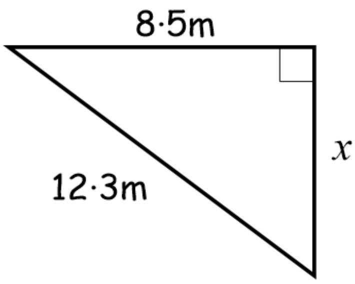
$x = \sqrt{51\cdot05}$

$x = 7\cdot1449....$

$x = \underline{7\cdot1\text{cm}}$

BASIC SKILL EXAMPLE 2: Pythagoras for a shorter side

Calculate the length of *x* in this triangle.



Solution

We are finding the length of *x*.

x is a smaller side, so we take away.

$x^2 = 12\cdot3^2 - 8\cdot5^2$

$x^2 = 79\cdot04$

$x = \sqrt{79\cdot04}$

$x = 8\cdot8904....$

$x = \underline{8\cdot9\text{ cm}}$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, Pythagoras questions should always:

- Be set in a real-life context.
- Ask you to do a **'two-step'** calculation. This means you will have to work out some (or all) of the lengths in the triangle before you can begin using Pythagoras. It is possible that you might have to use Pythagoras twice.

You **might** also have to:

- Round your answer to a specific number of significant figures (see **Assessment Style Example 1** on page 51).
- Use Pythagoras in a 3-dimensional situation (see **Assessment Style Example 1** on page 51).
- Calculate the area of the triangle (see **Assessment Style Example 2** on page 52).
- Calculate a related cost (see **Assessment Style Example 2** on page 52).
- Use Pythagoras with a triangle drawn inside a circle (see **Assessment Style Example 3** and **Assessment Style Example 4** beginning on page 53).

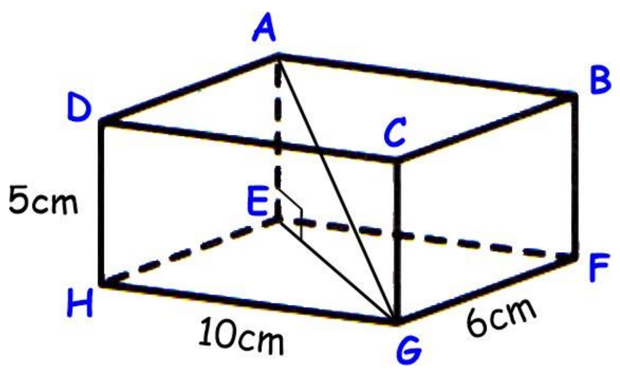
(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

The diagram shows a box that can be used for posting parcels.

(a) Calculate the diagonal length AG. Round your answer to 3 significant figures.

(b) A customer has bought a metal rod of length 13cm. Can the metal rod be posted inside this box? Explain your answer.



Solution

(a) To find length AG, we have to use Pythagoras in triangle AEG. However before we can do this, we must calculate length EG.
First we use Pythagoras in triangle EGH, in which EG is the hypotenuse, and the other sides are 10cm and 6cm.

$$EG^2 = 10^2 + 6^2$$
$$= 136$$
$$EG = \sqrt{136}$$
$$= 11\cdot 67$$ (no need to round to 3 s.f. yet as this is not the end of the question)

Now we use Pythagoras in triangle AEG, which has hypotenuse AG, and other sides of length 5cm, and 11·67cm.

$$AG^2 = 11\cdot 67^2 + 5^2$$
$$= 161\cdot 1889$$
$$AG = \sqrt{161\cdot 1889}$$
$$= 12.696...$$ (you MUST write your unrounded answer first)

$= \underline{\underline{12.7\text{cm}}}$ (3 s.f.)

b) The diagonal length is the longest length in the whole box, so if the rod cannot fit along the diagonal, it can definitely not fit inside the box any other way.

The *decision* is that No, the rod cannot be posted.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

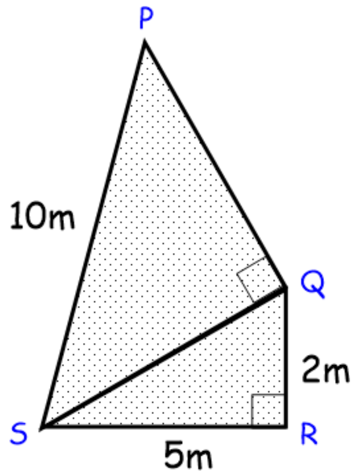
- No because the longest length in the box is 12·7cm which is less than 13cm.
- No, because 12·7cm < 13cm.
- No, she will need 0·3cm more space.

You would **not** get a mark for these explanations:

- No, she has less than 13cm *(no mention of the calculated number)*

Assessment Style Example 2

A sail for a boat is in the shape of two triangles as shown in the diagram.



(a) Calculate the total area of the sail.

(b) The sail is made out of material that costs £34·70 per square metre. Calculate the total cost of the material required for the sale. Round your answer to 2 significant figures.

Solution

(a) First calculate the Area of Triangle QRS:

$$A = \frac{BH}{2}$$
$$= \frac{2 \times 5}{2} = 5\text{m}^2$$

To calculate the Area of Triangle PQS, we must first obtain both the base QS and the height PQ. We can obtain QS using Pythagoras in triangle QRS. We can then use Pythagoras again in the second triangle.

$$QS^2 = 5^2 + 2^2$$
$$= 29$$
$$QS = \sqrt{29}$$
$$= 5 \cdot 385....$$
$$= 5 \cdot 39\text{m}$$

$$PQ^2 = 10^2 - 5.39^2$$
$$= 70.9479$$
$$PQ = \sqrt{70 \cdot 9479}$$
$$= 8 \cdot 423....$$
$$= 8 \cdot 42\text{m}$$

Now we use these values to calculate the area of PQS:

$$A = \frac{BH}{2}$$
$$= \frac{5 \cdot 39 \times 8 \cdot 42}{2}$$
$$= 22 \cdot 6919 = 22 \cdot 69\text{m}^2$$

Total Area = Area of PQS + Area of QRS
= 22·69 + 5
= 27·69m²

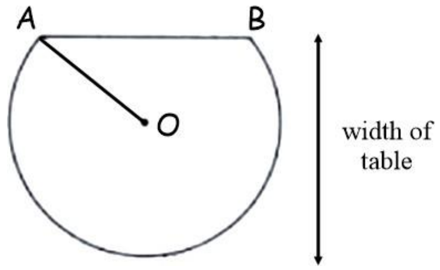
(b) Total cost = Area × Price per square metre
= 27·69 × 34·70
= 960·843 (MUST write the unrounded answer before rounding)
= £960 (2 s.f.)

If there is a right-angle inside a triangle in a diagram, then you can use Pythagoras to work out other lengths inside a right-angled triangle. The radius of the circle will be the hypotenuse of the triangle.

Assessment Style Example 3

A table top is in the shape of part of a circle.
The centre of the circle is O.
AB is a chord of the circle.
AB is 70cm.
The radius OA is 40cm.

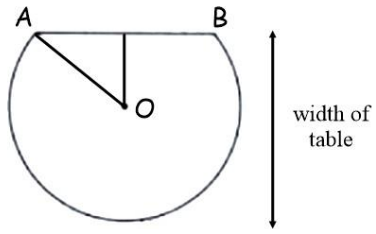
Calculate the width of the table.



Solution

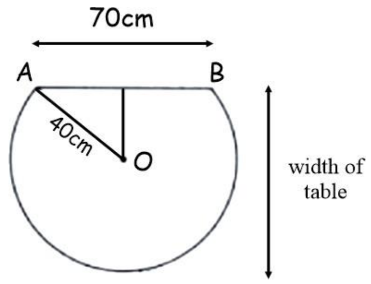
Step one – draw a new line in the diagram to make the diagram symmetrical.

In the diagram on the right, this is OC.
This means that triangle OAC is right-angled.



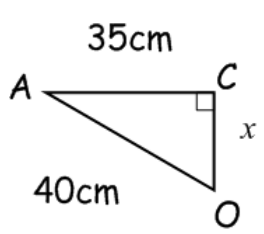
Step two – write in the length of the radius and any other lengths we are told.

OA is 40cm and AB is 70cm.



Step three – identify the lengths in the right-angled triangle, using the fact that the 'new' line splits the chord in half.

In this diagram, AB is 70cm, so AC must be 35cm.



Step four – use Pythagoras to calculate the other length in the triangle.

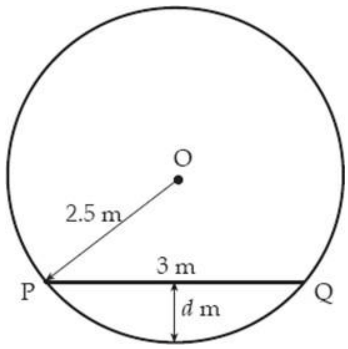
$$x^2 = 40^2 - 35^2$$
$$= 375$$
$$x = \sqrt{375} = \underline{19 \cdot 4\text{cm (1d.p.)}}$$

Step five – in this question, we were asked for the width of the table. The width of the table is OC + the radius. So in this diagram, the width is 19·4 + 40 = 59·4cm.

Assessment Style Example 4

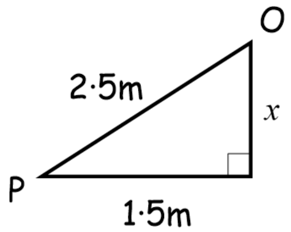
The diagram shows the cross-section of a cylindrical oil tank.
The centre of the circle is O.
PQ is a chord of the circle.
PQ is 3m.
The radius OP is 2·5m.

Find the depth, *d*, of the oil.



Solution

First identify the lengths in the right-angled triangle, using the fact that the diagram is symmetrical. (see previous example for more detail on how to do these steps).



Now use Pythagoras to calculate the other length in the triangle.

$$\begin{aligned} x^2 &= 2\cdot5^2 - 1\cdot5^2 \\ &= 4 \\ x &= \sqrt{4} \\ &= \underline{2\text{m}} \end{aligned}$$

Finally you need to check whether you have answered the whole question

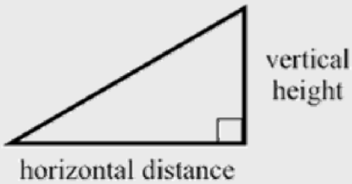
In this question, they wanted the depth of the water. From the diagram, *d* and *x* add together to make a radius (2·5m). Since x = 2 metres, *d* must be <u>0·5 metres</u>.

Gradient

The **gradient** of a slope is a measure of its steepness. The higher the gradient is, the steeper the slope is.

Formula: <u>given</u> on the formula sheet in National 5 Lifeskills Mathematics assessments

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$



In some situations it makes sense to talk about positive or negative gradients. A **positive** gradient (e.g. 2, $\frac{1}{2}$, 0·364) means the line slopes **upwards**. A **negative** gradient (e.g. –2, $-\frac{1}{2}$, –0·364) means the line slopes **downwards**.

- A gradient of 2 means ‘along 1, up 2’.
- A gradient of –3 means ‘along 1, down 3’
- A gradient of $\frac{3}{4}$ means ‘along 1, up $\frac{3}{4}$’, but it is more easily thought of as ‘along 4, up 3’

BASIC SKILL EXAMPLE: How to calculate the gradient of a slope

Calculate the gradient of the slope whose side view is pictured on the right.

Solution

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$
$$= \frac{4\cdot 2}{28\cdot 5}$$
$$= \underline{\underline{0\cdot 147}} \text{ (3 d.p.)}$$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, gradient questions should always:

- Be set in a real-life context.
- Expect you to calculate one or both of the horizontal or vertical distances before the calculation (e.g. using subtraction or Pythagoras' theorem).

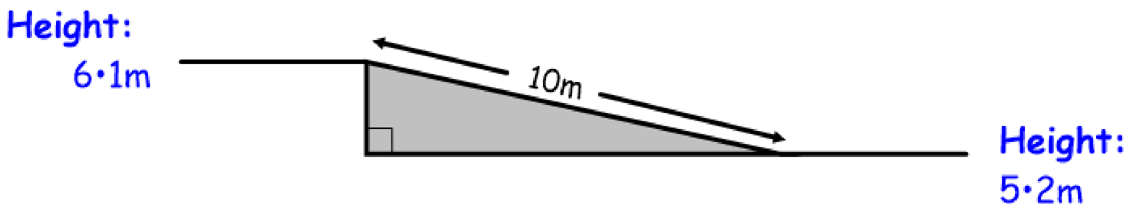
You **might** also have to:

- Make and explain a decision about whether a gradient is within a given limit (see **Assessment Style Example 1**).
- Work out dimensions when given the gradient (see **Assessment Style Example 2**).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

A 10 metre long wheelchair ramp is built between two levels of a sports stadium. Its cross section is shown in the diagram below.



(a) Calculate the gradient of the wheelchair ramp.

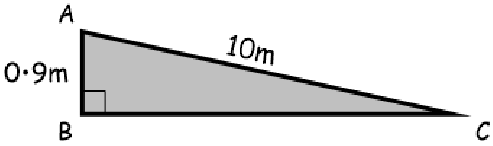
(b) Regulations say that the gradient of a wheelchair ramp must be less than 0·1. Does this ramp meet the regulations? Explain your answer.

Solution

a) We need to calculate the vertical distance and the horizontal distance.

Vertical distance = 6·1 – 5·2 = 0·9metres.

To find the horizontal distance (line BC In the diagram on the right), we use Pythagoras' Theorem:



$$BC^2 = 10^2 - 0\cdot 9^2$$
$$= 99\cdot 19$$
$$BC = \sqrt{99\cdot 19} = 9\cdot 96\text{m}$$

$$\begin{aligned}\text{Gradient} &= \frac{\text{Vertical height}}{\text{Horizontal distance}}\\ &= \frac{0 \cdot 9}{9 \cdot 96}\\ &= 0 \cdot 09036...\\ &= \underline{\underline{0 \cdot 09}} \text{ (2 d.p.)}\end{aligned}$$

b) The *decision* is that Yes, the ramp does meet regulations.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

- Yes because 0·09 is less than 0·1.
- Yes because it is 0·01 under the limit.

You would **not** get a mark for these explanations:

- No, it is less than 0·1 *(no mention of the calculated number).*

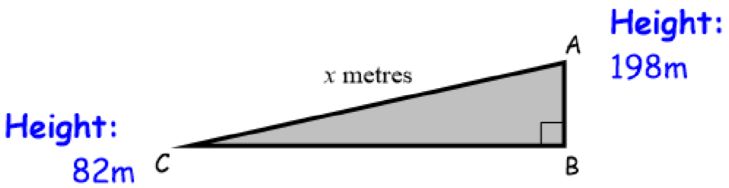
Assessment Style Example 2

To travel from the village of Pelton to the local tourist beauty spot, walkers have to walk along a road up a slope that had gradient $\frac{1}{3}$.

The walkers start from a height of 82 metres and the beauty spot is at a height of 198 metres. How long is the road?

Solution

Since the question did not include a diagram we should begin by making our own diagram containing the information given. The length we have been asked to calculate is the diagonal distance AC, which has been marked *x*.



The Vertical distance (AB) = 198 – 82 = 116m.

To calculate the horizontal distance we must use the gradient formula, remembering that the question told us that the gradient was $\frac{1}{3}$.

$$\text{Gradient} = \frac{\text{Vertical height}}{\text{Horizontal distance}}$$

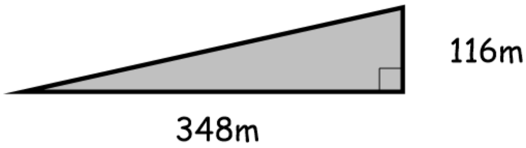
$$\frac{1}{3} = \frac{116}{BC}$$

by considering equivalent fractions

$$\begin{aligned}BC &= 116 \times 3\\ &= 348\text{m}\end{aligned}$$

Since we now know AB = 116 metres and BC = 348m, we can use Pythagoras to calculate AC:

$$AC^2 = 348^2 + 116^2$$
$$= 134560$$
$$AC = \sqrt{134560}$$
$$= 366 \cdot 824...$$
$$= \underline{\underline{366 \cdot 8\text{m}}} \text{ (1 d.p.)}$$



Area, Perimeter and Circles

At National 4 level you learnt to calculate the area or circumference of a circle. At National 5 level you will need to use the same formulae for *fractions* of a circle as part of a longer question investigating the area or perimeter or a composite shape.

Formulae: given on the formula sheet in National 5 Lifeskills Mathematics assessments:

Circumference of a circle: $C = \pi d$

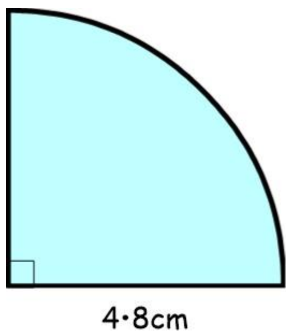
Area of a circle: $A = \pi r^2$

You are allowed to use 3·14 instead of π in any calculations. On a non-calculator paper, you will *have* to use 3·14.

Where a section of a shape is made from part of a circle, the length of a curved edge can be found using the formula for the circumference of a circle.

BASIC SKILL EXAMPLE 1: Circumference of a Fraction of a Circle

The diagram shows one quarter of a circle.
Calculate the length of the curved edge.



Solution

Radius is 4·8cm so diameter is double that (9·8cm).

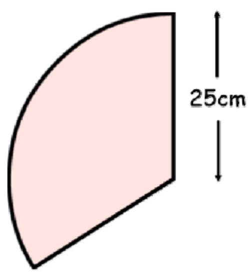
The formula for length of the curved edge is $C = \pi d$.
In this example, we have a quarter of a circle so we adapt the formula to be $C = \frac{1}{4}\pi d$.

$$C = \tfrac{1}{4}\pi d$$
$$= \pi \times 9 \cdot 6 \div 4 \qquad \text{(or } 1 \div 4 \times \pi \times 9 \cdot 6\text{)}$$
$$= 7 \cdot 5398...$$
$$= \underline{\underline{7 \cdot 5\text{cm}}} \text{ (1 d.p.)}$$

If a question asks for the **perimeter** of a shape, then all edges must be added together. Some may have to be calculated first.

BASIC SKILL EXAMPLE 2: Perimeter of Part of a Circle

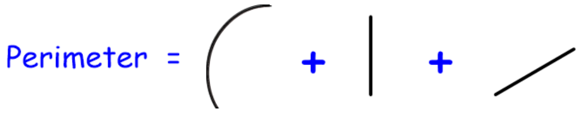
The shape pictured is made of one-third of a circle with radius 35cm. Find the perimeter of the shape.



(continued on next page…)

Solution (Basic Skill Example 2)

The perimeter is found by adding all three sides in the shape together.



Both of the straight lengths are 25cm. However we must calculate the curved length using the formula $C = \pi d$.

In this example, we have one-third of a circle so we adapt the formula to be $C = \frac{1}{3}\pi d$.

Radius is 25cm so diameter is double that (50cm).

$$C = \tfrac{1}{3}\pi d$$
$$= \pi \times 50 \div 3 \qquad \text{(or } 1 \div 3 \times \pi \times 50\text{)}$$
$$= 52 \cdot 3598...$$
$$= 52 \cdot 4\text{cm (1 d.p.)}$$

Perimeter = 52·4 + 25 + 25
= <u>102·4cm</u>

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, perimeter questions should always:

- Be set in a real-life context.
- Involve a **composite** shape (a shape with two or more shapes joined together) where one of the shapes is part of a circle.
- Expect you to calculate a curved length using the formula $C = \pi d$.

You **might** also have to:

- Round your answer to a number of significant figures (see **Assessment Style Example** on page 64, which is an example involving volume).
- Use the result of your area calculation to calculate a cost or other quantity (see **Assessment Style Example 2** on page 60, which is an example involving area).
- Make and explain a decision (see **Assessment Style Example 1** on page 58).
- Work out how many smaller objects can fit in to a larger one (see **Assessment Style Example** on page 65, which is an example involving volume).

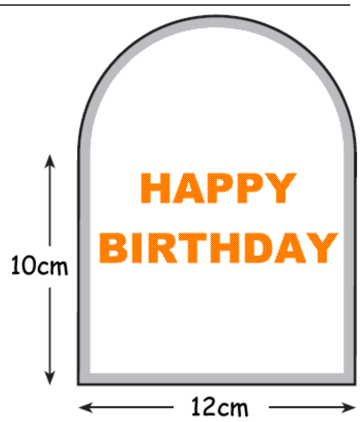
(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

Ellie is making a birthday card consisting of a rectangle and a semi-circle.

Ellie wants to put gold ribbon all around the border of the card.

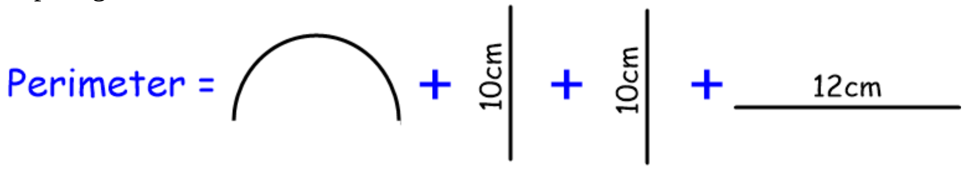
Ellie has 50cm of gold ribbon. Does she have enough to go all the way around the edge? Explain your answer.



Solution

The circle has a diameter of 12cm.

The perimeter is found by adding all four (three straight and one curved) sides of the shape together.



However we must calculate the curved length using the formula $C = \pi d$. In this example, we have half of a circle so we adapt the formula to be $C = \frac{1}{2}\pi d$ with diameter 12cm.

$$\begin{aligned} C &= \tfrac{1}{2}\pi d \\ &= \pi \times 12 \div 2 \qquad \text{(or } 1 \div 2 \times \pi \times 12\text{)} \\ &= 18\cdot849... \\ &= 18\cdot8\text{cm (1 d.p.)} \end{aligned}$$

Perimeter = 18·8 + 10 + 10 + 12
= <u>50·8cm</u>

The *decision* is that <u>No</u>, Ellie does not have enough border.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

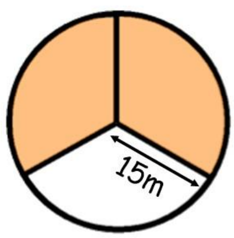
- No, because needs 50·8cm which is more than 50cm.
- No, she will need 0·8cm more.

You would **not** get a mark for these explanations:

- No, she needs more than 50cm *(no mention of the calculated number)*.

BASIC SKILL EXAMPLE 3: Area of part of a circle

The diagram shows a circle divided into three equal sections. The circle has radius 15 metres. Calculate the shaded area.



<u>Solution</u>

The formula for area of a circle is $A = \pi r^2$.
In this example, we have two-thirds of a circle so we adapt the formula to be $A = \frac{2}{3}\pi r^2$.

$$\begin{aligned} A &= \tfrac{2}{3}\pi r^2 \\ &= \pi \times 15^2 \div 3 \times 2 \qquad \text{(or } 2 \div 3 \times \pi \times 15^2\text{)} \\ &= 471\cdot2388... \\ &= \underline{\underline{471\cdot2\text{m}^2}}\text{ (1 d.p.)} \end{aligned}$$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, area questions should always:

- Be set in a real-life context.
- Involve a **composite** shape (a shape with two or more shapes joined together) where one of the shapes is part of a circle.
- Expect you to calculate the area of the circular part using the formula $A = \pi r^2$.

You **might** also have to:

- Round your answer to a number of significant figures (see **Assessment Style Example** on page 64, which is an example involving volume).
- Use the result of your area calculation to calculate a cost or other quantity (see **Assessment Style Example 2** on page 60, which is an example involving area).
- Make and explain a decision (see **Assessment Style Example 1** on page 58, which is an example involving perimeter).
- Work out how many smaller objects can fit in to a larger one (see **Assessment Style Example** on page 65, which is an example involving volume).

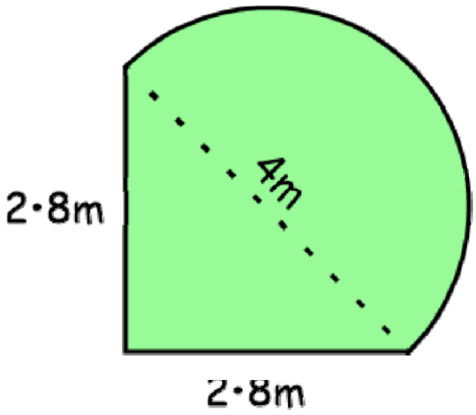
(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 2

Eddie and Jack have a garden in the shape of a triangle and a semicircle as shown in the diagram.

They want to buy grass seed to cover the garden. Grass seed is sold in packets that cost £4·99 each. Each packet of seed will cover an area of 1·8m².

How much will it cost Eddie and Jack to buy enough seed to cover the entire garden?



Solution

This is an area question. We must first calculate the area of the garden.

Area of triangle:

$$A = \frac{BH}{2} = \frac{2\cdot8 \times 2\cdot8}{2} = 3\cdot92\text{m}^2$$

Area of semicircle:

$$A = \pi r^2 \div 2 = \pi \times 2^2 \div 2 = 6\cdot28...\text{m}^2$$

Total Area = 3·92 + 6·28 = 10·2m².

One packet of seed covers 1·8m², so to cover the whole garden the number of packets Eddie and Jack need is given by 10·2 ÷ 1·8 = 5·66… (When the question requires rounding, you must state the unrounded answer first).

It is not possible to buy 0·6667 of a packet, so we must round up to the nearest whole number and Eddie and Jack must buy 6 packets. The total cost is 6 × £4·99 = **£29·94**.

Volumes of 3-d Shapes

Definitions
A **prism** is a 3d solid with a uniform cross-section. In everyday language, this means that is the same shape all the way along.
The **cross-section** is the shape at either end (and throughout the middle) of a prism.

Formula: given on the formula sheet in National 5 Lifeskills Mathematics assessments

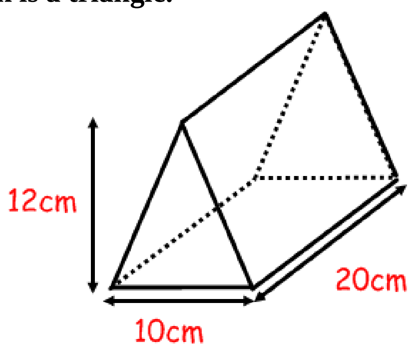
Volume of a prism: $V = \text{Area of cross-section} \times \text{height}$

$V = Ah$

You should also know the key formulae for areas of 2-d shapes given on page 20.

BASIC SKILL EXAMPLE 1: Volume of a (triangular) prism

Find the volume of this prism, whose cross section is a triangle.



Solution

The height of this prism is the distance from one (triangular) end to the other.

In this shape, the height is 20cm.

Step 1: Work out the area of the cross-section

In this shape, the cross-section is a triangle. The formula for the area of a triangle is $A = \frac{BH}{2}$

Important: you will use a different formula in each question, depending on whether the cross section is a rectangle, square, triangle, circle, semicircle etc.

$$A_{triangle} = \frac{BH}{2}$$
$$= 10 \times 12 \div 2 = 60\text{cm}^2$$

Step 2: Use the formula to find the volume

$$V = Ah$$
$$= 60 \times 20$$
$$= \underline{1200\text{cm}^3}$$

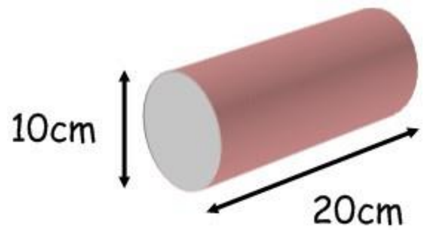
A cylinder is a special example of a prism with a circular cross-section. The method above can be adapted to derive a formula for the volume of a cylinder.

Formulae: given on the formula sheet in National 5 Lifeskills Mathematics assessments

Volume of a Cylinder:	$V = \pi r^2 h$
Volume of a Cone:	$V = \frac{1}{3}\pi r^2 h$
Volume of a Sphere:	$V = \frac{4}{3}\pi r^3$

BASIC SKILL EXAMPLE 2: Volume of a cylinder

Calculate the volume of this cylinder.



Solution

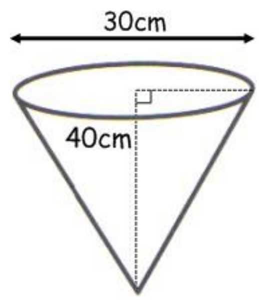
Diameter is 10cm so radius is 5cm

$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 20 \quad (\text{ or } \pi \times 5 \times 5 \times 20\ )$$
$$= 1570 \cdot 796327....$$
$$= \underline{1570 \cdot 8\text{cm}^3 (1 \text{ d.p.})}$$

In the cone formula, the 'height' refers to the <u>perpendicular</u> height (the one that goes straight up) and <u>not</u> any sloping heights.

BASIC SKILL EXAMPLE 3: Volume of a cone

Calculate the volume of this cone.



Solution

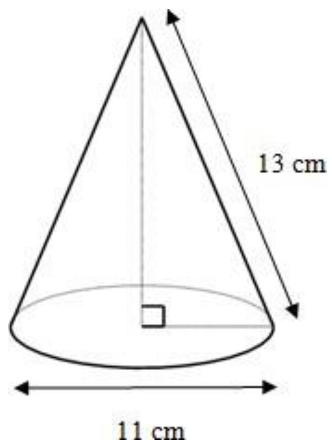
Diameter is 30cm so radius is 15cm

$$V = \tfrac{1}{3}\pi r^2 h$$
$$= \pi \times 15^2 \times 40 \div 3 \quad (\text{ or } 1 \div 3 \times \ \pi \times 15^2 \times 40\ )$$
$$= 9424 \cdot 777961....$$
$$= \underline{9424 \cdot 8\text{cm}^3 (1 \text{ d.p.})}$$

If a sloping height is given rather than the perpendicular height, Pythagoras must be used to obtain the perpendicular height.

<u>Assessment Style Example 1</u>

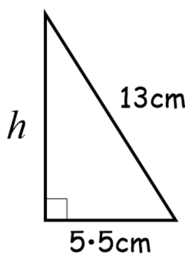
Metal components for a machine are made in the shape of a cone, with diameter 11 cm and slant height 13 cm, as shown in the diagram.
How many metal components can be made out of 16 litres of (melted) metal?



Solution

The radius of the cone is 5·5cm.

The radius, slant height and perpendicular height form a right-angled triangle as shown in the diagram below, in which the perpendicular height is labelled h.



We find h using Pythagoras:

$$h^2 = 13^2 - 5 \cdot 5^2$$
$$= 138 \cdot 75$$
$$h = \sqrt{138 \cdot 75}$$
$$= 11 \cdot 779...$$
$$= 11 \cdot 8\text{cm } (1 \text{ d.p.})$$

So the perpendicular height is 11·8cm

We now find the volume of a cone with radius 5·5cm and height 11·8cm.

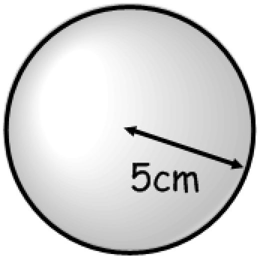
$$V = \tfrac{1}{3}\pi r^2 h$$
$$= \pi \times 5\cdot 5^2 \times 11\cdot 8 \div 3 \quad (\text{ or } 1 \div 3 \times \pi \times 5\cdot 5^2 \times 11\cdot 8\ )$$
$$= 373\cdot 797....$$
$$= \underline{373\cdot 8\text{cm}^3 (1 \text{ d.p.})}$$

We now need to calculate how many cones (with volume 373·8cm³) can be made out of 16 litres of metal.

16 litres = 16000cm³.
$16000 \div 373\cdot 8 = 42\cdot 803...$ (When the question requires rounding, you must state the unrounded answer first), so **42** (complete) cones can be made.

BASIC SKILL EXAMPLE 4: Volume of a sphere

Calculate the volume of this sphere.



Solution

Radius is 5cm

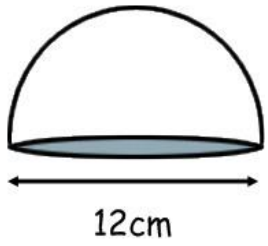
$$V = \tfrac{4}{3}\pi r^3$$
$$= \pi \times 5^3 \div 3 \times 4 \quad (\text{ or } 4 \div 3 \times \pi \times 5 \times 5 \times 5\ )$$
$$= 523\cdot 5987756....$$
$$= \underline{523\cdot 6\text{cm}^3}$$

Definition: A **hemisphere** is half of a sphere.

To find the volume of a hemisphere, find half of the volume of the corresponding sphere.

BASIC SKILL EXAMPLE 5: Volume of a hemisphere

Calculate the volume of this hemisphere.



Solution

Diameter is 12cm so radius is 6cm

$$V = \tfrac{4}{3}\pi r^3 \div 2$$
$$V = \pi \times 6^3 \div 3 \times 4 \div 2$$
$$V = 452\cdot 3893421....$$
$$V = \underline{452\cdot 4\text{cm}^3}$$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, volume questions should always:

- Ask you to work out the volume of a composite shape (more than one shape joined together).
- Be set in a real-life context.

You **might** also have to:

- Round your answer to a number of significant figures (see **Assessment Style Example** on page 64, which is an example involving volume).
- Use the result of your area calculation to calculate a cost or other quantity (see **Assessment Style Example 2** on page 60, which is an example involving area).
- Make and explain a decision (see **Assessment Style Example 1** on page 58, which is an example involving perimeter).
- Work out how many smaller objects can fit in to a larger one (see **Assessment Style Example 1** on page 62 and **Assessment Style Example 3** on page 65, which are examples involving volume).
- Convert between different units (see **Assessment Style Example** on page 65, which is an example involving volume).

(There are many ways questions may be adapted and so this list can never cover everything).

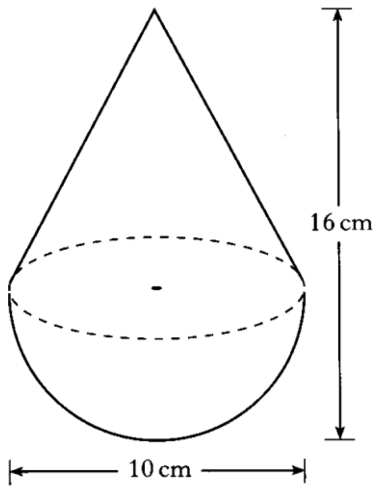
In the exam, you may be expected to deal with a shape formed from more than one other shape joined together. If the diagram is confusing and you are not sure what the shape in the question is, then read the question carefully as it usually tells you.

Assessment Style Example 2

A child's toy is in the shape of a hemisphere with a cone on top, as shown in the diagram.

The toy is 10 centimetres wide and 16 centimetres high.

Calculate the volume of the toy. Give your answer correct to 2 significant figures.



Solution

The cone and the hemisphere have the same radius, 5cm.

The 16cm line in the picture is made of the height of the cone plus the radius of the sphere.

Height of cone + 5cm = 16cm
i.e. height of cone is 16 – 5 = 11cm.

Volume of cone:

$$V = \frac{1}{3}\pi r^2 h$$
$$= \pi \times 5^2 \times 11 \div 3$$
$$(\text{or } 1 \div 3 \times 5^2 \times 11)$$
$$= 287 \cdot 98...$$

Volume of hemisphere

$$V = \frac{4}{3}\pi r^3 \div 2$$
$$= \pi \times 5^3 \div 3 \times 4 \div 2$$
$$(\text{or } 4 \div 3 \times \pi \times 5^3 \div 2)$$
$$= 261{\cdot}8...$$

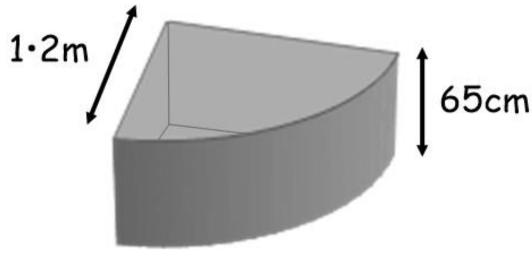
Total volume = 287·98 + 261·8
= 549·7…

(When the question requires rounding, you must state the unrounded answer first)

= 550cm^3 (2 s.f.)

Assessment Style Example 3

A water trough on a farm is in the shape of a quarter cylinder with radius 1·2m and height 65cm.



Smaller animal feeding bowls are in the shape of cylinders with diameter 40cm and height 14cm.

If the trough is full of water, how many feeding bowls can be completely filled?

Solution

First note that all the units in the question are in centimetres except one. We need to change 1·2m to 120cm.

Volume of the large trough:
The cross-section of the large trough is a quarter cylinder with radius 120cm and height 65cm. We use the cylinder formula with r = 120, h = 65 and in which we divide our answer by 4.

$$
\begin{aligned}
V &= \pi r^2 h \div 4 \\
&= \pi \times 120^2 \times 65 \div 4 \\
&= 735132 \cdot 68 ... (\text{cm}^3)
\end{aligned}
$$

Volume of the smaller bowls:
The bowls are cylinders with diameter 40cm (and so radius 20cm) and height 14cm. So we use the cylinder volume formula with r = 20 and h = 14.

$$
\begin{aligned}
V &= \pi r^2 h \\
&= \pi \times 20^2 \times 14 \\
&= 17592 \cdot 9 ... (\text{cm}^3)
\end{aligned}
$$

To find how many feeding bowls can be filled from the large trough, we divide the volumes.

Note that this question requires rounding (down) to the nearest whole number. Even though it doesn't explicitly ask us to round, the fact that it asks how many bowls can be "completely" filled means the answer we give must be a whole number.

735132·68 ÷ 17592·9 = 41·785…
(When the question requires rounding, you must state the unrounded answer first)
= 41.

Answer: 41 bowls can be completely filled.

Managing Finance and Statistics Unit

Finance

Budgets, Profit and Loss

When talking about money, every company, family, event and individual will have money that comes in (**income**) and money that goes out (**expenditure**). A **financial statement** can be used to compare income and expenditure. A blank financial statement may look like the one below:

Financial Statement

INCOME		EXPENDITURE	
Item	**Amount (£)**	**Item**	**Amount (£)**
Total Income		**Total Expenditure**	

To find **Total Income** or **Total Expenditure**, you add the amounts in the relevant column.

You need to be able to determine the financial position of an event or an individual. There are two possible financial positions:

- You make a **profit** or have a **surplus** if your income is greater than your expenditure (if Income – Expenditure is *positive*).
- You make a **loss** or have a **deficit** if your income is less than your expenditure (if Income – Expenditure is *negative*).

BASIC SKILL EXAMPLE: Budget

Iain Asghar earns £1150 from his job and £60 interest from his savings every month. Every month he spends £580 on rent, £260 on food, £86 on TV/phone/internet, £120 on petrol and £140 on council tax.
Does he have a surplus or a deficit each month? How much is his surplus/deficit?

Solution
Total Income = 1150 + 60 = £1210
Total Expenditure = 580 + 260 + 86 + 120 + 140 = £1186
Income – Expenditure = 1210 – 1186 = £24
This is positive, so Iain has a *surplus* of £24 per month.

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, budgeting questions should always:

- Contain multiple sources of income or expenditure, some of which you will have to calculate yourself.
- Require you to make a decision related to profit/loss (or surplus/deficit).
- Be set in a real-life context.

You **might** also have to:

- Calculate somebody's pay using the techniques from the section beginning on page 68.
- Calculate percentage increases or decreases in some of the amounts involved.

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1 – household income

Mr David McEwan earns a salary of £10 272 per annum.
Ms Angela Clark works part-time and earns a salary of £370 per month.
They have three children: Kayleigh, Taylor and Jack.

They are eligible for Family Tax Credits of £50·20 per month and receive child benefit of £68·40 per month for the eldest child (Kayleigh) and £52·10 per month for their other children.

Their monthly expenditure is: £140 for council tax (with a family discount of 25%), £354 for food, £140 for petrol, £423·50 for their mortgage, £86·25 for gas/electricity and £78 for insurance.

Describe the financial position of the family each month.

Solution

We can begin by drawing up a monthly financial statement and writing in all the monthly figures that we already know. Some will still need to be calculated.

Monthly Financial Statement

INCOME		EXPENDITURE	
Item	**Amount (£)**	**Item**	**Amount (£)**
David's Wages		Council Tax	
Angela's Wages	£370	Food	£354
Family Tax Credits	£50·20	Petrol	£140
Child Benefit		Mortgage	£423·50
		Gas/Electricity	£86·25
		Insurance	£78
Total Income		**Total Expenditure**	

We can now calculate the remaining figures:

- David's annual wage is £10 272, so his *monthly* wage is 10272 ÷ 12 = <u>£856</u>
- Child benefit = £68·40 for the oldest child + £52·10 × 2 for the two younger children, which gives a total of <u>£172·60</u>.
- Council Tax = 140 with a 25% discount. 25% of 140 = 140 ÷ 4 = £35, so the total tax payable is £140 - £35 = <u>£105</u>.

The completed financial statement, with totals added, now looks like this:

Monthly Financial Statement

INCOME		EXPENDITURE	
Item	**Amount (£)**	**Item**	**Amount (£)**
David's Wages	£856	Council Tax	£105
Angela's Wages	£370	Food	£354
Family Tax Credits	£50·20	Petrol	£140
Child Benefit	£172·60	Mortgage	£423·50
		Gas/Electricity	£86·25
		Insurance	£78
Total Income	**£1448·80**	**Total Expenditure**	**£1186·75**

Total Income – Total Expenditure = £1448·50 – £1186·75 = £261·75.

The *decision* is that the family have a **surplus of £261·75** per month.

Assessment Style Example 2 – profit or loss from a sale (adapted from 2015 exam question)

Megan buys 300 shares for a total price of $1020.

When she decides to sell them, the price is $3·45 per share, and Megan also has to pay a fee of 2½% of the selling price.

Has she made a profit or a loss, and how much is this?

Solution

Income (when selling):	**Expenditure (when buying):**
Selling price = 300 × 3·45 = $1035.	$1020
Tax = 2½% of $1035	
= 0·025 × 1035	
= 25·875	
= $25·88.	
Income = 1035 – 25·88	
= $1009·12	

Income – Expenditure = 1009·12 – 1020 = –$10·88.
The decision is that Megan makes a loss, and that the loss is $10·88.

Pay

Definition: deductions are amounts of money that get taken off your pay. The most common deductions are:

- **Income tax**: tax taken off your pay and paid directly to the Government. See page 68 for more details about how tax is calculated.
- **National Insurance**: money taken off your pay to go towards paying for services such as hospitals. This is usually a percentage of pay.
- **Superannuation** (pension): money taken off your pay and kept in a fund to pay you a pension when you eventually retire. This is usually a percentage of pay.

A **bonus** is paid to staff when they reach certain targets within a specified period of time. One example of a type of bonus is commission.

Definition: commission is a bonus paid to a salesperson based on their sales. It is usually a percentage of their sales.

BASIC SKILL EXAMPLE 1: Commission

Andrew Holmes is a car salesman. He is paid monthly commission of 2·4% on all his sales over £24 000. How much does he gets paid in a month where he sells £118 500 worth of cars?

Solution

Sales on which commission is payable = 118 500 – 24 000 = £94 500
Commission = 2·4% of £94 500
= 0·024 × 94 500
= **£2268**

Definition: gross pay is the money you get paid *before* deductions are taken off
Definition: net pay is the money you get to take home *after* deductions are taken off.

Formula: not given on the formula sheet in National 5 Lifeskills Mathematics assessments

Net Pay = Gross Pay – Total Deductions

If a worker works extra hours (often in the evening, nights or weekends), this is called **overtime**. When overtime is worked, the hourly wage that the worker get paid is usually higher than normal.

Definition: your **basic wage** is the rate a worker gets paid at for their normal hours.
Definition: double time is where a worker gets paid double their basic rate for overtime.
Definition: time-and-a-half is where a worker gets paid half as much again for overtime. It is also possible to have other overtime rates such as '**time-and-a-quarter**' or '**time-and-a-third**'.

Overtime is found by calculating the basic wage and then using a method from this table:

Type of Overtime	Method 1	Method 2
Double Time	× 2	
Time-and-a-half	× 1·5	÷ 2 and add on to original amount
Time-and-a-quarter	× 1·25	÷ 4 and add on to original amount
Time-and-a-third	÷ 3 × 4	÷ 3 and add on to original amount
Time-and-three-quarters	× 1·75	÷ 4 × 3 and add on to original amount

BASIC SKILL EXAMPLE 2: Calculating overtime

Calculate the overtime wage for:

(a) **Amanda who is paid a basic wage of £8·35 per hour and works 8 hours overtime at time-and-a-half.**

(b) **Jamie who is paid a basic wage of £6·24 per hour and works 10 hours overtime at time-and-a-third.**

Solution

(a) Using method 1 from the table above:

Basic wage = 8·35 × 8 = £66·80

£66·80 × 1·5 = **£100·20**

(b) Using method 2 from the table above:

Basic wage = 6·24 × 10 = £62·40

£62·40 ÷ 3 = £20·80

£62·40 + £20·80 = **£83·20**

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, pay questions should always:

- Ask you to work out somebody's take home (net) pay given a variety of information.
- Be set in a real-life context.

You **might** also have to:

- Calculate overtime payment (see **Assessment Style Example 1** on page 70).
- Calculate commission payments (see **Assessment Style Example 2** on page 70).

- Calculate the tax and/or pension payable on the income (see **Assessment Style Example 2** on page 70 for pension payments; and the next section for tax).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

Rachel Frost is a radiographer. She works a basic 40 hour week and is paid a basic rate of £9·50 per hour. Any overtime is paid at time and a half, except when she works on Sundays when she gets paid Double Time instead.
One week she worked a total of 48 hours, plus another 5 hours on Sunday.
What was her gross pay?

Solution

She works a 48 hour week and her basic hours are 40 hours, so she worked 8 hours overtime.

Basic pay: £9·50 × 40 = £380
Overtime pay: £9·50 × 8 × 1·5 = £114
Sunday pay (double time): £9·50 × 5 × 2 = £95
Gross pay: £380 + £114 + £95 = **£589.**

Assessment Style Example 2

Harvey Robertson is a salesman. Part of his wage-slip is shown below.
He gets paid 4·5% commission on all items he sells over £850.
He pays 3% of his gross salary into his pension.
How much does he get paid in a month when he sells £3200 worth of items?

Name	Employee number	Week	Date	Tax Code	NI Number
Harvey Robertson	4303	26	26/06/2012	570H	BW 91 44 65 C

Basic Pay	Overtime	Bonus	Commission	[redacted]	Gross Pay
£325.40	£26.00	£50.00			
National Insurance	Income Tax	Pension	Other		Total Deductions
£45.30	£75.20		£0.00		
					Net Pay

Solution

Sales on which commission is payable = 3200 – 850 = £2350.
Commission = 4·5% of £2350 = 0·045 × 2350 = £105·75.

Gross Pay = £325·40 + £26 + £50 + £105·75 = £507·15.

Pension = 3% of £507·15 = 0·03 × 507·15 = 15·2145 = £15·21
Deductions = £45·30 + £75·20 + £15·21 = £135·71

Net Pay = Gross Pay – Total Deductions = £507·15 – £135·71 = **£371·44**

You have to pay tax on the income you earn. However in order to try to keep the system fair and to ensure that lower earning people pay less (or zero) tax, and high earning people pay more tax, everybody is allowed to earn a certain amount of money tax-free before they start getting any money taken off them.

Definition: The amount of money you are not taxed on is called your **personal allowance** or your **tax allowance**. This is usually shown as a tax code on a wage slip.

Definition: The rest of your money that you *are* taxed on is called your **taxable income**.

Tax is taken off as a percentage of your wage – however there are also different rates of tax. As you earn more, this percentage increases.

BASIC SKILL EXAMPLE 3: Income Tax

Alison McNeill works in a laboratory. She gets an annual salary of £29 800 and her annual tax allowance is £10 000. The income tax rate is 20%.

Calculate Alison's annual salary after tax has been deducted.

Solution

Alison's taxable income is £29 800 – £10 000 = £19 800

Tax = 20% of taxable income
= 0·2 × 19800
= £3960

Annual salary with taken off tax = £29800 – £3960 = **£25 840**

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, tax or National Insurance questions should always:

- Ask you to work out somebody's take home pay given the personal allowance and the rate of tax.
- Be set in a real-life context.

You **might** also have to:

- Calculate tax when more than one tax rate applies (see **Assessment Style Example 3** on page 71).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 3

Michael Hall is a chief executive. He earns £65000 per annum.
His annual tax allowance is £10000.

The income tax rates are:

On the first £31865 of taxable income	20%
On the rest of taxable income	40%

Calculate Michael's annual salary after tax has been deducted.

Solution

Michael's taxable income = £65000 – £10000 = £55000. This is more than £31865, so some of his income must be taxed at 40%.

How much tax he pays at each rate:

20%: £31865 — 20% of £31865 = 0·2 × 31865 = £6373

40%: £55000 – 31865 = £23135 — 40% of £23135 = 0·4 × 23135 = £9254

Total tax: £6373 + £9254 = £15627
Net pay: £65000 – £15627 = **£49373.**

National Insurance (NI) is a form of tax used to fund Welfare State services such as hospitals, doctors, public pensions and unemployment benefit. It is paid by employers and employees as a percentage of a worker's wage.

BASIC SKILL EXAMPLE 4: National Insurance

In October 2014 the rates of weekly National Insurance (NI) contributions set by Government are:

Weekly earnings	National Insurance
£0 - £153	0%
£153·01 – £805	12%

Amanda earns £650 a week. How much NI does she pay?

Solution

Income on which National Insurance is payable = 650 – 153 = £497.
National Insurance = 12% of £497 = 0·12 × 497 = £59·64.

Best Deal

You will be expected to compare up to three financial deals and to decide which one is the best deal. A straightforward numerical way to do this is to calculate and compare rates (the price per____[e.g. kilogram, litre, person]), for example:

- If we know the price and the weight (in grams), then we can compare by calculating the *price per gram*.
- If we know the price and the volume (in litres), we can calculate the *price per litre*.

BASIC SKILL EXAMPLE: comparing rates

Jackie is a lorry driver who works in Europe.

- **In Belgium she fills up with 80 litres of diesel and it costs her €206·40.**
- **In Germany she fills up with 124 litres of diesel and it costs her €332·32.**
- **In Austria she fills up with 95 litres of diesel and it costs her €222·30.**

In which country does she get the best deal?

Solution

Calculate the price per litre (price ÷ litres) for each country:

Belgium:	206·40 ÷ 80	= €2·58 per litre.
Germany:	332·32 ÷ 124	= €2·68 per litre.
Austria:	222·30 ÷ 95	= €2·34 per litre.

The *decision* is that she gets the best deal in Austria.

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, 'best deal' questions should always:

- Expect you to make (and possibly explain) a decision.
- Compare at least three products with at least three pieces of information on each.
- Be set in a real-life context.

You **might** also have to:

- Take account of discounts, taxes or interest rates (see **Assessment Style Example 1** on page 77).

(There are many ways questions may be adapted and so this list can never cover everything).

It is possible that, out of the three products, one may be better on one factor (e.g. it may be the cheapest), but another may be better on another factor (e.g. it may be the quickest or the healthiest). The question might give you some guidance on what the buyer's priorities are (e.g. what is more important to them: Price? Speed? Size?), such as the next example.

Assessment Style Example

Mohammed is planning a holiday. He has decided to fly from Glasgow. He has a choice of three destinations.

Copenhagen	**Prague**	**Berlin**
Distance = 650 miles	Distance = 870 miles	Distance = 595 miles
Speed of Flight = 300mph	Speed of Flight = 450mph	Speed of Flight = 350mph
Price = £120	Price = £90	Price = £180

Mohammed wants to choose a flight that is quick and that is good value for money. Which destination should he choose? Explain your answer.

Solution

We calculate the time taken for each flight using the formula $T = \frac{D}{S}$.

We also calculate the price per mile (price ÷ mile).

Copenhagen	**Prague**	**Berlin**
$T = \frac{D}{S} = \frac{650}{300}$	$T = \frac{D}{S} = \frac{870}{450}$	$T = \frac{D}{S} = \frac{595}{350}$
$= 2\cdot17$ hours	$= 1\cdot93$ hours	$= 1\cdot7$ hours
Price per mile = 120 ÷ 650 $= £0\cdot18$ / mile	Price per mile = 90 ÷ 870 $= £0\cdot10$ / mile	Price per mile = 180 ÷ 595 $= £0\cdot30$ / mile

The best value flight is the one to Prague (10p per mile).
The worst value flight is the one to Berlin (30p per mile).

The quickest flight is the one to Berlin (1·7 hours).
The longest flight is the one to Copenhagen (2·17 hours).

Based on this information, either Prague or Berlin should be chosen because they are the best value and quickest respectively. However Berlin is also the worst value. So it would make most sense for Mohammed to choose Prague.

The best *decision* is Prague, and the reason is that it is the best value (10p per mile, which is less than Berlin (30p per mile) and Copenhagen (18p per mile); and not the longest flight (1·93 hours, which is less than Copenhagen (2·17 hours); and even though it is more than Berlin, Berlin is the worst value for money).

Currencies

When converting from one currency to another, an **exchange rate** is used. An exchange rate explains how many units of one currency you get for another. For example, the exchange rate for Pounds (£) into Euros (€) might be £1 = €1·27 *(for every one pound you exchange, you get*

1.27 Euros in return).

In the UK, exchange rates are usually expressed in terms of pounds (i.e. £1 =______). However for people in other countries the exchange rate is likely to be expressed in terms of their own currency. For example in France the exchange rate above would be likely to be expressed as €1 = £0·79 *(for every one Euro you exchange, you get 79 pence in return).*

Exchange rates change regularly from day to day or even hourly depending on global events.

To do calculations involving foreign exchange we must either multiply or divide by the exchange rate. Which one we choose depends on which way around we are converting.

In the exchange rate £1 = \$1·61, the **base currency** is pounds. In the exchange rate \$1 = £1·62, the base currency is dollars.

- To change from the base currency to the other currency, **multiply** by the exchange rate.
- To change from the other currency to the base currency, **divide** by the exchange rate.

BASIC SKILL EXAMPLE 1: changing from the base currency to the other currency

Janet changes £250 into Euros. The exchange rate is £1 = €1·37. How much money does she get?

Solution

Pounds are the base currency. We are changing from the base currency to the other currency, so we multiply.

1·37 × 250
= 342·5
= **€342·50** (money must be expressed with two decimal places)

BASIC SKILL EXAMPLE2: changing from the other currency to the base currency

Harry is returning from the USA with \$800. The exchange rate is £1 = \$1·57. Harry changes his money back into pounds. How much money does he get?

Solution

Pounds are the base currency. We are changing from the other currency back into the base currency, so we divide.

800 ÷ 1·57
= 509·554….
= **£509·55** (money must be expressed with two decimal places)

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, foreign currency questions should always:

- Involve at least **three** currencies and have a number of steps.
- Require conversions involving either or both multiplication and division.
- Be set in a real-life context.

You **might** also have to:

- Read the rates from a table showing many different exchange rates (see table below).
- Take account of commission (see **Assessment Style Example 2** on page 75).
- Make a decision after a calculation (see **Assessment Style Example 1** on page 75).
- Include a percentage increase or decrease to the price (see **Assessment Style Example 1** on page 75).

(There are many ways questions may be adapted and so this list can never cover everything).

Every world currency has a three letter abbreviation. The UK currency is abbreviated as **GBP** (Great British Pounds), Euros are **EUR**, US Dollars are **USD**.

Exchange Rates 1GBP buys		
Japanese Yen (¥)	JPY	172·419
Euros (€)	EUR	1·2666
US Dollars ($)	USD	1·6054
Swedish Krona (kr)	SEK	11·5082
Indian Rupee (₹)	INR	97·9293
Korean Won (₩)	KRW	1712·97
Ukraine Hryvnia (₴)	UAH	20·8023

Some other currencies and their symbols are shown in the table on the right, which is used in the next two Assessment Style examples. The exchange rates shown were correct in October 2014. You are not expected to learn all the currency names and symbols.
Note that in the table above, some exchange rates are given to more than two decimal places. This is allowed so long as any final answers are rounded to two decimal places.

Assessment Style Example 1

Lili is going to buy a new pair of shoes on the Internet. She can buy them from the USA (in US Dollars/USD) or from Italy (in Euros/EUR).

- **The price in Euros is €209·99 plus €30 postage.**
- **The price in US Dollars is $255 plus $20 postage, but Lili's credit card provider charges an extra 2% on the total cost when buying from the USA.**

Which currency should Lili buy in?

Solution

Total cost in **Euros** = 209·99 + 30 = €239·99.

The exchange rate is £1 = 1·2666. Changing into GBP gives:
239·99 ÷ 1·2666 = £189·475… = £189·48.

Total cost in **US Dollars** *before* adding on 2% = 255 + 20 = $275.
We now calculate and add on the extra 2% charge:
0·02 × 275 = $5·50, so total cost = 275 + 5·50 = $280·50.

The exchange rate is £1 = 1·6054. Changing into GBP gives:
280·50 ÷ 1·6054 = £174·7225… = £174·72.

The *decision* is that Lili should buy from the USA as it is (£14·76) cheaper.

Assessment Style Example 2

Jenna is a sales executive from the USA. She is going to the UK and South Korea on a business trip.

Jenna takes $8000 spending money with her to the UK. She changes her money into GBP and is charged 2% commission.

Whilst in the UK she spends £2500. She then travels to South Korea and changes her money into KRW and is charged 1·2% commission.

Use the exchange rate table on the previous page to work out how many Korean Won Jenna receives. Round your answer to 3 significant figures.

Solution

First change $8000 to GBP and subtract 2% commission. The exchange rate is £1 = $1·6054. The base currency is GBP and we are changing into the base currency, so we divide: 8000 ÷ 1·6054 = £4983·18

Commission: 2% of £4983·18 = 0·02 × 4983·19 = £99·66
Taking off the 2%: 4983·18 – 99·66 = £4883.52

Jenna spends £2500 in the UK, so she leaves the UK with:
4883·52 – 2500 = £2383·52.

Now we change £2383·52 to KRW and subtract 1·2% commission. The exchange rate is £1 = 1712·97 Won (we could also use the symbol ₩ for Won, but it is not expected that you would know this). The base currency is GBP and we are changing out of the base currency, so we multiply:

2383·52 × 1712·97 = 4 082 898·254 Won

Commission: 1·2% of 4 082 898·254 = 0·012 × 4 082 898·254 = 48994·78
4 082 898·254 – 48994·78 = 4 033 903·47 Won

(When the question requires rounding, you must state the unrounded answer first)

The answer is that Jenna will receive **4 030 000 Won** rounded to 3 significant figures.

Savings

When money is deposited in a bank account, it earns **interest**. Interest is an extra amount added on at the end of the year as a reward for letting the bank have your money. Interest is usually expressed as a percentage of the amount in the bank account: this is called the **interest rate**, and is usually given per annum (p.a.), which means "per year". For example an account may have an interest rate of **2·3% per annum** or **0·75% p.a.** .

There are two methods to calculate interest:

- In **simple interest**, the interest is calculated based on the original amount in the bank and the interest is the same every year.
- In **compound interest**, the interest is calculated based on the actual amount in the bank at the end of each year. In real-life, this is the method that is almost always used.

If the question asks for the **balance**, it is asking for the amount of money in the account. This is found by adding the interest on to the original amount.

BASIC SKILL EXAMPLE 1: Simple interest

Karen invests £1500 in a bank account paying 1·5% simple interest per annum. If she leaves the money in the account for 3 years, what is the
(a) Interest
(b) Balance
after three years?

Solution

(a) The simple interest for one year is 1·5% of £1500 = 0·015 × 1500 = £22·50.
The simple interest for three years is £22·50 × 3 = **£67·50**

(b) The balance after three years is 1500 + 67·50 = **£1567·50**

Compound interest is an example of Appreciation, covered in the *Numeracy* unit on page 18. This is because the amount in the account is always going up, so you always add the amount on each time.

Each year the amount in the bank account grows as the previous year's interest is added, and so the next year more interest is earned.

When calculating compound interest, the balance has to be calculated first. The interest is calculated afterwards by subtracting the original amount.

BASIC SKILL EXAMPLE 2: Compound Interest

Duncan puts £2500 into a savings account paying 2·4% compound interest per annum. If he leaves the money in the account for 3 years, what is the
(a) Balance
(b) Interest
At the end of the 3 years?

Solution
There are two methods that can be used. The theory behind each one is outlined in the Numeracy unit on page 17.

	Longer method	Quicker method
(a)	**Year 1:** 0·024 × 2500 = £60 2500 + 60 = £2560 **Year 2:** 0·024 × 2560 = £61·44 2560 + 61·44 = £2621·44 **Year 3:** 0·024 × 2621·44 = £62·91 2621·44 + 62·91 = £2684·35	$2500 \times 1{\cdot}024^3 = 2684{\cdot}35$
(b)	**Interest:** 2684·35 – 2500 = £184·35	**Interest:** 2684·35 – 2500 = £184·35

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, savings questions should always:
- Involve an interest rate calculation.
- Require you to round your (final) answer to 2 decimal places automatically.
- Be set in a real-life context.

You **might** also have to:
- Compare different bank accounts with different rates of interest or tax rates (see **Assessment Style Example 1** on page 77).
- Make and explain a decision based on your calculation (see **Assessment Style Example 2** on page 78).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

Marion wants to put £6000 into a savings account. She plans to invest it for 1 year and not to withdraw any money until the end of the year. She has investigated bank interest rates and has found the following information:

ScotBank	**e-Bank**	**Tax Free Saver**
1·8% interest p.a.	1·62% interest p.a.	1·3% interest p.a.
Interest taxed at 20%	**Plus a £50 bonus if no withdrawals are made during the year.**	No tax
	Interest and bonus taxed at 20%	

Which bank should Marion choose? Explain your answer.

Solution

We calculate the interest for each bank:

ScotBank:
1·8% of £6000 = 0·018 × 6000 = £108
Tax = 20% of £108 = 0·2 × 108 = £21·60
Total Interest = 108 – 21·60 = £86·40.

e-Bank:
1·62% of £6000 + £50 = 0·0162 × 6000 + 50 = £147·20
Tax = 20% of £147·20 = 0·2 × 147·20 = £29·44
Total Interest = 147·20 – 29·44 = £117·76.

Tax-Free Saver:
1·3% of £6000 = 0·013 × 6000 = £78.

The *decision* is that Marion should choose **e-Bank** because it pays £117·76 interest which is more than the £86·40 or £78 paid by the other two banks.

When calculating compound interest over a long time period, the longer method becomes impractical. The quicker method is essential in these situations.

Assessment Style Example 2

Linda invests $3000 into a trust fund for her daughter that pays 2·5% compound interest p.a.
After 15 years she needs $5000 to put her daughter through college. Will Linda have enough money in the account after 15 years to afford this?

Solution

The longer method for appreciation is not a good method here unless you want to do 15 lines of calculations. We use the quicker method.

Interest is appreciation. For 2·5% interest, the multiplier is 1·025.

$3000 \times 1.025^{15} = 4344{\cdot}8945$

$= \underline{\underline{\$4344{\cdot}89}}$ (must show units ($) and round to 2 decimal places for money)

The *decision* is that No, Linda will not have enough money.

The following *explanations* **should** get a mark. See the examples on page 7 for further guidance on how to write an explanation.

- No, because she will have $4344·89 which is less than $5000.
- No, she will need $655·11 more.

You would **not** get a mark for these explanations:

- No she has less than $5000 *(no mention of the calculated number)*

Borrowing: Loans and Credit

When you borrow money as a **loan** from a finance company, there are three things that determine how much you pay back:

- The **amount of money** you borrow.
- The **rate of interest**. This might be expressed as a monthly interest rate or an annual interest rate known as the **Annual Percentage Rate** (APR).
- **Length of time:** you will pay more money overall if you borrow the money for a longer time.

Loans may also include other charges such as **Payment Protection Insurance** (PPI). This is where you pay a little bit more each month, but where you will get some help with your payments if you have a sudden unexpected drop in income (e.g. you lose your job or fall ill). Payment Protection Insurance has been in the news a lot recently as it has been sold dishonestly in the past.

In some examples the interest rate and the monthly payment might have already been calculated for you.

Definition: the **cost of the loan** is how much more you end up paying back than you originally borrowed.

BASIC SKILL EXAMPLE 1: cost of a loan

The Carlyle family borrowed £5000 with loan protection from the Scottish Bank over a period of 36 months.
Their monthly repayment is £176·39. What is the cost of the loan to the Carlyle family?

Solution

The total amount they had to repay over 36 months was £176·39 × 36 = £6350·04.

They paid back £6350·04 in total, and they originally borrowed £5000. Therefore the cost of the loan is £6350·04 – £5000 = **£1350·04.**

In other examples, you will have to calculate the monthly repayment yourself using the interest rate. These examples are most likely to be based on simple interest (as opposed to compound interest).

BASIC SKILL EXAMPLE 2: calculating monthly repayments

Liam takes out a £5000 loan with a simple interest rate of 11·2% per annum.
He chooses to pay the loan back over 2 years.
Calculate Liam's monthly repayment.

Solution

The interest for one year = 11·2% of £5000 = 0·112 × 5000 = £560.
The interest for two years = 560 × 2 = £1120
The total amount repayable = the original amount + the interest
= 5000 + 1120 = £6120

2 years is 24 months, so the monthly repayment is 6120 ÷ 24 = **£255**.

When you borrow money on a **credit card** (or a store card), you do not have to pay it all back at once. Instead you can choose how much you pay back and when. You can pay the entire balance off at once if you want, but you can pay a lot less if you want to so long as you pay at least the minimum payment set down by the company.

Definition: the **balance owed** on a credit card statement is how much money you owe the company at the current date.
Definition: the **Annual Percentage Rate (APR)** is the interest rate that you pay on your balance each year. By law, the APR must be stated for all credit cards, store cards and loans.

The Annual Percentage Rate is based on compound interest and is not equal to the monthly rate multiplied by 12 because it not based on simple interest.

BASIC SKILL EXAMPLE 3: calculate the APR

A credit card charges a monthly interest rate of 2%. Calculate the APR.

Solution
The multiplier for a monthly interest rate of 2% is 1·02.
The multiplier for compound interest for a year (12 months) is given by:

$1{\cdot}02^{12} = 1{\cdot}268\ldots$, which is the multiplier for a 26·8% interest rate, so the APR is **26·8%**.

If you can't work out that 1·268 means a 26·8% interest rate, then you can convert the multiplier to an interest rate by:

1. Subtracting 1
2. Multiplying the answer by 100.

i.e. $1{\cdot}268 - 1 = 0{\cdot}268$
$0{\cdot}268 \times 100 = 26{\cdot}8\%$

BASIC SKILL EXAMPLE 4: calculate the APR and convert into percentage

A credit card charges a weekly interest rate of 1·8%. Calculate the APR.

Solution
The multiplier for a weekly interest rate of 1·8% is 1·018.
The multiplier for compound interest for a year (52 weeks) is given by:

$$1{\cdot}018^{52} = 2{\cdot}5286\ldots = 2{\cdot}529 \text{ (rounded to 3 d.p.)}$$

To change this to an APR, we subtract 1 and multiply by 100:

$$2\cdot 529 - 1 = 1\cdot 529$$

$$1\cdot 529 \times 100 = \underline{\underline{152\cdot 9\%}}.$$

The credit card company decide the **minimum payment** you have to pay each month. This is often expressed as a percentage of the balance owed. Sometimes it may be expressed in a way such as '3% of the balance owed or £5, whichever is greater'.

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, loans and credit questions should always:

- Expect you to calculate (monthly or yearly) payments.
- Require you to round your (final) answer to 2 decimal places automatically.
- Be set in a real-life context.

You **might** also have to:

- Calculate an APR (see **Basic Skill Example 3** and **Basic Skill Example 4** above).
- Make and explain a decision based on your calculation (see **Assessment Style Example 2** on page 78).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example 1

The Liu family are going to take out a £8400 loan to buy a new car.

The loan will have a fixed simple interest rate of 18·5% per annum and they will pay the loan back over 24 months.

(a) Calculate the Liu family's monthly repayment.
(b) The Liu family can only afford to pay £400 per month. How much of a deposit will they have to pay in order to be able to do this?

Solution

(a) 24 months = 2 years, so we calculate two years' simple interest.
One year's interest = 18·5% of £8400 = 0·185 × 8400 = £1554
Two years' interest = 1554 × 2 = £3108.
Total amount repayable = 8400 + 3108 = £11508.

Dividing into 24 equal monthly repayments:
11508 ÷ 24 = **£479·50** per month.

(b) The Liu family's loan ought to cost them £479·50 per month, but they want to reduce this to £400 per month.

Monthly payment of £479·50 needs to be reduced to £400.

So they need to pay off £79·50 of each of the £479·50 payments up front in order to reduce their payments, meaning that they need to pay off $\frac{79 \cdot 50}{479 \cdot 50}$ of the full £8400 cost of the car up front.

So deposit needs to be:

$$\frac{79 \cdot 50}{479 \cdot 50} \times 8400 = 8400 \div 479 \cdot 50 \times 79 \cdot 50$$
$$= 1392 \cdot 7007....$$
$$= \underline{\underline{£1392 \cdot 70}}$$

Assessment Style Example 2

Shown below is Jane Rose's credit card statement.

(a) Complete the credit card statement to show the interest and balance owed.
(b) Jane makes the minimum payment that month. Calculate how much money she pays.

Credit Card Statement **Scottish Bank**

Name: Jane Rose

Date: 29 May 2012 **Credit limit:** £2000

Account number: 4948 2057 4927

29 April	Balance brought forward	£240.00
30 April	Payment received - thank you	– 70.00
	SUBTOTAL	£170.00
1 May	Interest at 3.9%	
5 May	Sunshine Holidays Ltd	£950.00
17 May	Green's supermarkets	£75.36
21 May	Lo-price televisions	£249.99
	BALANCE OWED	

Minimum payment: 5% of final balance owed or £40, whichever is greater

Solution

(a) The interest at 3·9% = 0·039 × 170 = £6·63

The final balance = £170 + £6·63 + £950 + £75·36 + £249·99 = £1451·98

(b) 5% of the balance owed is = 0·05 × 1451·98 = 72·599 = £72·60

This is greater than £40, so the minimum payment she makes is £72·60.

Statistics

All content relating to the topic of **Probability** is covered in the notes for the *Numeracy* unit starting on page 28.

Scatter Graphs and Line of Best Fit

A scatter graph is a way of displaying information and looking for a connection between two sets of numbers.

At National 4 level, you learnt to plot points on a scatter graph and to draw a line of best fit. For National 5 Lifeskills Mathematics, you will need to use the same skills with the difference that the questions will always be set in a context and you will be expected to use your graphs to draw conclusions. At National 4 the axes would usually have been drawn for you. At National 5 level you may have to draw the graph from scratch, including the axes.

When drawing a scatter graph:

- The top row (or the left-hand column) usually goes on the horizontal axis.
- The bottom row (or the right-hand column) usually goes on the vertical axis.
- The axes must be labelled.
- There is no need for the axes to start from zero, though they can if you wish.
- It is enough to plot the dots. There is no need to label each dot with a name or letter, although it is allowed to do so if you wish.

BASIC SKILL EXAMPLE 1: Drawing a Scatter Graph

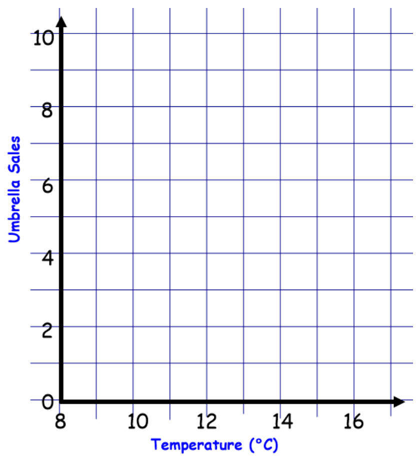
A gift shop records the temperature each day for 12 days. The table below shows the temperature and the number of umbrellas sold each day. Show this information on a scatter graph.

Day	A	B	C	D	E	F	G	H	I	J	K	L
Temperature (°C)	13	12	11	13	15	13	15	9	9	15	10	8
Umbrella Sales	1	6	5	4	3	2	0	8	7	2	6	9

Solution

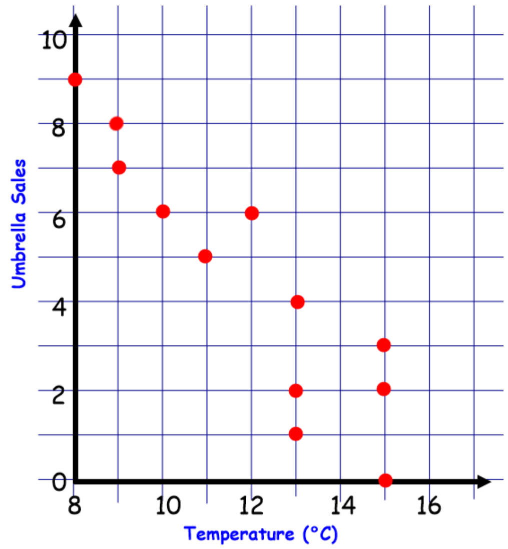
Step One – Draw and label the axes

The diagram on the right shows one possible set of axes. There are others. The sentences below explain they key features.

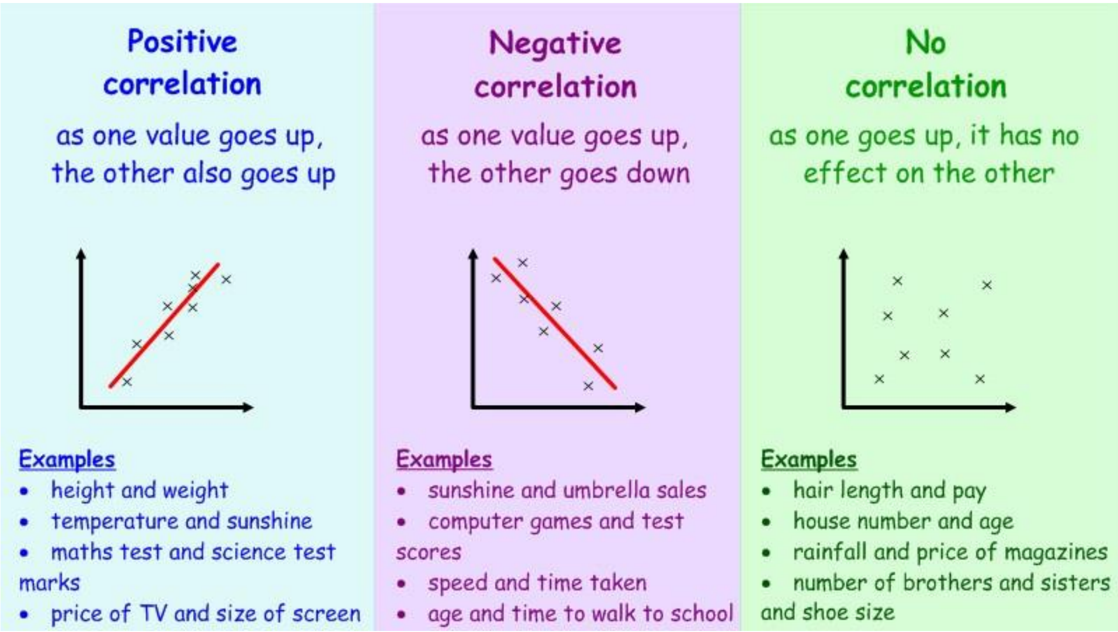


- **Temperature** is the top row of the table, so it goes on the **horizontal** axis.
- The numbers must go from at least 8 up to 15 and the axis must be labelled 'Temperature (°C)'.
- **Umbrella sales** is the bottom row of the table, so it goes on the **vertical** axis.
- The numbers must go from at least 0 up to 9.
- The axis must be labelled 'Umbrella Sales'.

Step Two – plot the points on the scatter graph. Each column in the table corresponds to one point on the graph The completed graph looks like this.



Definition: the **correlation** between two sets of numbers refers to the relationship (if any) between the numbers. A scatter graph is good for showing correlation. Correlation can be **positive** (going up), **negative** (going down), or **none**.



Definition: a **line of best fit** is line drawn on to a scatter graph that shows the correlation of the graph. The straight lines drawn above for positive and negative correlation are examples of lines of best fit.

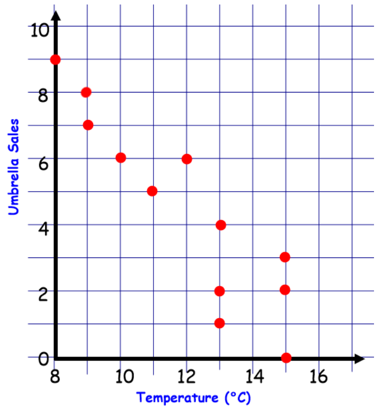
The line of best fit should show where an 'average' point on the graph should go. It should go:

- through the middle of the points, with *roughly* the same number of points above and below the line.
- in the same direction that the points are laid out on the page. **Do not "join the dots"!**

You will *always* be asked to draw the line of best fit in a scatter graph question in an assessment question. Once you have drawn the line, you will always be asked to *use it*.

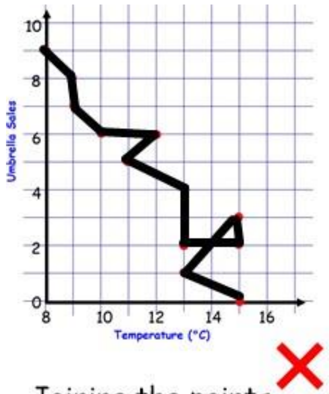
BASIC SKILL EXAMPLE 2: Drawing a Line of Best Fit

Draw a line of best fit on this scatter graph (this is the graph from the previous example)

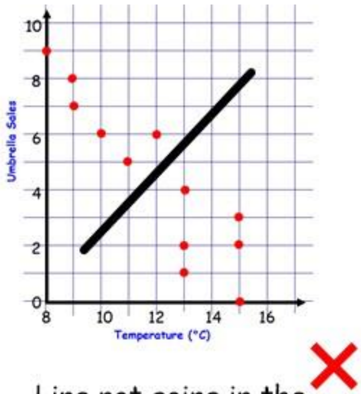


Solution

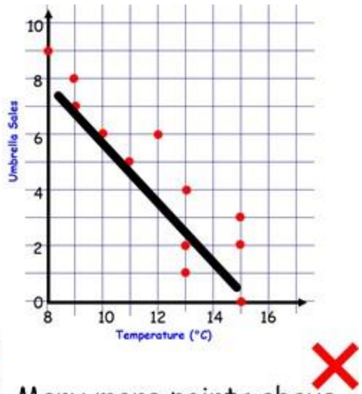
These three lines of best fit would be marked **wrong**



Joining the points. WRONG

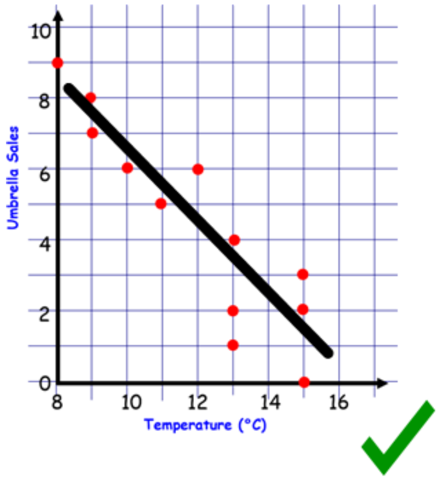


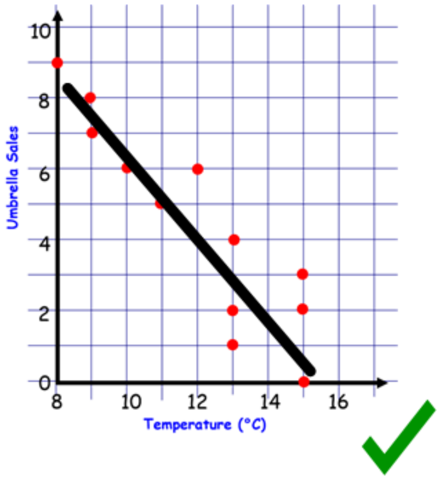
Line not going in the same direction as the points. WRONG.



Many more points above the line than below the line: too low. WRONG

Any of these answers would be **acceptable** as they are in the correct direction and have roughly the same number of points above and below the line.





What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, scatter graph questions should always:

- Be set in a real-life context.
- Ask you to plot points from a table onto a scattergraph.
- Ask you to draw a line of best fit for some given points.
- Ask you to use the line of best fit to estimate one value given the other.

You **might** also have to:

- Draw the axes for the scatter graph (more likely in unit assessments than the final exam) (see **Basic Skill Example 1** below).

(There are many ways questions may be adapted and so this list can never cover everything).

BASIC SKILL EXAMPLE 3: Using the line of best fit to estimate

This example uses the scatter graph from the previous question.

On the next day, the temperature is 14°C. Estimate how many scarves the shop will sell.

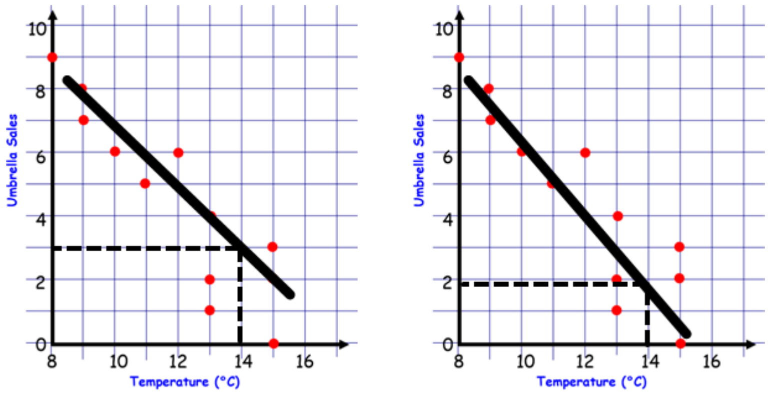
Solution

If your answer matches with your line, you get the mark. If it doesn't match with your line, you don't get any marks. Simple as that.

The correct answer will depend on your graph.

You need to draw lines on your graph at 14°C, and to see where they meet the line of best fit.

For the first two examples above, this would look like the graphs on the right.



If your line of best fit was the one on the left, your answer would be 3 umbrellas. If your line of best fit was the one on the right, your answer would be 2 umbrellas.

It does not matter that these answers are different – remember the question only asked for an *estimate*. The key thing is that it matches your line of best fit.

Median, Quartiles and Box Plots

Definition: the **median** is the number that divides an ordered list of numbers into two equally-sized parts.

Definition: the **quartiles**,(the **lower quartile** and **upper quartile**) along with the median, are the numbers that divide an ordered list into four equally-sized parts. The list must be written in order.

The lower quartile can be abbreviated as LQ or Q_1. The upper quartile can be abbreviated as UQ or Q_3. The median could be abbreviated as Med. or Q_2.

BASIC SKILL EXAMPLE 1: Calculating Median and Quartiles

Find the median and quartiles of the following numbers:

19 22 45 35 42 4 6 18 23 25 26 27 20 30 26

Solution

Step One – rewrite the list in order

4 6 18 19 20 22 23 25 26 26 27 30 35 42 45

Step Two – draw an arrow to show the middle of the list. This arrow shows the **median**.

4 6 18 19 20 22 23 25 26 26 27 30 35 42 45

The arrow is pointing to the number 25, so **the median is 25**.

Check that the arrow is in the correct place by counting how many numbers are to the left and right. In this case, there are 7 numbers on the left-hand side and 7 numbers on the right-hand side so we are correct.

Note: Sometimes the arrow does not point directly to a number, it might have to point in between two numbers. When this happens, we choose the number in the middle of the two numbers. See **Basic Skill Example 3** and **Assessment Style Example 1**.

Step Three – draw two further arrows to indicate the median of the left-hand side of the list (not including the original median) and the right-hand side of the list (also not including the original median). These are the **quartiles**.

4 6 18 19 20 22 23 25 26 26 27 30 35 42 45

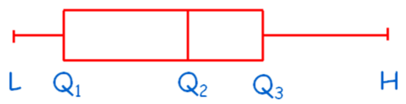
The arrows are pointing to the numbers 19 and 30, so **the lower quartile is 19** and the **upper quartile is 30**.

Check that the three arrows divide the list into four equal-sized quarters. In this case, each of the four section contains exactly 3 numbers.

Definition: a **five-figure summary** of a list of numbers is the lowest (L), lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and highest (H).

Definition: a **boxplot** is a way of showing five-figure summaries in a diagram.

The shape of a boxplot always has this general shape:



BASIC SKILL EXAMPLE 2: Drawing a Box Plot

Draw a boxplot for the following numbers:

2 3 4 4 4 5 5 6 7 8 9

Solution

Step One – write down the lowest and highest values.

Lowest = 2. Highest = 9

(Basic Skill Example 2 continued…)

Step Two – calculate the median. This list is already in order.

2 3 4 4 4 5 5 6 7 8 9

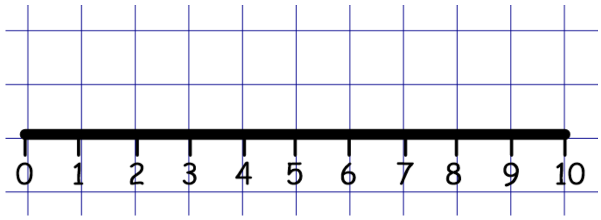
The median is 5.

Step Three – calculate the upper and lower quartiles using the method from the previous example.

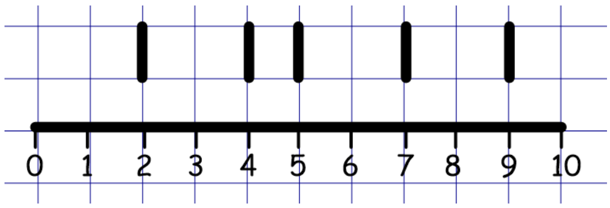
2 3 4 4 4 5 5 6 7 8 9

The lower quartile is 4 and the upper quartile is 7.

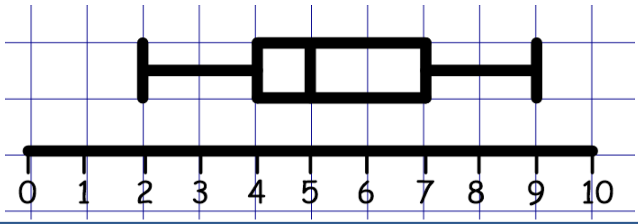
Step Four – draw and label a horizontal axis.



Step Five – draw five vertical lines corresponding to the five numbers calculated in steps 1-3. (the lowest, lower quartile, median, upper quartile and highest).



Step Six – join the middle three lines together to create a rectangle, and join the end points to create the full box plot shape.



At National 4 you learnt that the range was the difference between the highest and lowest numbers. At National 5 you are expected to be able to calculate the **InterQuartile Range** (IQR), which is the difference between the upper and lower quartiles. There is also the Semi-InterQuartile Range (SIQR), which is half of the IQR.

Formulae: not given on the formula sheet in National 5 Lifeskills Mathematics assessments

IQR = upper quartile – lower quartile

$$\text{SIQR} = \frac{\text{upper quartile} - \text{lower quartile}}{2}$$

BASIC SKILL EXAMPLE 3: calculating the IQR or SIQR

For the data set:

5 6 7 9 11 12 15 17 19 22

Calculate

(i) The InterQuartile Range.

(ii) The Semi-InterQuartile Range.

Solution

Step One – calculate the quartiles.

The arrows for the median and quartiles are drawn as follows:

5 6 7 9 11 12 15 17 19 22

The upper quartile is 17 and the lower quartile is 7.

Step Two – use the formulae for IQR or SIQR

(i) $\text{IQR} = Q_3 - Q_1$
$= 17 - 7$
$= 10$

(ii) $\text{SIQR} = \dfrac{Q_3 - Q_1}{2}$
$= \dfrac{17-7}{2} = \dfrac{10}{2} = 5$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, median and quartile questions should always:

- Be set in a real-life context.
- Ask you to calculate the median and quartiles for a list of numbers.

You **might** also have to:

- Obtain the numbers from a graph or other diagram (e.g. stem and leaf or dotplot) first.
- Calculate the InterQuartile Range or Semi InterQuartile Range.
- Write comments comparing two data sets (see part (b) of **Assessment Style Example** below).

(There are many ways questions may be adapted and so this list can never cover everything).

Assessment Style Example

The list below shows the number of children in each class at Stoneygate Primary School in 2010:

31, 14, 22, 20, 16, 17, 15, 15, 28, 19, 12, 17, 21, 19, 26, 31

(a) Draw a box plot to represent this data

(b) The same data was obtained in 2014, and the boxplot below was drawn. Write two comments comparing the class sizes in 2010 and 2014.

Solution

(a) Step one – write the list in order, and make a five-figure summary.

L lower Q Median upper Q H

12, 14 , 15, 15, 16, 17, 17, 19, 19, 20, 21, 22, 26, 28, 31, 31

15·5 19 24

Lowest value	L = 12 Q_1
Lower Quartile	= 15·5 Q_2
Median	= 19
Upper Quartile	Q_3 = 24
Highest value	H = 31

Step two – draw the box plot for 2010, including an appropriate scale. The scale should be accurate and labelled.

(b) When writing comments, there should be one comment relating to how high or low the data is on average; and a second comment about which data is more consistent/more varied.

A possible comment is:
In 2014 class sizes were higher on average, and in 2010 the class sizes were more varied.

Standard Deviation

Definition: the standard deviation of a list of numbers is a measure of how spread out the numbers are from the mean.

- A lower standard deviation indicates numbers that are **more consistent**.
- A higher standard deviation indicates numbers that are **more varied**.

Formula: given on the formula sheet in National 5 Lifeskills Mathematics assessments

$$\text{standard deviation} = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} \quad \text{or} \quad \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$

The following symbols are used in these formulae:

- n stands for 'how many numbers are in the list'
- $\bar{x}$ stands for 'the mean' ($\bar{x}$ is read out loud as 'x bar')
- Σ means "add together" (Σ is **sigma**, the Greek capital 'S')

You only need to know how to use <u>one</u> of these formulae. In general, it is more helpful to just know the method rather than memorising the formula. The following two examples show how the <u>same</u> question is done using each method.

BASIC SKILL EXAMPLE 1a: Standard Deviation using the formula $s = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$

Find the mean and standard deviation of these five numbers: 2, 3, 9, 6, 5

<u>Solution</u>

<u>Step 1</u> – Calculate the Mean.

Mean: $\dfrac{2+3+9+6+5}{5} = \dfrac{25}{5} = 5$, so the mean is 5.

<u>Step 2</u> - Draw up a table with column headings x, $x - \bar{x}$ and $(x-\bar{x})^2$.

x	$x-\bar{x}$	$(x-\bar{x})^2$
2		
3		
9		
6		
5		

<u>Step 3</u> – Complete the table, remembering that $\bar{x}$ = the mean = 5 .

- *In the middle column, take away the mean from each number in the left-hand column.*
- *In the right-hand column, square each number in the middle column.*

x	$x-\bar{x}$	$(x-\bar{x})^2$
2	–3	9
3	–2	4
9	4	16
6	1	1
5	0	0
	TOTAL	**30**

<u>Step 4</u> – find the total of the final column

In this example, $\sum(x-\bar{x})^2 = 30$.

<u>Step 5</u> – use the formula, remembering that n = 5 as there were five numbers.

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{30}{5-1}}$$

$$= \sqrt{\frac{30}{4}}$$

$$= 2\cdot 74 \text{ (2 d.p.)}$$

If you don't like using the formula, you can just remember these two steps:

- Divide by n – 1.
- Square root.

The next example is the same question as in the previous example, but using the other formula.

BASIC SKILL EXAMPLE 1b: Standard Dev. using the formula $s = \sqrt{\dfrac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$

Find the mean and standard deviation of these five numbers: 2, 3, 9, 6, 5

Solution

Step 1 – Calculate the Mean.

Mean: $\dfrac{2+3+9+6+5}{5} = \dfrac{25}{5} = 5$, so the mean is 5.

Step 2 - Draw up a table with column headings x and x^2.

x	x^2
2	
3	
9	
6	
5	

Step 3 – Complete the table.

	x	x^2
	2	4
	3	9
	9	81
	6	36
	5	25
TOTAL	**25**	**155**

Step 4 – Obtain the totals from the table.

In this example, $\sum x = 25$, $\sum x^2 = 155$.

Step 5 – use the formula, remembering that $n = 5$ because there were 5 numbers.

$$s = \sqrt{\dfrac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$

$$= \sqrt{\dfrac{155 - \frac{25^2}{5}}{5-1}}$$

$$= \sqrt{\dfrac{155-125}{4}} = \sqrt{\dfrac{30}{4}} = 2 \cdot 74 \text{ (2 d.p.)}$$

What should an exam question look like?

In assessments for National 5 Lifeskills Mathematics, standard deviation questions should always:

- Ask you to work out the standard deviation for a list of (probably 7 or fewer) numbers.
- Be set in a real-life context.

You **might** also have to:

- Write a comment about what the mean and/or standard deviation show us (see page 93).

(There are many ways questions may be adapted and so this list can never cover everything).

Drawing Pie Charts

Interpreting a pie chart was covered in the notes for the *Numeracy* unit on page 29. As part of the *Managing Finance and Statistics* unit, you may be required to draw a pie chart yourself.

You need to be able to work out the angles you would use to draw the slices in a pie chart. You have to work out what fraction of the circle each slice has to be.

BASIC SKILL EXAMPLE: Calculating the Angles in a Pie Chart

In a school survey 200 pupils were asked what their favourite takeaway food was. The results were:

Food	Frequency
Pizza	55
Fish and Chips	64
Chinese	81

How many degrees would you need for each sector to represent this data on a pie chart?

Solution

Step One – Work out the **fraction** for each slice

There were 200 pupils in total, so the fraction has to be out of 200

Pizza: $\frac{55}{200}$ Fish and chips: $\frac{64}{200}$ Chinese: $\frac{81}{200}$

Step Two – work out each slice as a fraction of 360°

Pizza: $\frac{55}{200}$ of $360° = 99°$ $(360 \div 200 \times 55)$

Fish and chips: $\frac{64}{200}$ of $360° = 115.2°$ $(360 \div 200 \times 64)$

Chinese: $\frac{81}{200}$ of $360° = 145.8°$ $(360 \div 200 \times 81)$

Step Three – check that your answer adds up to 360°

Comparing Statistics

The **mean**, **median** and **mode** are averages. They say whether a list of numbers is on average higher or lower.

- A lower mean, median or mode means the numbers are **lower** on average.
- A higher mean, median or mode means the numbers are **higher** on average.

The **range**, **(semi-)InterQuartile range** and **standard deviation** are measures of spread. They say whether a list of numbers is more or less varied/consistent.

- A lower range, IQR or standard deviation means the numbers are more **consistent**.
- A higher range, IQR or standard deviation means the numbers are more **varied**.

Assessment Style Example

The temperature in Aberdeen has a mean of 3°C and a standard deviation of 5.
In London it has a mean of 9°C and a standard deviation of 3.
Make two comments comparing the temperatures in London and Aberdeen.

Solution

You would get NO MARKS (because you are stating the obvious) for:

- *"Aberdeen has a lower mean"*.
- *"London has a higher mean"*.
- *"Aberdeen has a higher standard deviation"*.
- *"London has a lower standard deviation"*.

You would get NO MARKS (because your sentence makes no sense) for:

- *"Aberdeen is lower"* (no mention of temperature)
- *"The first one has a lower temperature"* (no mention of Aberdeen or London)
- *"In London it is more consistent"* (no mention of what 'it' is)
- *"The standard deviation in London is more consistent"* (it is the temperature that is more consistent, not the standard deviation).

You WOULD get marks for sentences such as:

- *"The temperature in Aberdeen is lower than London and the temperature is less consistent"*
- *"The temperature in London is higher and more consistent than Aberdeen"*

Further examples of assessment style questions requiring comparing statistics can be found in:

- Medians, Quartiles and Boxplots: see **Assessment Style Example** on page 89.

Index of Key Words