Coatbridge High School
Mathematics Department

National 5 Mathematics Revision Notes

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FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sine Rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $\text{Area} = \dfrac{1}{2} ab \sin C$

Volume of a sphere: $\text{Volume} = \dfrac{4}{3}\pi r^3$

Volume of a cone: $\text{Volume} = \dfrac{1}{3}\pi r^2 h$

Volume of a pyramid: $\text{Volume} = \dfrac{1}{3} A h$

Standard deviation: $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\Sigma x^2 - (\Sigma x)^2/n}{n-1}}$, where n is the sample size.

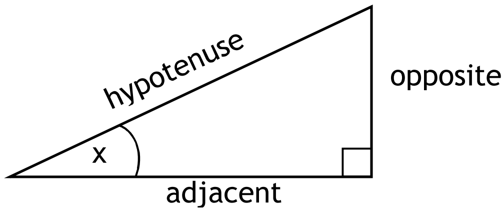
Topics By Unit

Expressions & Formulae	Relationships	Applications
Surds & Indices Factors & Brackets Completing the Square Algebraic Fractions Gradient Arcs & Sectors Volume	Straight Line Graphs Straight Line Equations Equations & Inequations Simultaneous Equations Change the Subject Quadratic Graphs Quadratic Equations Uses of Pythagoras Similar Shapes Trig Graphs Trig Equations	Trig Formulae Vectors Fractions & Percentages Statistics

Basic Trigonometry (Nat 4)

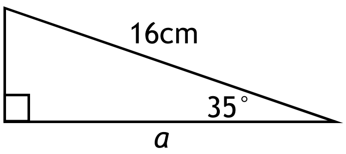
$\sin x = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos x = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan x = \dfrac{\text{opposite}}{\text{adjacent}}$

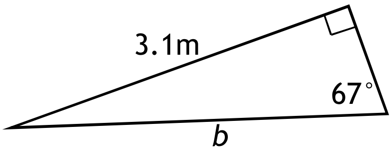
$\sin x = \dfrac{O}{H}$, $\cos x = \dfrac{A}{H}$, $\tan x = \dfrac{O}{A}$

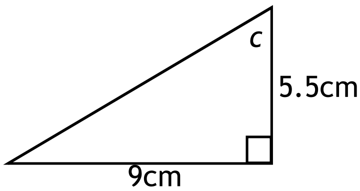


SOH CAH TOA

Find a, b and c in the examples below:

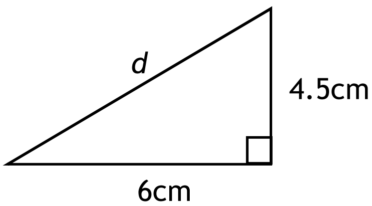


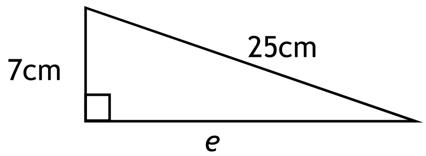




Pythagoras' Theorem (Nat 4)

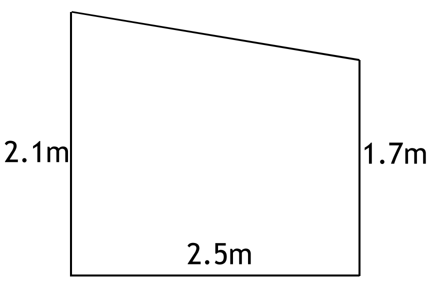
Find d and e in the examples below:





If the question has an angle: SOHCAHTOA	If there isn't an angle: PYTHAGORAS

The side wall of a shed has dimensions as shown below. Find:



a) the length of the roof

b) the angle the roof makes with the ground

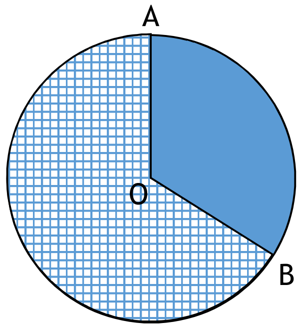
An **arc** is a part of the circumference of a circle.

If the ends of the arc are joined to the centre of the circle, then a **sector** is formed.

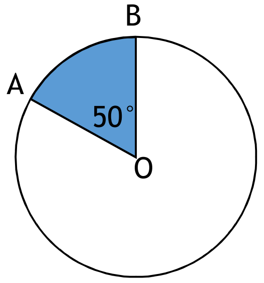
In the diagram opposite, AB is an arc and AOB is a sector.

When the angle at the centre is less than 180°, then AB is a **minor arc** and AOB is a **minor sector** (e.g. the shaded sector in the diagram)

When the angle at the centre is more than 180°, then AB is a **major arc** and AOB is a **major sector** (e.g. the checked sector in the diagram)



Look at the shaded sector in the circle below:



Angle AOB = 50°. This is $\frac{50}{360}$ of the full circle.

This means that arc AB has $\frac{50}{360}$ of the circumference of the full circle.

This also means that sector AOB has $\frac{50}{360}$ of the area of the full circle.

If the angle inside the sector is x° then:

$$\frac{x}{360} = \frac{\text{arc}}{\pi D} = \frac{\text{sector}}{\pi r^2}$$

NOTE:

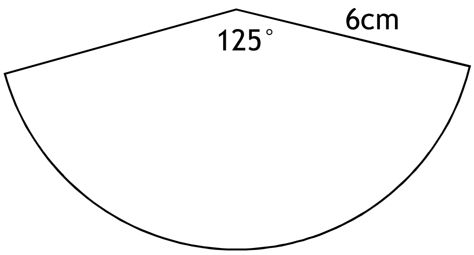
there is no need to simplify the fraction before we use it.

unless told otherwise, round answers to three significant figures.

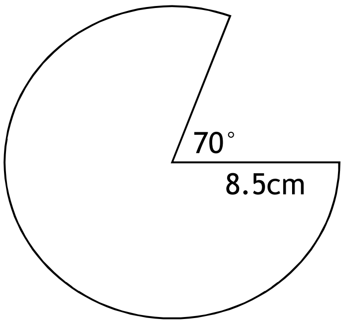
Example 1: In each sector of a circle below, find: (i) the length of the arc

(ii) the area of the sector.

a)



b)



We can also be asked to work backwards from the arc length or sector area.

Example 2: A sector of a circle has an angle of 26° at the centre.

If the arc length is 5cm, find the radius of the original circle.

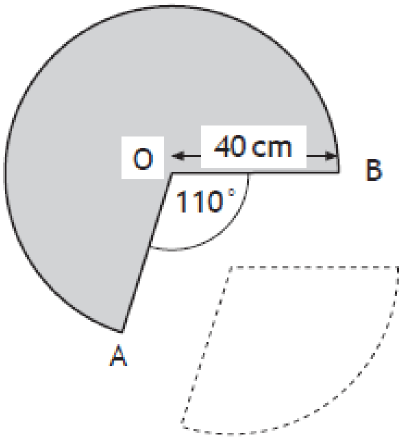
Example 3: A sector is made from a circle with a diameter of 3.2m.

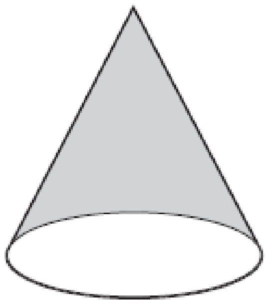
If the sector has an area of 6.37m^2, find the angle at the centre of the sector.

Past Paper Question.

A cone is formed from a paper circle with a sector removed as shown.

The radius of the paper circle is 40 centimetres. Angle AOB is 110°.



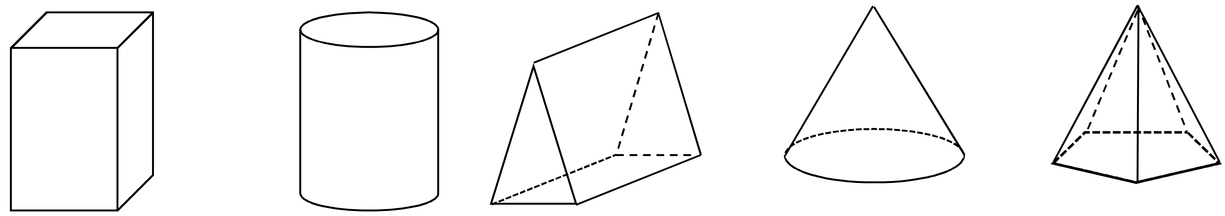


a) Calculate the area of the sector removed from the circle (**3 marks**).

b) Calculate the circumference of the base of the cone (**3 marks**).

Volumes of Prisms (E&F)

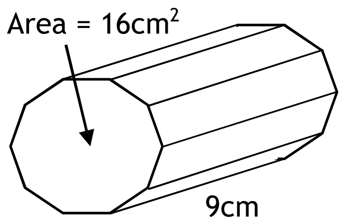
A **prism** is a regular solid object which has two parallel **congruent** end faces: if it was put in a bacon slicer, then all of the slices would be identical in size and shape.

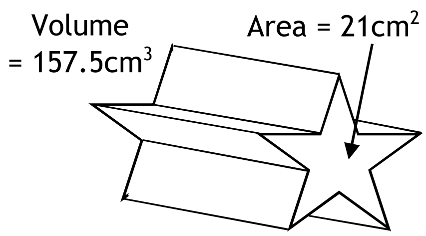


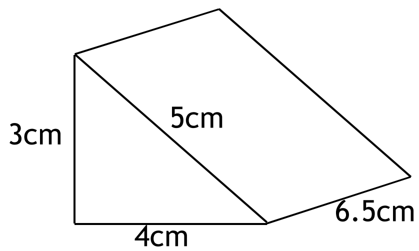
These are prisms **These are not prisms**

The volume of a prism is given by: $\boxed{V = Ah}$ where A = area of the end face
h = distance between end faces

Example 4: consider the prisms below:





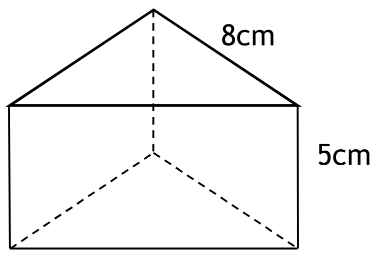


a) find the volume.

b) Find the height.

c) Find the volume.

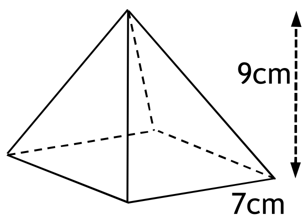
Example 5: A box is made in the shape of an equilateral triangular prism.



The triangular faces have side 8cm and the prism is 5cm high.

Find the volume of the box.

To find the volume of a **pyramid**, treat it like a prism but divide by three: $\boxed{V = \frac{1}{3}Ah}$

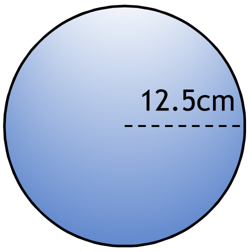


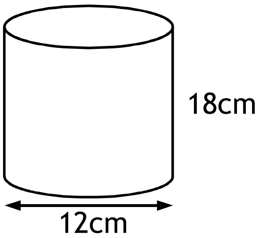
Example 6: A square-based pyramid has a base 7cm wide and a height of 9cm. Find its volume.

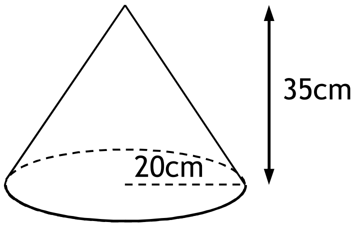
Example 7: Use the given formula to find the volume of each solid below (round to 3 s.f.).

Note: The formula for the volume of a cylinder is **not** given in the formula list!

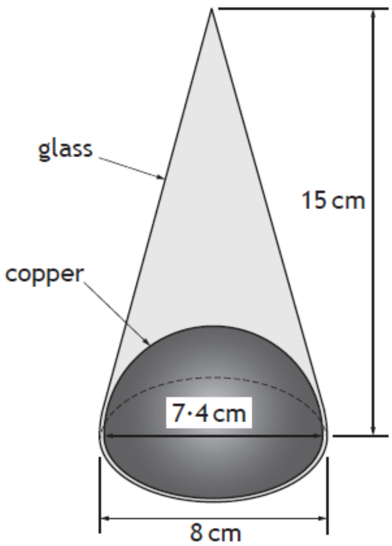
Sphere	Cylinder	Cone
$V=\frac{4}{3}\pi r^3$	$V=\pi r^2 h$	$V=\frac{1}{3}\pi r^2 h$







Past Paper Question



An ornament is in the shape of a cone with diameter 8 centimetres and height 15 centimetres.

The bottom contains a hemisphere made of copper with diameter 7.4 centimetres. The rest is made of glass, as shown in the diagram opposite.

Calculate the volume of the glass part of the ornament.

Give your answer correct to 2 significant figures. **(5 marks)**

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Example 8: Multiply out the brackets and collect like terms:

a) $3x(2x - 7) + 2(6 - x)$

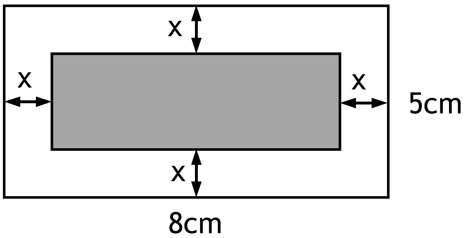
b) $(4p + 3)(p - 6)$

c) $(x + 1)^2 - 2(x + 1)$

d) $(b + 2)(b^2 - 3b - 2)$

Example 9: A rectangle has sides 8cm and 5cm. A border xcm wide is drawn inside it.

Find an expression for the area of the smaller rectangle created inside the original.



When factorising algebraic expressions, look for the following **in this order:**

1. COMMON FACTOR
2. DIFFERENCE OF TWO SQUARES
3. TRINOMIAL

Example 10: Factorise the following expressions **fully**:

a) $3g^2 - 6g^3$

b) $p^2 - 100$

c) $4t^2 - 25$

d) $5x^4y - 3x^2y^3$

e) $28 - 7y^2$

f) $3x^4 - 48x^2$

Example 11: Factorise the following trinomial expression, writing down the method used.

$x^2 + 10x + 24$

Method

Example 12: Use the method shown above to factorise:

a) $x^2 - 7x + 12$

b) $x^2 - 5x - 36$

c) $18 + 3x - x^2$

d) $4x^2 - 16x - 9$

Surds	**(E&F)**

A **rational number** is a number which can be written as a fraction of two integers. Any whole number, integer or decimal which repeats itself is a rational number.

e.g. 5 (= 10 ÷ 2), 6.2 (= 62 ÷ 10), 0.1111111… (= 1 ÷ 9), etc.

An **irrational number** is a number which can't be written as a fraction of two integers. They are decimal numbers with infinite digits with no repeated pattern: π is an irrational number.

Surds are irrational numbers which come from roots, e.g. $\sqrt{2}$ = 1.4142135… , $\sqrt[3]{6}$ = 1.817120…

Since neither of these decimal numbers are exact, when left written in root form they are called **exact values**.

Surds can be combined like x or y terms in algebraic expressions.

e.g. since 3x + 5y - x + 2y = 2x + 7y, $3\sqrt{2}+5\sqrt{3}-\sqrt{2}+2\sqrt{3} =$ ____________________

When asked to **simplify** a surd we need to make the number inside the root sign as small as possible. To do this, remember the rule:

$$\sqrt{\mathbf{ab}} = \sqrt{\mathbf{a}} \times \sqrt{\mathbf{b}}$$

If we rewrite the number inside the root sign as a square number multiplied by another number, then the square number inside a root sign will simplify to a whole number.

e.g. $\sqrt{8} \quad = \sqrt{4}\times\sqrt{2}$

$= 2\sqrt{2}$ ($\sqrt{4}$ becomes 2)

e.g. $\sqrt{12} \quad = \sqrt{4}\times\sqrt{3}$ (choose **4** x 3 as **4** is a square number: 2 x 6 has no square numbers)

$= 2\sqrt{3}$ ($\sqrt{4}$ becomes 2)

We should always look for the **largest** square number that we can divide by.

e.g. $\sqrt{72} \quad = \sqrt{36}\times\sqrt{2}$ (**9** x 8 would also give 72 but $\sqrt{8}$ would need to be simplified more)

$= 6\sqrt{2}$ ($\sqrt{36}$ becomes 6)

It is useful to try to memorise as many of the square numbers as possible; the first fifteen are:

1	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**	**121**	**144**	**169**	**196**	**225**

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Example 13: Simplify:

a) $4\sqrt{5}+5\sqrt{5}-6\sqrt{5}$

b) $\sqrt{7}+2\sqrt{10}-4\sqrt{10}-\sqrt{7}$

c) $\sqrt{18}$

d) $\sqrt{48}$

e) $\sqrt{450}$

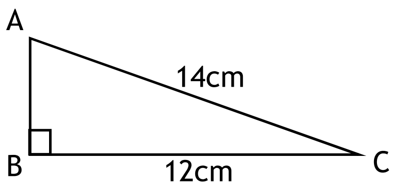
f) $\sqrt{50}+\sqrt{32}-\sqrt{2}$

g) $\sqrt{3}\times\sqrt{15}$

h) $\sqrt{6}\left(\sqrt{6}-\sqrt{2}\right)$

i) $\left(3\sqrt{2}+1\right)\left(\sqrt{2}-3\right)$

Example 14: Find the exact length of side AB, giving your answer in its simplest form.



To simplify surds in fractions, remember the rule: $\boxed{\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}}$

Example 15: Simplify:

a) $\dfrac{\sqrt{21}}{\sqrt{3}}$

b) $\dfrac{2\sqrt{10}}{\sqrt{5}}$

c) $\dfrac{6\sqrt{3}}{8\sqrt{12}}$

We already know that $\frac{1}{2}=\frac{2}{4}$ because the top and bottom of the first fraction were both multiplied by the same number (in this case 2).

In the same way, we can make equivalent fractions by multiplying the top and bottom of a fraction by the same surd value: we often do this to get rid of a surd on the bottom of our original fraction.

This process is known as **rationalising the denominator**.

Example 16: Express each of the following with a rational denominator in simplest terms.

a) $\frac{2}{\sqrt{3}}$ b) $\frac{10}{\sqrt{2}}$ c) $\frac{14}{3\sqrt{6}}$ d) $\frac{4\sqrt{10}}{\sqrt{2}}$

Past Paper Question 1:

Express $\frac{2}{\sqrt{5}}$ with a rational denominator in its simplest form. **(2 marks)**

Past Paper Question 2:

Express $\sqrt{40} + 4\sqrt{10} + \sqrt{90}$ as a surd in its simplest form. **(3 marks)**

Indices (E&F)

Indices is another word for powers: in 3^5, 3 is called the **base** and 5 is called the **index**.

There are six rules of indices for us to memorise:

$a^m \times a^n = a^{m+n}$

When we **multiply** we **add the powers**

$a^2 \times a^3 = a^5$
$g^7 \times g^{-4} = g^3$
$d^{1/5} \times d^{2/5} = d^{3/5}$

$a^m \div a^n = a^{m-n}$

When we **divide** we **subtract the powers**

$a^6 \div a^2 = a^4$
$g^2 \div g^{-3} = g^5$
$d^{3/7} \div d^{2/7} = d^{1/7}$

$a^0 = 1$

Anything to the power of **zero** equals **one**

(since $a^5 \div a^5 = a^0$ and anything divided by itself is one)

$5^0 = 1$, $1.74^0 = 1$, etc.

$a^{-1} = \frac{1}{a}$

Negative powers become **positive** on the **denominator**

$a^2 \div a^4 = a^{-2}$

AND $a^2 \div a^4 = \frac{a \times a}{a \times a \times a \times a} = \frac{1}{a^2}$

SO $a^{-2} = \frac{1}{a^2}$

$3a^{-2} = \frac{3}{a^2}$, $\frac{4}{5}a^{-7} = \frac{4}{5a^7}$

$(a^m)^n = a^{mn}$

For **powers of powers** we **multiply them together**

$(a^2)^3 = a^6$

$(3a^4)^2 = 9a^8$

$\left(\frac{2}{3a^3}\right)^3 = \frac{8}{27a^9}$

$a^{m/n} = \sqrt[n]{a^m}$

Fractional powers can be rewritten as **roots**

$\sqrt{a} \times \sqrt{a} = a$ (or a^1)

AND $a^{1/2} \times a^{1/2} = a^1$

SO $a^{1/2} = \sqrt{a}$

$a^{1/3} = \sqrt[3]{a}$, $a^{5/4} = \sqrt[4]{a^5}$

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Questions on indices usually require the use of more than one rule!

Example 17: Simplify the following, leaving your answers with positive indices:

a) $\dfrac{3p^4q^2r}{6p^2r^3}$

b) $3a^{1/3}\left(a^{-1/3}+2a^{-2/3}\right)$

c) $\left(a^{12}\right)^{-2/3}$

When asked to evaluate a number with a fractional power:

Rewrite as a root
Do the root part first
Do the power part last

Example 18: Evaluate:

a) $16^{3/2}$

b) $8^{2/3}$

c) $81^{5/4}$

d) $32^{-1/5}$

Past paper Question 1: Simplify $\dfrac{5p^7 \times 4p^{-2}}{2p}$

(2 marks)

Past Paper Question 2: Evaluate $8^{5/3}$

(2 marks)

Scientific Notation Calculations (E&F)

Scientific notation is used to write very large or very small numbers easily.

Revision: Write in scientific notation:

The distance from Earth to the Sun is 147 000 000 km

The width of a Hydrogen atom is 0.000000053 mm

To write numbers in scientific notation on a calculator, use the button marked either: **$x10^x$** or **Exp**

e.g. to write 2.97×10^5 we would type 2.97, press $\boxed{x10^x}$, then type 5.

Example 19: The Voyager 1 space probe is the only man-made object which has left our solar system. It is currently moving at a speed of 17.6 kilometres per second.

Calculate the distance travelled by Voyager 1 in a year, giving your answer in scientific notation accurate to 3 significant figures.

Straight Line Graphs (Rel)

Example 20: On the axes opposite, sketch:

a) $y = 2x$

x	0	1	2	3
2x				

b) $y = 2x + 1$

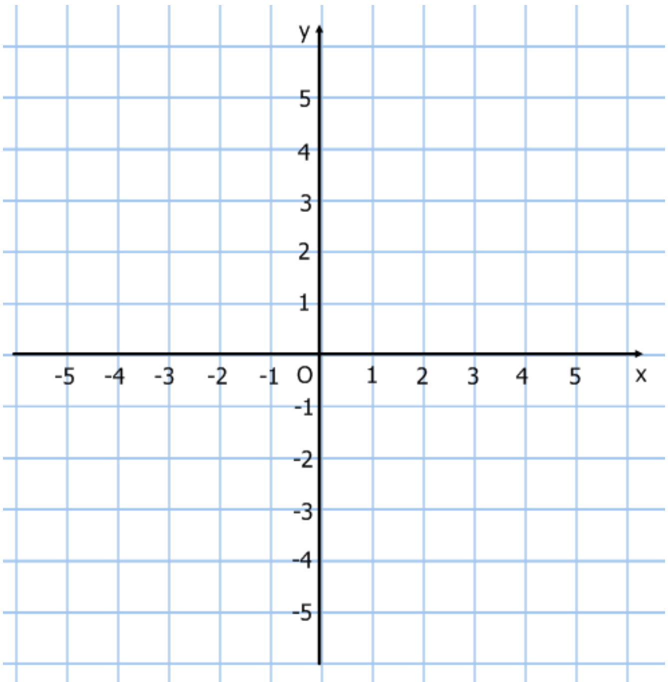
x	0	1	2	3
2x + 1				

c) $y = 2x - 4$

x	0	1	2	3
2x – 4				

d) $y = -2x + 3$

x	0	1	2	3
-2x + 1				



This shows that a formula of the form

$\boxed{y = mx + C}$

produces a **straight line graph**.

In each case above, m (the number with the x) is the **gradient** of the line, and c (the other number) describes the point the line cuts the y-axis (known as the **y-intercept).**

Example 21: Write down the gradient and y-intercept of the lines:

a) $y = 4x - 2$

b) $y = \frac{3}{2}x$

c) $y = 5 - 3x$

To find the gradient of the line joining any two points, use the formula:

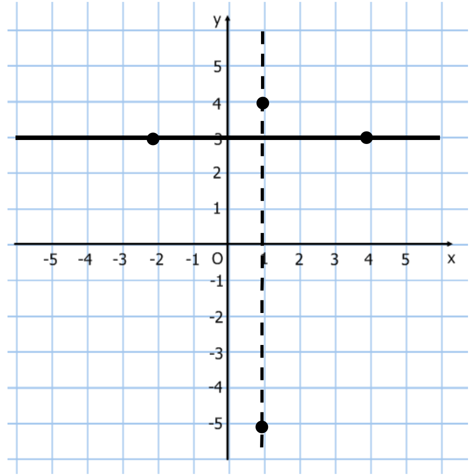
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 22: Find the gradient of the line joining the points:

a) A (3, 5) and B (6, 11)

b) C (-3, -1) and D (1, -3)

Calculate the gradient of the solid line in the diagram opposite, which passes through (-1, 3) and (5, 3).



Now calculate the gradient of the dotted line, which passes through (2, 4) and (2, -5)

The **solid** line is **horizontal:** its gradient is **zero** and its equation is **y = 3**.

The **dotted** line is **vertical**: its gradient is **undefined** and its equation is **x = 2**.

To summarise what we have seen so far:

a) lines sloping **up** from left to right have **positive** gradients
b) lines sloping **down** from left to right have **negative** gradients
c) lines with **equal** gradients are parallel
d) **horizontal** lines have gradient **zero** and equation y = a
e) **vertical** lines have gradient **undefined** and equation x = b

To find the equation of a straight line, we need its gradient and a point on the line.

If the point is on the y-axis, use

y = mx + c

If the point is not on the y axis, use

y - b = m(x - a)

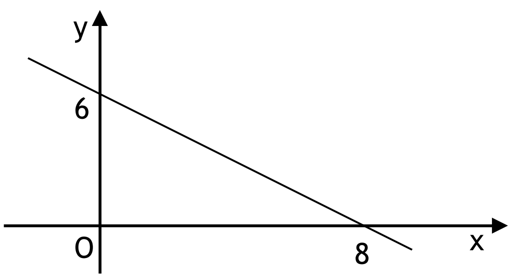
Example 23: Find the equation of:

a) the line through (0, 4) with gradient 4

b) the line through (2, 6) with gradient -2

c) the line through (-3, 2) and (0, -4)

d) the line through (-2, -4) and (8, 1)



e) the line shown in the diagram opposite.

Straight line graphs can be used to describe real life situations where something changes by a fixed amount.

Example 24: Craig is an electrician. He charges a call out fee plus an hourly rate. Craig's charges are represented on the graph below.

a) How much is the call out fee?

b) Find the equation of the line in terms of H and C.

c) Use your formula to find the cost of an 8 hour job.

The line $y = -x + 5$ could also be written as $y + x - 5 = 0$. This is called the **general form** of the straight line equation.

Straight line equations can be written as:	$y = mx + c$	$y - b = m(x - a)$	$Ax + By + C = 0$

When given an equation in the general form, the gradient is **NOT** always the number in front of x! We have to rearrange the equation until it says "y = ….." first.

Example 25: Find the gradient and y-intercept of the line $6x + 2y - 12 = 0$.

Example 26: A line has as its equation $2x - 4y + 4 = 0$.

a) Find the coordinates of the points where the line meets the x- and y-axes.

b) Sketch the graph $2x - 4y + 4 = 0$.

Past Paper Question 1: Find the equation of the line joining the points (-2, 5) and (3, 15).

Give the equation in its simplest form.

(3 marks)

Past Paper Question 2: A straight line has equation $4x + 3y = 12$.

a) Find the gradient of this line. **(2 marks)**

b) Find the coordinates of the point where this line crosses the x-axis. **(2 marks)**

Function Notation (E&F)

A **function** is a set of instructions which changes one group of numbers into another. For example, $y = 3x + 1$ takes an x-coordinate, multiplies it by three then adds one to change it into a y-coordinate.

In **function** notation, the rule above would be written as $\mathbf{f(x) = 3x + 1}$ and "f(x)" is read as "f of x".

In a function, the numbers going in are called the **domain**; those coming out are called the **range.**

Example 27: For $f(x) = 2x + 3$, find:

a) $f(5)$

b) $f(-1)$

c) $f(a)$

d) the value of x for which $f(x) = 17$.

Example 28: For $g(x) = 12 - 3x^2$, find $g(-2)$.

Example 29: $h(x) = x^3 + 1$.

Find the value of p if the point (p, -7) lies on the graph of $y = h(x)$.

A **mixed fraction** is a whole number and a fraction together. A **top-heavy** fraction is one with the numerator (top number) greater than the denominator (bottom number).

Example 30: Convert:

a) $\frac{35}{8}$ into a mixed fraction

b) $4\frac{2}{7}$ into a top-heavy fraction

To add or subtract fractions there must be a **common denominator**. If there isn't, then we need to make **equivalent fractions** first.

Example 31: Find:

a) $\frac{5}{7}+\frac{1}{7}$

b) $\frac{5}{8}+\frac{1}{4}$

c) $\frac{3}{4}-\frac{2}{3}$

d) $3\frac{1}{4}+1\frac{2}{5}$

e) $6\frac{3}{5}-4\frac{1}{3}$

f) $5\frac{1}{5}-3\frac{1}{3}$

To multiply fractions together:

multiply the top numbers
multiply the bottom numbers

To divide one fraction by another:

flip the second fraction upside down
change divide into multiply

Before multiplying or dividing, mixed fractions **must be made top-heavy first**.

Example 32: Find in simplest form:

a) $\frac{5}{8}\times\frac{4}{9}$

b) $3\frac{1}{2}\times2\frac{2}{5}$

c) $\frac{10}{21}\div\frac{2}{7}$

d) $6\frac{4}{5}-\frac{3}{4}$ of $2\frac{6}{7}$

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When simplifying algebraic fractions, we can only cancel out **factors** on the top and bottom.

i.e. $\dfrac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x-3)} = \dfrac{x-1}{x-3}$ $\qquad \dfrac{(x+2)\cancel{(x-1)}}{(x-1)^{\cancel{3}\,2}} = \dfrac{x+2}{(x-1)^2}$ $\qquad$ **BUT:** $\dfrac{x+2}{x+5} \neq \dfrac{2}{5}$ **this can't be simplified any further!**

Example 33: Simplify:

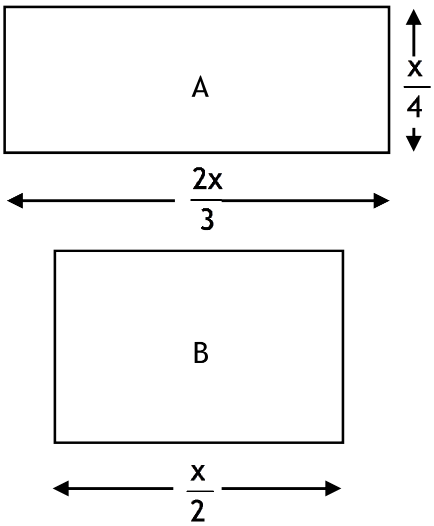
a) $\dfrac{7xy}{14y^2}$ $\qquad$ b) $\dfrac{12x-4xy}{12x}$ $\qquad$ c) $\dfrac{2x+2}{x^2+3x+2}$ $\qquad$ d) $\dfrac{x^2+3x-4}{x^2-x-20}$

Algebraic fractions follow **exactly** the same rules as fractions with only numbers!

Example 34: Express each as a single fraction in its simplest form:

a) $\dfrac{2x}{3}+\dfrac{x}{4}$ $\qquad$ b) $\dfrac{2x-1}{3}-\dfrac{x}{2}$ $\qquad$ c) $\dfrac{5}{x+1}-\dfrac{3}{x-2}$

d) Two rectangles with equal areas have sides as shown below. Find expressions for:



(i) The area of rectangle A

(ii) The height of rectangle B.

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Functions where the highest power is 2 are called **quardatic functions.** The graph of a quadratic function is known as a **parabola.**

The point where the parabola changes direction is called its **turning point**.

Example 35: Complete the tables and sketch the quadratic functions on the axes below.

a) $y = x^2$

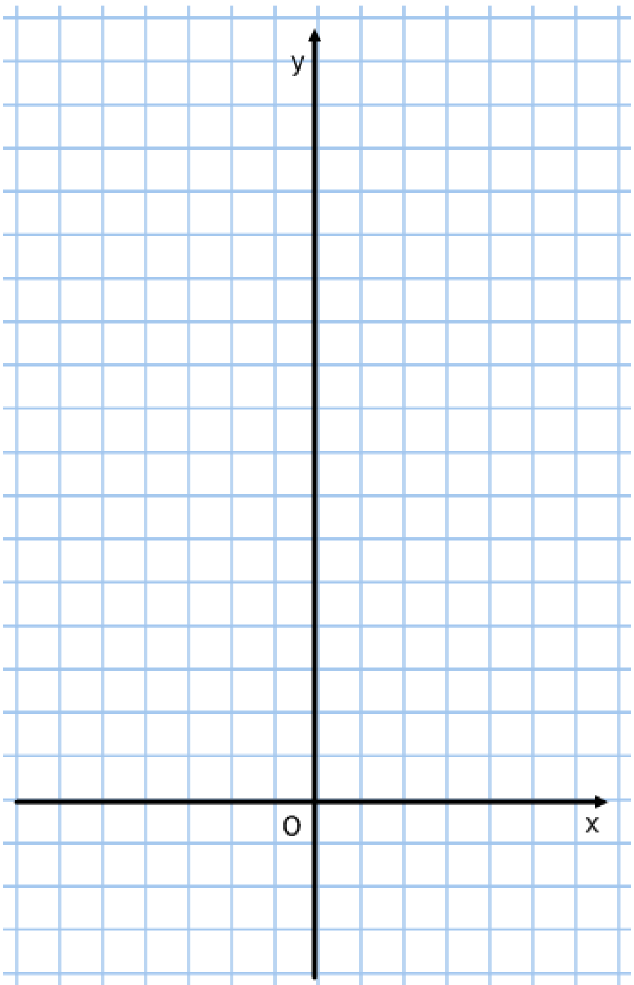
x	-3	-2	-1	0	1	2	3
y							

b) $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

c) $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
y							



For $y = x^2 + c$,

If $c > 0$, the parabola cuts **above** the x-axis

If $c < 0$, the parabola cuts **below** the x-axis

the **y-axis** is a **line of symmetry**

d) $y = 2x^2$

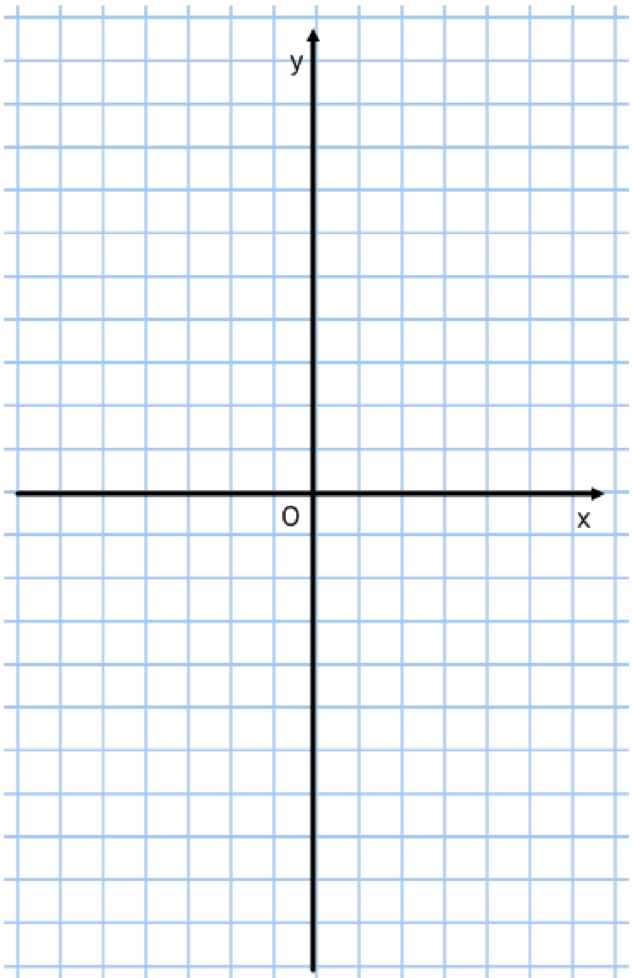
x	-3	-2	-1	0	1	2	3
y							

e) $y = ½x^2$

x	-3	-2	-1	0	1	2	3
y							

f) $y = - x^2$

x	-3	-2	-1	0	1	2	3
y							



For $y = k x^2$,

If $k > 0$, the parabola is **stretched**

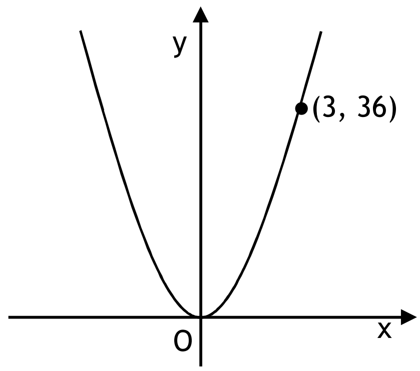
If $0 < k < 1$, the parabola is **squashed**

If $k < 0$, the parabola has a **maximum TP**

the **y-axis** is a **line of symmetry**

Example 36: The point (3, 36) lies on the graph $y = k x^2$ as shown.

Find the value of k.



Example 37: Complete the tables and sketch:

a) $y = (x - 1)^2$

x	-1	0	1	2	3
y					

b) $y = -(x + 2)^2$

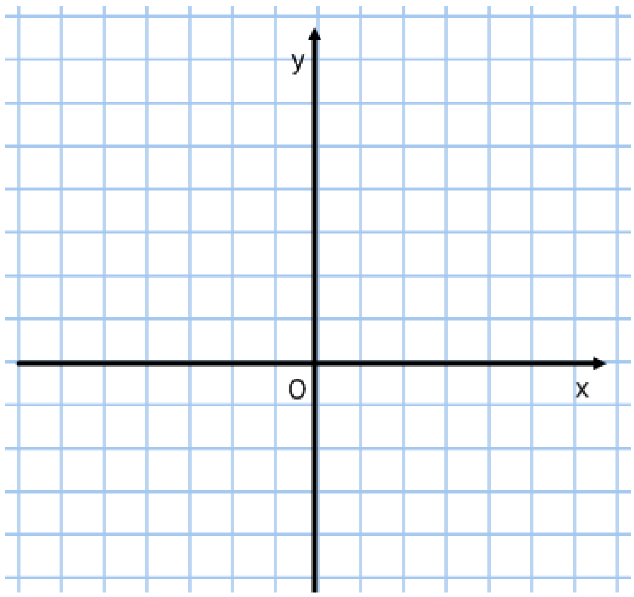
x	-4	-3	-2	-1	0
y					

c) $y = (x - 3)^2 + 2$

x	1	2	3	4	5
y					

d) $y = -(x - 2)^2 + 4$

x	0	1	2	3	4
y					



a) $y = (x - 1)^2$ has a ________ TP at __________ and x = ______ is the axis of symmetry

b) $y = -(x + 2)^2$ has a ________ TP at __________ and x = ______ is the axis of symmetry

c) $y = (x - 3)^2 + 2$ has a ________ TP at __________ and x = ______ is the axis of symmetry

d) $y = -(x - 2)^2 + 4$ has a ________ TP at __________ and x = ______ is the axis of symmetry

The graph of $y = (x - a)^2 + b$ has:

its TP at the point (a, b)

x = a as its axis of symmetry

a MINIMUM TP if the bracket has a positive number at the front

a MAXIMUM TP if the bracket has a negative number at the front

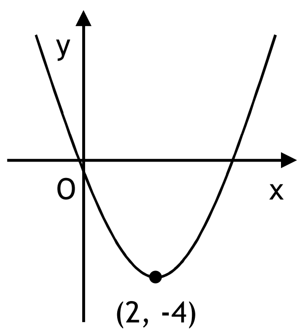
Example 38: Write down the position and nature of the TP and the axis of symmetry of:

a) $y = (x - 5)^2 + 6$

b) $y = 5 - (x + 3)^2$

Example 39: The graph shown below has as its equation $y = (x - p)^2 + q$.

State the values of p and q.



Completing the Square (E&F)

A quadratic function written $f(x) = (x - a)^2 + b$ is in its **completed square form**.

Changing a quadratic funtion written as a trinomial into this form is called **completing the square**.

To write a trinomial in completed square form:

divide the number in front of x by 2

put this inside a squared bracket with x

square it and subtract from the number term

Example 40: For each quadratic function below: (i) write in the form $y = (x - a)^2 + b$

(ii) state the coordinates of the TP

a) $y = x^2 + 6x + 1$

b) $y = x^2 - 10x - 15$

c) $y = x^2 + 3x$

Solving Quadratic Equations (Rel)

Linear equations (i.e. when there is only an x term) give one answer. Quadratic equations (i.e. when there is an x^2 term) are equations which give **two** answers.

To solve a quadratic equation, we use the fact that if the answer to a multiplication problem was **zero**, then at least one of the things we multiplied must have been zero.

In other words, if $(x - 2)(x - 3) = 0$ then either $(x - 2) = 0$ or $(x - 3) = 0$.

We then solve each of those new equations to get that either $x = 2$ or $x = 3$.

Example 41: Solve:

a) $(x - 5)(x + 3) = 0$

b) $x(x + 1) = 0$

c) $(2x + 5)(3 - x) = 0$

If the equation is not already written in brackets, we have to:

Check that the right-hand side is zero

Factorise the left-hand side

Solve for each bracket

Example 42: Solve:

a) $x^2 - 11x = 0$

b) $9 - x^2 = 0$

c) $x^2 + 6x + 5 = 0$

d) $x^2 + 3x - 4 = 0$

e) $3x^2 + 11x - 4 = 0$

f) $x^2 = 3x + 40$

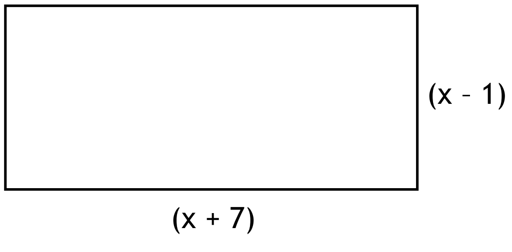
g) $(x - 2)(x + 3) = 6$

h) $\frac{14}{x} + 5 = x$

If the question is in a "real-life" context, then we should solve as normal but **discard negative answers** where appropriate.

Example 43: A rectangle has sides (x + 7) cm and (x – 1) cm as shown below.

Find the value of x, given that the rectangle has an area of $48cm^2$.



Sketching Graphs of Quadratic Functions (Rel)

On the graph of a quadratic function, there are four things to consider:

if/where it cuts the x-axis (the roots)	where it cuts the y-axis (the y-intercept)	where the TP and axis of symmetry are	what type of TP is it (min or max)

The steps we need to take to get these pieces of information will change depending on the way the quadratic function is written.

If the function is in completed square form, we do not normally have to include the roots.

If the function is written as a standard or factorised trinomial, then we should include the roots.

Example 44: Sketch the graph of $y = (x + 4)^2 + 3$, labelling the TP, axis of symmetry and y-intercept.

Example 45: Sketch the graph of $y = x^2 – 6x – 16$, clearly labelling the TP, axis of symmetry and points where the graph cuts the x- and y-axes.

The Quadratic Formula (Rel)

Not every quadratic equation can be solvedby factorising.

For example, consider the graph of $y = (x - 3)^2 - 2$. We know that because the turning point is (3, -2) that the graph must cut the x-axis.

However, if we remove the brackets and try to find the roots, we need to solve $x^2 - 6x + 7 = 0$. The problem here is that there are no whole numbers which multiply to give 7 but which also add up to -6.

To solve equations like this, we use the **quadratic formula**.

For $ax^2 + bx + c = 0$,

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Questions which need the use of the quadratic formula will ask for answers given to some number of decimal places or significant figures.

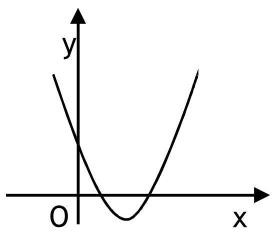
Example 46: Find the roots of the equation $x^2 - 6x + 7 = 0$, giving your answers to 1 d.p.

The Discriminant and the Nature of Roots (Rel)

Shown below are three quadratic graphs where only the number term has changed.

Use the quadratic formula to find the roots of each graph.

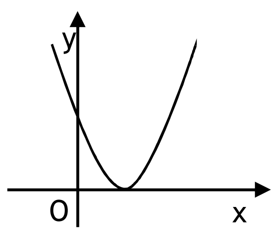
(i) $y = x^2 - 4x + 3$



a = 1, b = -4, c = 3

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

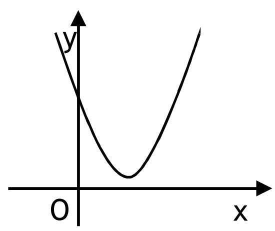
(ii) $y = x^2 - 4x + 4$



a = 1, b = -4, c = 4

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

(iii) $y = x^2 - 4x + 5$



a = 1, b = -4, c = 5

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

In each equation, only the value of c changes, yet the roots are all different. The **nature** of the roots is determined purely by the part of the formula inside the square root sign, i.e. $b^2 - 4ac$.

In equation (i), $b^2 - 4ac > 0$, and the equation had two **unequal** (i.e. different or distinct) roots.
In equation (ii), $b^2 - 4ac = 0$, and the equation had two **equal roots** (as $x^2 - 4x + 4 = (x - 2)(x - 2)$).
In equation (iii), $b^2 - 4ac < 0$, and the equation had **no real** roots.

For $ax^2 + bx + c = 0$, $b^2 - 4ac$ is known as the **discriminant.**

- **If $b^2 - 4ac > 0$ the roots are real and unequal**
- **If $b^2 - 4ac = 0$ the roots are real and equal**
- **If $b^2 - 4ac < 0$ there are NO real roots**

If $b^2 - 4ac$ gives a perfect square, the roots are RATIONAL
If $b^2 - 4ac$ does NOT give a perfect square, the roots are IRRATIONAL (i.e. surds)

Whenever we are asked about the nature of the roots of a quadratic equation, we should _always_ use the discriminant!

Example 47: State the nature of the roots of the equations below:

a) $x^2 - 6x + 9 = 0$

b) $2x^2 - 4x + 5 = 0$

c) $x^2 - 4 = 7x$

Past Paper Question 1: Solve $2x^2 + 7x - 15 = 0$ **(3 marks)**

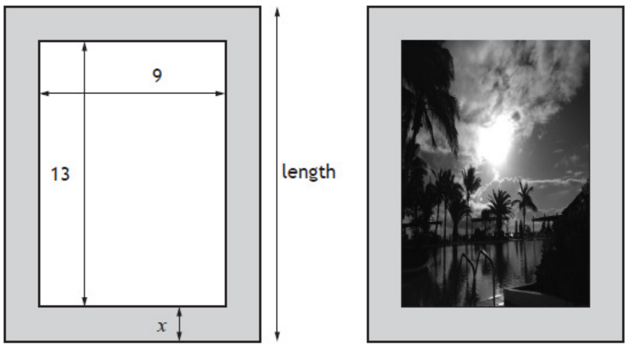
Past Paper Question 2: A parabola has equation $y = x^2 - 8x + 19$.

a) Write the equation in the form $y = (x - p)^2 + q$. **(2 marks)**

b) Sketch the graph of $y = x^2 - 8x + 19$, showing the coordinates of the turning point and the point of intersection with the y-axis. **(3 marks)**

Past Paper Question 3: Find the range of values of p such that $px^2 - 2x + 3 = 0$, $p \neq 0$, has no real roots. **(4 marks)**

Past Paper Question 4: A rectangular picture measuring 9 centimetres by 13 centimetres is placed on a rectangular piece of card.



The area of the card is 270 square centimetres.

There is a border x centimetres wide on all sides of the picture.

a) Write down an expression of the length of the card **(1 mark).**

b) Hence show that $4x^2 + 44x - 153 = 0$. **(2 marks)**

c) Calculate x, the width of the border. Give your answer to one decimal place. **(4 marks)**

Example 48: Solve the following equations:

a) $5(3x - 2) - 8 = 6 + 7x$

b) $15 - (8 - x) = 4(2x - 3) + 2$

c) $\frac{x - 4}{5} + 3 = -1$

d) $\frac{x - 3}{2} + \frac{4x}{3} = 15$

Example 49: Paul is paid £x in wages. He puts £250 into his savings account and spends $\frac{2}{3}$ of what is left. After this, he has £180 left.

Make an equation describing this information, and solve it to find Paul's wages.

Inequations use the signs <, >, ≤ and ≥. They are solved exactly the same as equations, with one difference: if we have to multiply or divide by a negative number, **the arrow changes direction.**

Example 50: Solve:

a) $4(3x - 1) < 8 - 3(2x - 1)$

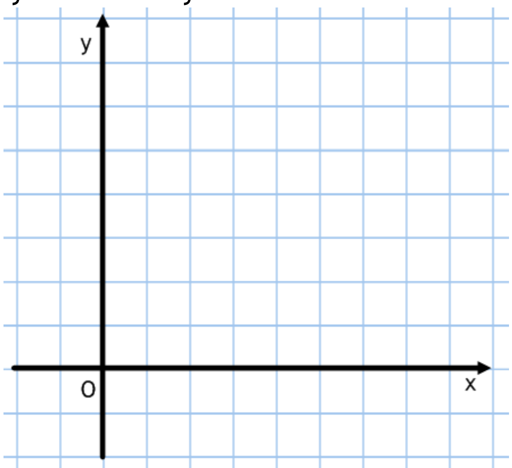
b) $20 - 2(3x + 8) \geq 8 - 5x$

Simultaneous Equations (Rel)

We can be asked to find the coordinates of the point where two straight line graphs meet. There are two ways this can be done: **graphically** (i.e. by drawing the lines) or **algebraically** (i.e. solving both equations at the same time).

As straight line equations have **two** unknowns (x and y), there are **two** numbers in our answer. As the point lies on both lines at the same time, these are called **simultaneous equations.**

Example 51: Find graphically the point of intersection of the lines $y = x - 2$ and $y = -x + 6$.



Most of the time however, we are asked to solve these types of equations **algebraically**, i.e. without drawing a graph.

There are two methods we can use to solve simultaneous equations:

By Substitution

Since $y = x - 3$ AND $y = -x + 5$, we can say

$$\begin{aligned} x - 3 &= -x + 5 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

(use x = 4 in either of the original equations)

$$\begin{aligned} y &= 4 - 3 \\ y &= 1 \end{aligned}$$

So the solution is x = 4, y = 1

By Elimination

Add the equations and the x terms cancel.

$$\begin{array}{lrcll} & y &=& x - 3 & (1) \\ & y &=& -x + 5 & (2) \\ \hline \text{ADD} & 2y &=& 2 & \\ & y &=& 1 & \end{array}$$

Substitute 1 for y in equation (1)

$$\begin{aligned} 1 &= x - 3 \\ 4 &= x \end{aligned}$$

So the solution is x = 4, y = 1

Whilst both methods give the same answer, the substitution method can often become complicated by having to rearrange equations which result in fractions. Elimination is more commonly used.

Example 52: Solve:

a)
$$\begin{aligned} x + y &= 7 \\ 2x - y &= 8 \end{aligned}$$

b)
$$\begin{aligned} 4x - y &= 20 \\ 3x - y &= 17 \end{aligned}$$

Some equations do not have terms which will immediately cancel and need to be changed first.

Example 53: Solve:

a) $$\begin{aligned} 3f + 2g &= -12 \\ f - g &= 1 \end{aligned}$$

b) $$\begin{aligned} 2a + 5b &= 3 \\ 3a - 4b &= 16 \end{aligned}$$

Simultaneous equations can be used to solve problems in real life contexts.

Example 54: A builder buys bags of cement and sand to mix into concrete.

Four bags of cement and six bags of sand cost £39. Three bags of cement and four bags of sand cost £28.

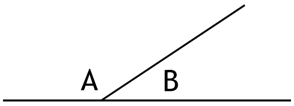
Find the cost of one bag of cement and one bag of sand.

Past Paper Question: Two groups of people go to a theatre.

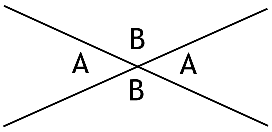
Tickets for 5 adults and 3 children cost £158.25. Tickets for 3 adults and 2 children cost £98.

Calculate the cost of a ticket for an adult and the cost of a ticket for a child. **(6 marks)**

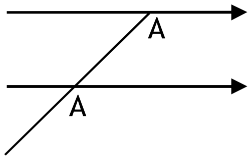
Remember the following types of angle:



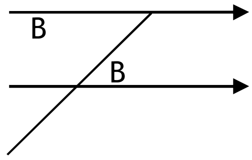
Supplementary
A + B = 180°



Opposite (X)
angles are EQUAL

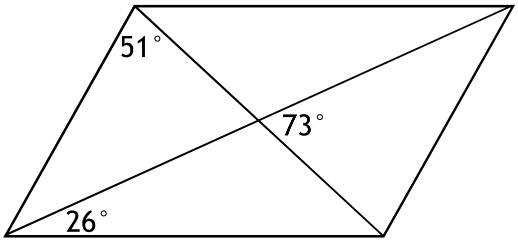


Corresponding (F)
angles are EQUAL

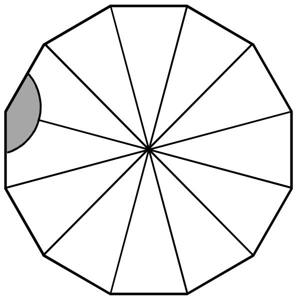


Alternate (Z)
angles are EQUAL

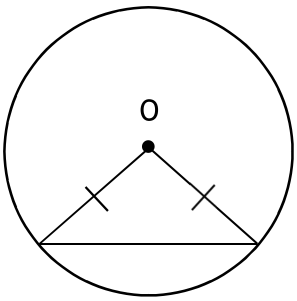
Example 55: Fill in all the missing angles in the parallelogram below.



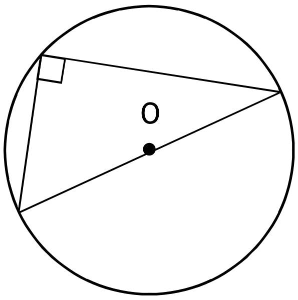
Example 56: A regular 12-sided polygon is shown. Calculate the size of the shaded angle.



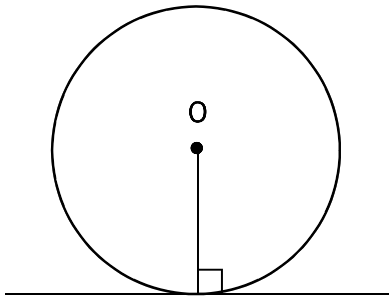
There are **three** angle facts to remember when it comes to circles:



Triangles including the centre and a chord are ISOSCELES



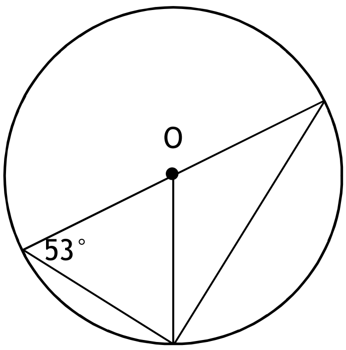
Triangles including the diameter and a point on the circumference are RIGHT ANGLED



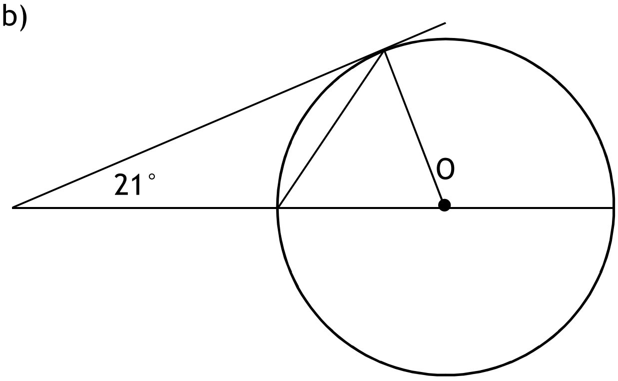
At the point where a tangent meets the circle, the radius and the tangent meet at 90°

Example 57: Fill in all missing angles in the diagrams below.

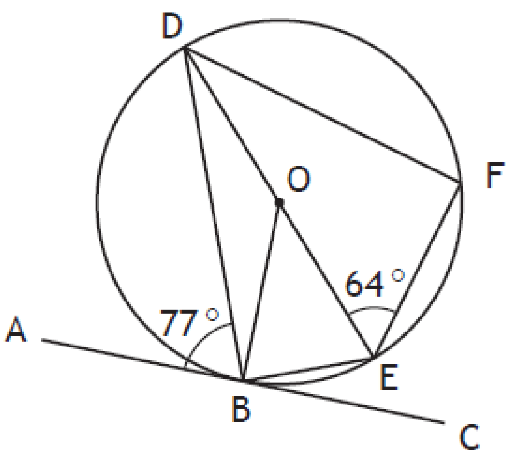
a)



b)



Exam questions on angles within circles will need **at least two** steps to complete.



Past Paper Question 1:

AC is a tangent to the circle, centre O, with point of contact B.

DE is a diameter of the circle and F is a point on the circumference.

Angle ABD is 77 and angle DEF is 64.

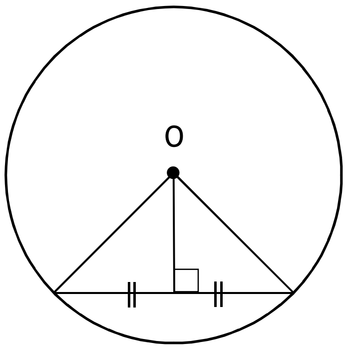
Calculate the size of angle BDF. **(3 marks)**

A special type of question occurs when we have an isosceles triangle inside a circle and we are asked to find or use its height.

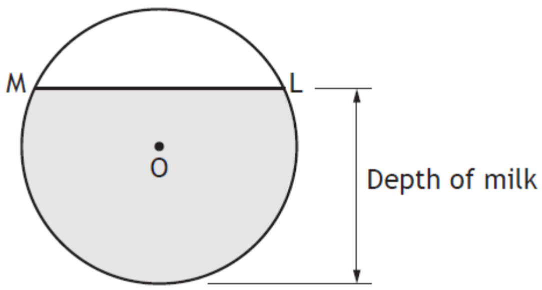
This is another way of asking us to use **Pythagoras' Theorem.**

To do this, we cut the chord in **half** to make a **right angled triangle**.

These questions are **very common in the exam** and will often be in the context of a tunnel, a tanker, a pipe, or anything where part has been "cut out" of a circle.



Past Paper Question 2: The diagram below shows the circular cross-section of a milk tank.



The radius of the circle, centre O, is 1.2 metres.

The width of the surface of the milk in the tank, represented by ML in the diagram, is 1.8 metres.

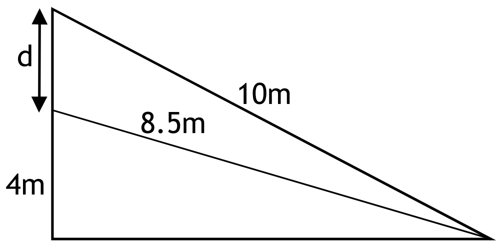
Calculate the depth of milk in the tank. **(4 marks)**

Pythagoras' Theorem can be used in complex problems requiring more than one step.

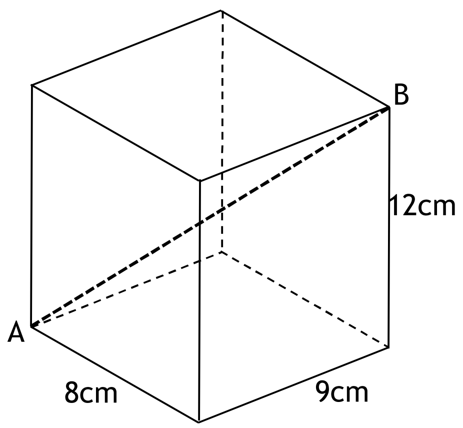
Example 58: A wall in a derelict building is propped up with two metal poles as shown below.

The poles are secured to the ground at the same point. The shorter pole is 8.5 metres long and reaches 4 metres up the wall. The longer pole is 10 metres long.

Find the distance between the points where each pole is attached to the wall, giving your answer to the nearest centimetre.



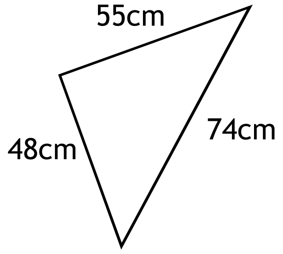
Example 59: Find the length of the space diagonal AB in the cuboid shown below.



If we are asked to **prove** whether or not a triangle is right angled, then we need to use the **Converse of Pythagoras.**

It is very important to show consistent working in these questions! It is easy to change methods halfway through, which will result in not being awarded all of the available marks.

State whether the triangle below is right angled.



Method 1: Converse

$74^2 = 5476$

$48^2 + 55^2 = 5329$

Since $74^2 \neq 48^2 + 55^2$, the triangle is **not** right angled (by the Converse of Pythagoras Theorem)

Method 2: Standard Pythagoras

$a^2 = b^2 + c^2$

$a^2 = 55^2 + 48^2$

$a^2 = 5329$

$a = \sqrt{5329}$

$a = 73\text{cm}$

Since a ≠ 74cm, the triangle is **not** right angled.

CHOOSE ONE OF THESE METHODS AND STICK TO IT!

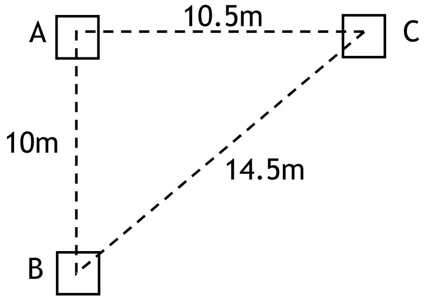
It is also important to give a statement at the end of a Converse of Pythagoras problem which is **in the context of the question**.

Example 60: Humza is building a raised deck in his back garden. He cements three posts into the ground which will have decking boards nailed to them: to meet building regulations, the boards need to form a 90° angle at Post A.

Post B and Post C are 10 metres and 10.5 metres from Post A respectively.

Post B is 14.5 metres from Post C.

Will the posts meet building regulations? Justify your answer.

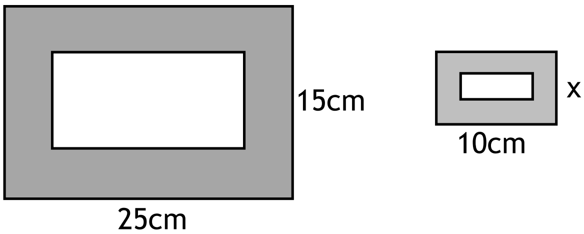


Similarity (Rel)

In mathematics, two objects are said to be **similar** if one is a **scaled version** of the other.

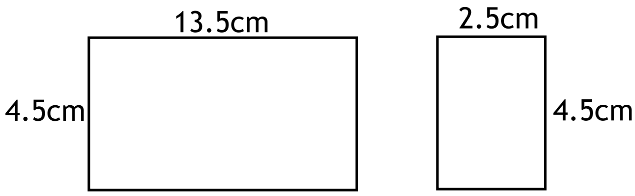
Revision Question 1: the two picture frames below are similar.

Find the length of side x.

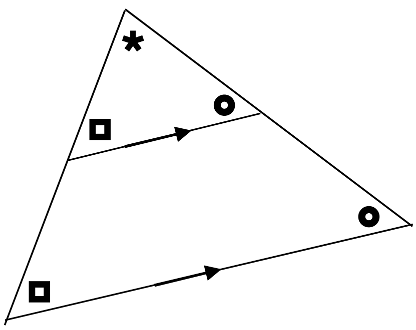


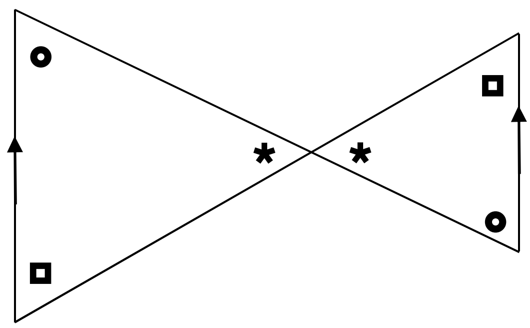
Revision Question 2: Are the rectangles shown below similar?

Justify your answer.



When two triangles have the same three angles then they are similar. This often happens when we have triangles appearing within parallel lines.

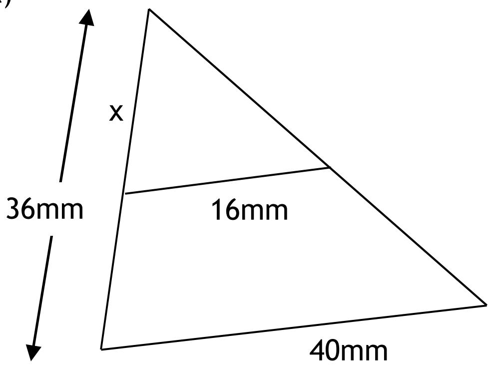




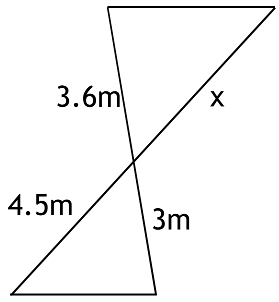
Equivalent sides are across from equivalent angles!

Example 61: Find the length marked x in each diagram below.

a)

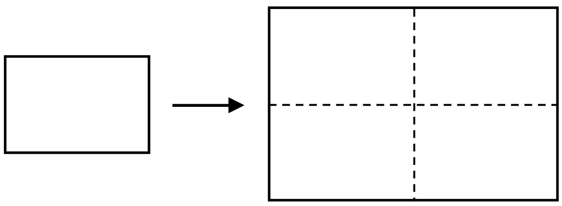


b)



A rectangle has sides of 5cm and 8cm.

A similar rectangle has sides **twice** as long.



Small

Area = 5 x 8

= $40cm^2$

Large

Area = 10 x 16

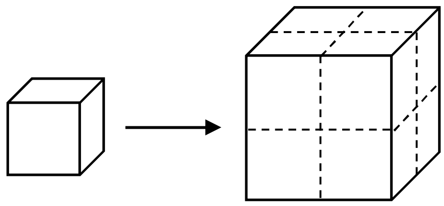
$160cm^2$

The larger rectangle has an area **four times** larger than the small rectangle

Length SF = 2 Area SF = 4

A cube has sides of 2cm.

A similar cube has sides **twice** as long.



Small

Volume = 2 x 2 x 2

= $8cm^3$

Large

Volume = 4 x 4 x 4

= $64cm^3$

The larger cube has a volume **eight times** larger than the small cube

Length SF = 2 Volume SF = 8

When finding areas or volumes of similar shapes:

Area = Original area X SF^2

Volume = Original volume X SF^3

Example 62: Two fridge magnets are mathematically similar. The small magnet has a height of 6cm and has an area of $22.5cm^2$.

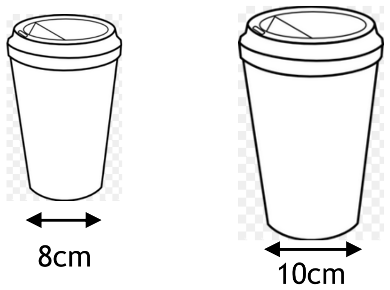
The large magnet has a height of 15cm. Find the area of the large magnet.

Area = $22.5cm^2$

Example 63: Coffee is sold in cups which are mathematically similar.

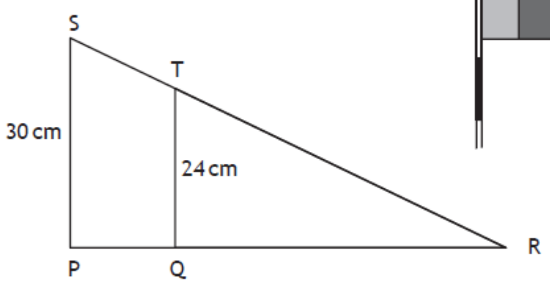
A medium cup has a base 8cm wide and holds 320ml. A large cup has a base 10cm wide.

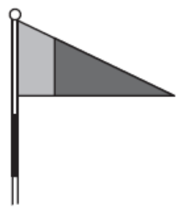
Find the volume of a large coffee cup.



Past Paper Question: The flag at each hole on a golf course is coloured red and blue.

The diagram below represents a flag. Triangle QRT represents the red section. PQTS represents the blue section.





Triangles PRS and QRT are mathematically similar.

The area of QRT is 400 cm^3.

Calculate the area of PQTS, the blue section of the flag. **(4 marks)**

Changing the Subject of a Formula (Rel)

The **subject** of a formula is the letter on the left hand side. To change the subject is to rewrite the formula so that a specified letter from the right hand side is alone on the left.

This can be useful if we need to work backwards using a formula, e.g. if given the area of a circle and sked to find is radius (or diameter).

To change the subject, think of BODMAS in reverse: undo the **least** important thing each time.

Worked Example: Change the subject of y = mx + c to x.

$mx + c =$	y	**Swap to place the letter we want on the left**
$mx =$	$y - c$	**Add is less important than multiply: becomes subtract on the left**
$x =$	$\frac{y - c}{m}$	**Divide everything by m**

Example 64: Change the subject of each formula to the specified letter.

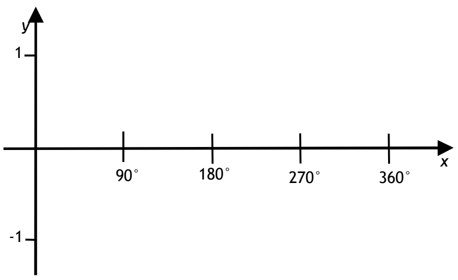
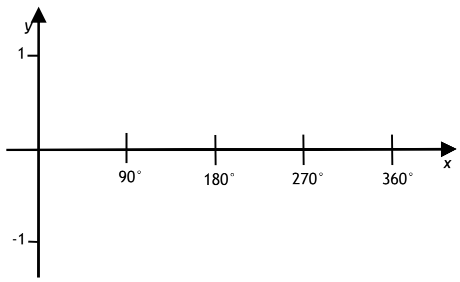
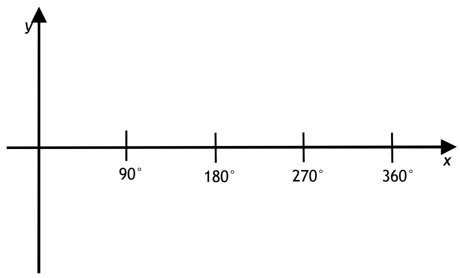
a) $P = 2L + 2B$ (L)

b) $F = \frac{9}{5}C + 32$ (C)

c) $V = \pi r^2 h$ (r)

Trigonometric Graphs (Rel)

Sketch the graphs of y = sinx °, y = cosx ° and y = tanx ° below.

		
y = sinx °	**y = cosx °**	**y = tanx °**

For trig graphs, how soon the graph repeats itself horizontally is known as the **period**, and half of the vertical height is known as the **amplitude.**

Function	Period	Amplitude
y = sinx °		
y = cosx °		
y = tanx °		

For the graphs of:

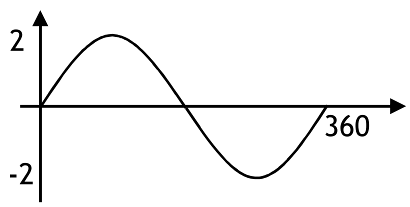
y = a sin bx ° + c
and
y = a cos bx ° + c :

a = amplitude
b = waves in 360°
c = vertical shift

y = tan bx ° + c:

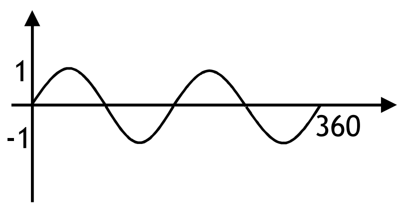
b = "waves" in 180°
c = vertical shift

y = 2 sin x°



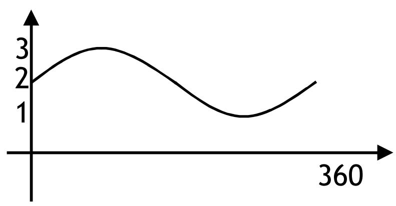
Max = 2, Min = -2

y = sin 2x°



2 waves in 360°

y = sin x° + 2



Slides 2 places up

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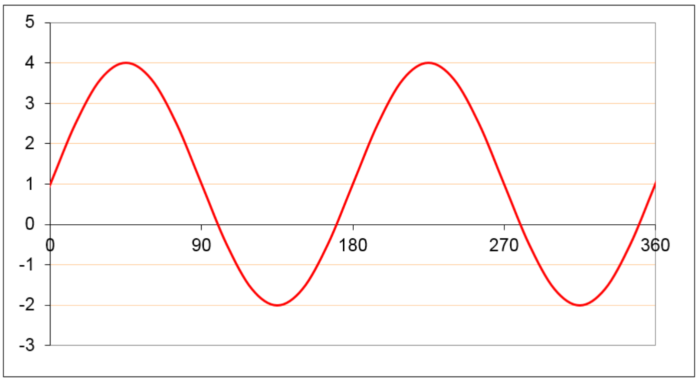
To find the equation from a graph of $y = a \sin bx + c$ or $y = a \cos bx + c$, follow these steps **in this order:**

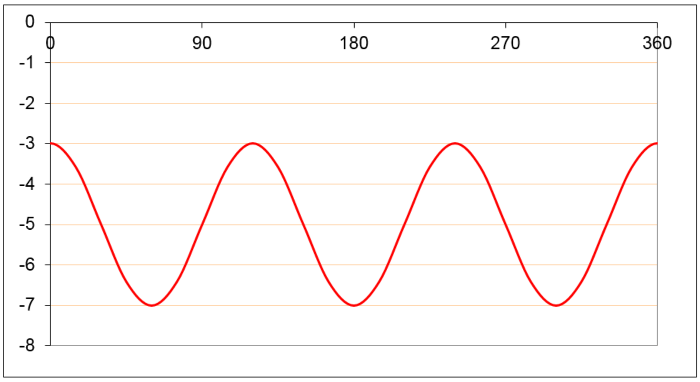
- **Sine graph or cosine graph?**
- **How many waves in 360° → "sin bx"**
- **Half the difference between max and min → "a sin bx"**
- **Compare y-intercept to where "a sin bx" should be → a sin bx + c**

Example 65: State the equations of the following trigonometric graphs:

a) $y = a \sin bx° + c$



b) $y = a \cos bx° + c$

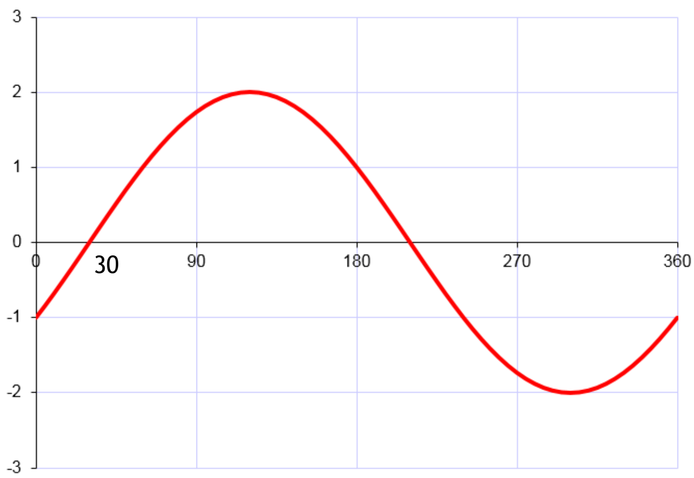


We saw in the quadratic graphs topic that the graph of $y = (x - 2)^2$ is like the graph of $y = x^2$ moved two spaces right: similarly, the graph of $y = \sin(x - 30)°$ is like the graph of $y = \sin x°$ but moved 30° (or spaces on the x-axis) to the right. The 30° in this case is called the **phase angle**.

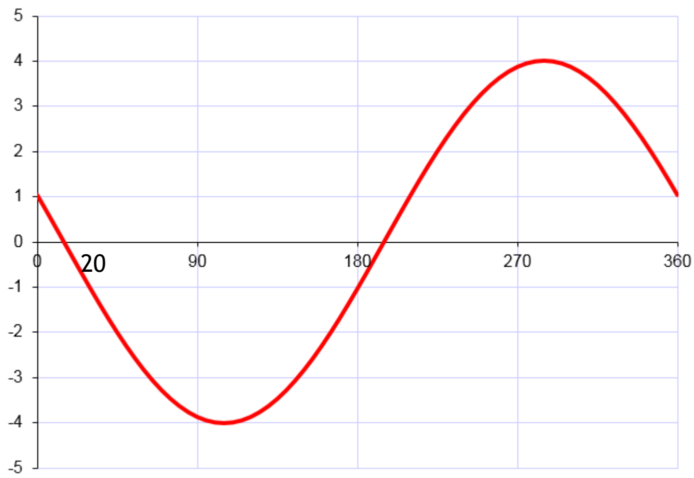
To find the equation of a trig function from such a graph, look for a point we would expect on the standard graph and work out how this point has moved to its position on the "new" graph.

Example 66: State the value of a and b in each graph below:

a) $y = a \sin(x - b)°$



b) $y = a \cos(x - b)°$



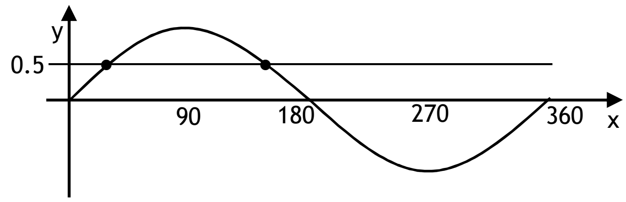
Trigonometric Equations (Rel)

To solve a trig equation, think of the graphs of y = sin x°, y = cos x° and y = tan x°.

Solve sin x° = 0.5, 0 ≤ x ≤ 360°

is asking

where in the first wave of the sine graph is the y-coordinate equal to 0.5?

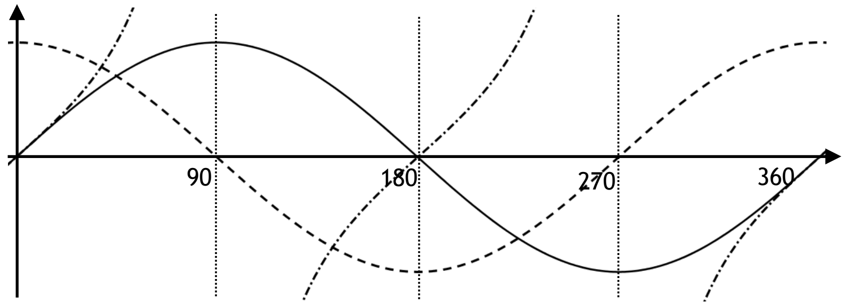


The graph shows us that this happens **twice:** once between 0° and 90° and again between 90° and 180°.

BUT: a calculator only gives one answer for $\sin^{-1}0.5$ (30°). We need a way to find the second answer!

The diagram below shows y = sin x°, y = cos x° and y = tan x° on the same graph.

The sine graph is a solid line, the cos graph is a dashed line and the tan graph is a dot/dash line.



Consider the y-coordinates on each graph

Between 0° and 90°
ALL ARE POSITIVE

Between 90° and 180°
ONLY SIN IS POSITIVE

Between 180° and 270°
ONLY TAN IS POSITIVE

Between 270° and 360°
ONLY COS IS POSITIVE

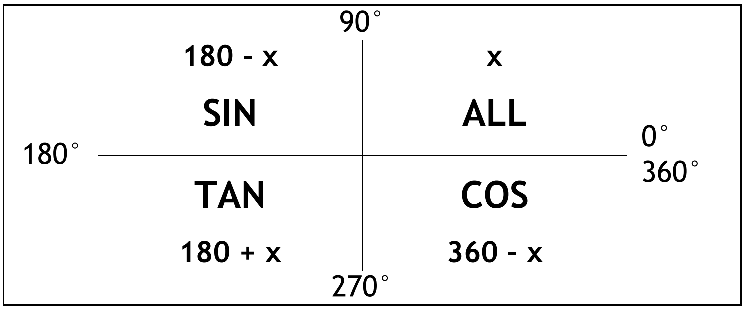
Because of the symmetry in the graphs, if x is an acute angle (i.e. between 0° and 90°) then:

sin x° = sin(180 - x)° | **tan x° = tan(180 + x)°** | **cos x° = cos(360 - x)°**

We can summarise all of this information in one diagram.

On a set of x-y axes, start from the x-axis and measure anti-clockwise.

This gives:



To complete the example at the top of the page, we would do the following:

sin x = 0.5
x = $\sin^{-1} 0.5$
x = 30°
x =

S	A
T	C

circle where sin is positive
change x into the second angle

Example 67: Solve for 0 ≤ x ≤ 360°:

a) cos x° = $\frac{2}{3}$

c) tan x° = - 1.5 (careful here!)

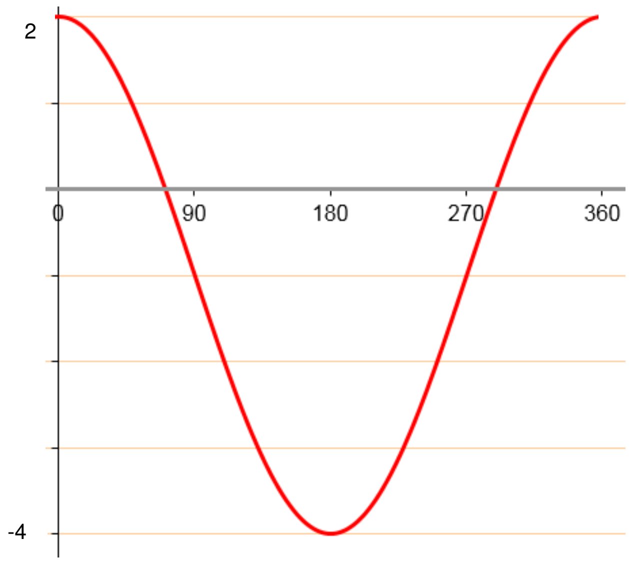
If necessary, rearrange the equation first until you get something that looks like Example 67 above.

Example 68: Solve for $0 \leq x \leq 360°$:

a) $4 \cos x° = 1$

b) $5 \sin x° + 4 = 3$

Past Paper Question: Part of the graph of $y = a \cos x + b$ is shown below.



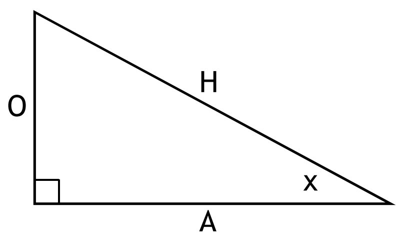
a) Explain how you know that $a = 3$ and $b = -1$ **(2 marks)**

b) Calculate the x-coordinates of the points where the graph cuts the x-axis. **(4 marks)**

Trigonometric Identities (Rel)

An **identity** is a mathematical statement which is true for any value of x. There are two that we need to remember using trig functions; these are called **trig identities**.

One of the identities uses the terms $\sin^2 x$ and $\cos^2 x$: these mean $(\sin x)^2$ and $(\cos x)^2$



By Pythagoras, $O^2 + A^2 = H^2$

$$\frac{\sin x}{\cos x} = \frac{O}{H} \div \frac{A}{H} = \frac{O}{H} \times \frac{H}{A} = \frac{OH}{AH} = \frac{O}{A} = \tan x$$

$$\sin^2 x + \cos^2 x = \left(\frac{O}{H}\right)^2 + \left(\frac{A}{H}\right)^2 = \frac{O^2}{H^2} + \frac{A^2}{H^2} = \frac{O^2 + A^2}{H^2} = \frac{H^2}{H^2} = 1$$

$$\boxed{\frac{\sin x}{\cos x} = \tan x}$$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

From the second identity, we can also say that

$\sin^2 x = 1 - \cos^2 x$	$\cos^2 x = 1 - \sin^2 x$

When asked to prove an equation using trig identities, try to change one side into the other.

Example 69: Prove that:

a) $5\cos^2 x + 5\sin^2 x = 5$

b) $\frac{\sin x \cos x}{\cos^2 x} = \tan x$

c) $\frac{\sin^2 x}{1 - \cos^2 x} = 1$

Statistics (Apps)

Revision: we have already used the following terms in statistics:

MEAN: average found by dividing the sum by the number of numbers

MEDIAN: average found by finding the middle number when in order

MODE: average found by choosing the most common number

RANGE: difference between the smallest and largets number

Mean, median and mode are types of average: range is a measure of how consistent the numbers are.

Example 70: Find the mean, median, mode and range of: 30, 32, 23, 41, 55, 36, 27, 30

Any set of data can be reduced to a **five-figure summary.** The five figures used are the lowest and highest numbers, the median and the upper and lower **quartiles**.

The lower quartile (Q_1) is the median of the first half of the data set, the upper quartile (Q_3) is the median of the second half (the median is also referred to as Q_2).

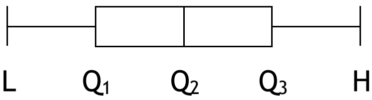
The quartiles can be used to find a more accurate measure of the spread of data than the range (if the highest and/or lowest numbers are different from the middle, then the range is useless). The **interquartile range** (IQR) is the difference between Q_3 and Q_1, and shows the spread of the middle 50% of the data. The **semi-interquartile range** (SIQR) is the IQR divided by 2.

Example 71: The ages of a group of people waiting in a queue at a bank were written in a stem and leaf diagram. Write down the five figure summary for these ages and find the interquartile range.

2	0	1	1	2	4	7	
3	1	7	9				
4	2	2	3	6	6	8	8
5	3	3	3	5	9		
6	0	5	6	8			

3 | 1 means 31 n = 25

The five figure summary can be shown graphically on a **boxplot**:



Example 72: The maximum tepmperatures in the UK during each month in 1911 and 2011 are shown.

Month	J	F	M	A	M	J	J	A	S	O	N	D
1911 Temp (˚C)	6	7	7	10	16	18	21	21	17	11	8	7
2011 Temp (˚C)	6	8	10	16	15	18	19	18	15	15	12	8

On the same diagram, draw a boxplot for each data set and make **two** valid comparisons between maximum temperatures in 1911 and 2011.

The IQR and SIQR are a better indication of consistency than the range, but are still only calculated using two numbers of the data set. A better measure would be one which uses all of the numbers.

The distance from any number to the mean is called a **deviation**. The average of all of these distances is called the **standard deviation**. The standard deviation has the **same units** as the numbers in the set.

The standard deviation is given by the formula:

$$s = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{n - 1}}$$

where

Σ means "the sum of"
x is any number in the set
$\bar{x}$ is the mean ("x bar")
n is how many numbers are in the set

The **smaller** the value of the standard deviation, the **more consistent** the set of data.

Example 73: The scores of six pupils taking a test are: 73 47 59 71 48 62

Find the mean and standard deviation of the scores.

x	$x - \bar{x}$	$(x - \bar{x})^2$
73		
47		
59		
71		
48		
62		
use to find the mean	this column should add up to **zero**	the total here is $\sum\left(x - \bar{x}\right)^2$

← the sum of each column goes at the bottom

If **every** number in a set is increased or decreased by the same amount, the **mean** changes by than amount but the standard deviation is **exactly the same**, for example:

4, 5, 8, 10, 18	5, 6, 9, 11, 19	1004, 1005, 1008, 1010, 1018
Mean = 9, S = 5.57	**Mean = 10, S = 5.57**	**Mean = 1009, S = 5.57**

In exam-style questions, we are often asked to make comparisons between our calculated mean and standard deviation with a mean and standard deviation given to us. It is important to do this **in the context of the question** and by **comparing the numbers**.

Past Paper Question: A runner has recorder her times, in seconds, for six different laps of a track.

53 57 58 60 55 56

a) Calculate: (i) the mean
(ii) the standard deviation for these times. **(4 marks)**

b) She changes her training routine hoping to improve her consistency. After this change, she records her times for another six laps.

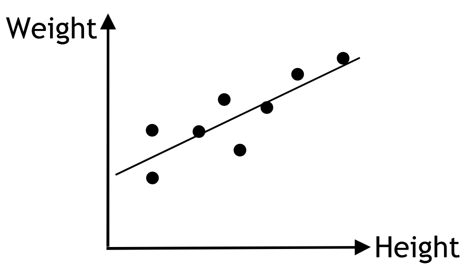
The mean is 55 seconds and the standard deviation 3.2 seconds.

Make two valid comparisons between her sets of lap times. **(2 marks)**

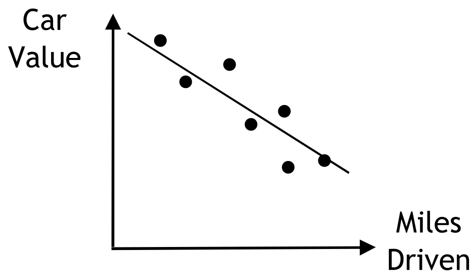
Scattergraphs (Apps)

Scattergraphs are used to compare two categories which are related to each other but cannot be described perfectly by a formula: they are used to show **correlation.**

If both increase, this is called a **positive correlation**.

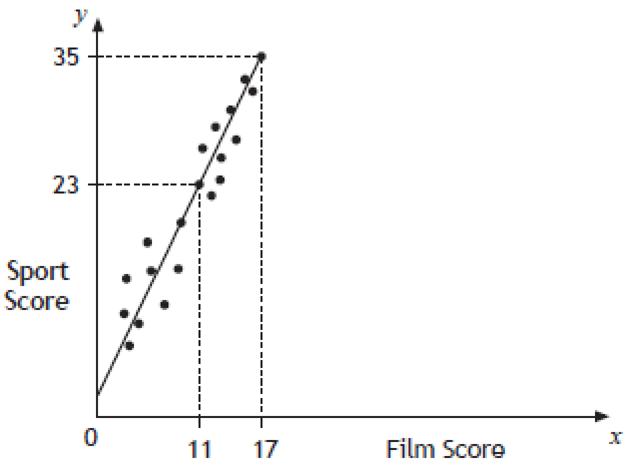


If one increases as the other decreases, this is called a **negative correlation.**



The line through the "middle" of the points is called the **line of best fit.**

Past Paper Question: Teams in a quiz answer questions on film and sport. The scattergraph shows the scores of some teams.



A line of best fit is drawn as shown.

a) Find the equation of this straight line. **(3 marks)**

b) Use this equation to estimate the sports score for a team with a film score of 8. **(1 mark)**

Percentages (Apps)

Revision: Find without a calculator:

a) In a plant food study, a sunflower had increased in height by 22%.

If the sunflower was 20cm at the start of the study, find its height at the end.

b) Joe bought a car for £5500. Two years later he sold it for £4840.

Express the loss as a percentage of the original price.

In "word" questions relating to percentages, be sure to read carefully to see whether the question is asking for a percentage change to be done (rare at Nat 5 level) or **reversed** (much more common).

Remember in these questions **not to simply add or take away the given percentage.**

Example 74: In a sale, all items were reduced by 15%. Alice bought a jacket in the sale for £42.50.

Find the original price of the jacket.

Example 75: Toothpaste is on special offer: each tube has 20% extra free.

The special offer tube contains 420ml of toothpaste. How much toothpaste does a tube usually contain?

When a quantity increases it is said to have **appreciated**. When it decreases it has **depreciated.**

In questions where an initial quantity changes by a fixed percentage over regular periods of time, it is easier to use a formula rather than make repeated calculations.

Be very careful in finding the decimal multiplier in these formulae!

If an amount INCREASES by 5% multiply it by 1.05	If an amount DECREASES by 5% multiply it by 0.95

Example 76: Mary puts £15 000 into her savings account which pays 2.1% interest p/a.

Find the amount in her account after 5 years.

Example 77: A council aims to reduce the amount of waste they send to landfill by 8% each year.

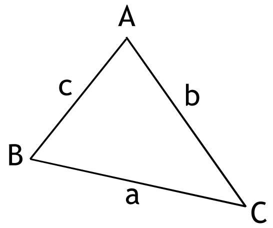
In 2012 they sent 355 000 tonnes of waste to landfill. If they meet their target, how much waste will be sent to landfill in 2020?

Answer to 3 significant figures.

Trigonometric Formulae (Apps)

Previously, we needed a right-angled triangle to use trigonometry to find a missing side or angle.

If asked to find missing information in a triangle which is **not** right angled, we use:



The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Cosine Rule

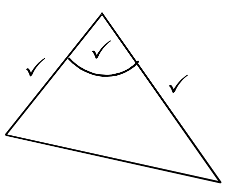
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Exam questions will **never** say whether we need to use either the Sine Rule or Cosine Rule: we have to know when to use each one!

ALWAYS ASSUME THE QUESTION USES THE SINE RULE, UNLESS:

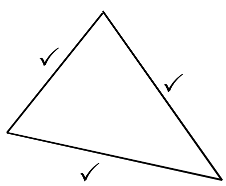
when finding a side we have

TWO SIDES AND THE INCLUDED ANGLE



when finding an angle we have

ALL THREE SIDES WITH NO ANGLES



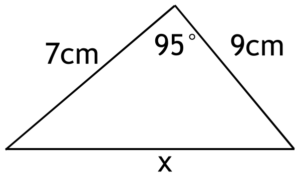
THEN USE THE COSINE RULE!

To find an angle,rearrange the cosing rule formula above to give:

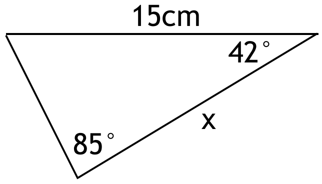
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Example 78: Find the side or angle marked x in each triangle below:

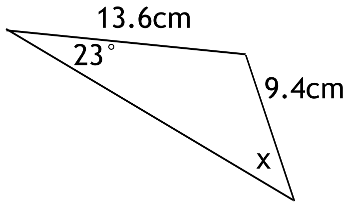
a)



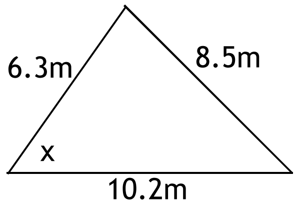
b)



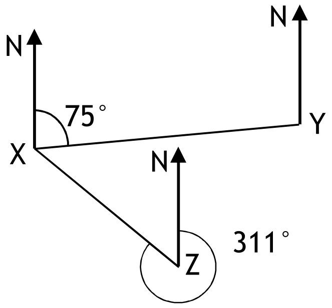
c)



d)



Questions involving the Sine Rule or Cosine Rule in real life contexts often use **three figure bearings.**



To measure a three figure bearing:

Start from the North line	Measure the angle CLOCKWISE

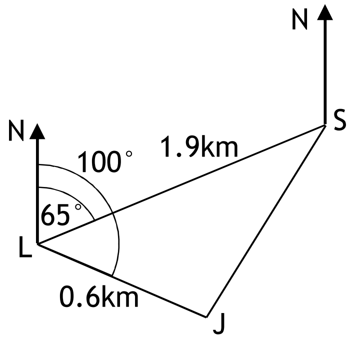
In the diagram, the bearing of X from Z is 311° and the bearing of Y from X is 075° (add zeroes to make three figures when needed).

Look for Z angles and F angles in bearings questions!

Example 79: A ship is 1.9km from a lighthouse on a bearing of 065°.

A jetski is 0.6km from the lighthouse on a bearing of 100°.

a) Calculate the distance between the ship and the jetski.



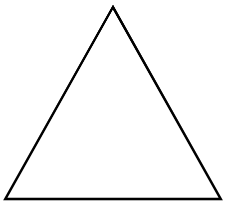
b) Calculate the bearing of the jetski from the ship.

The area of any triangle can be calculated using the formula:

$$A = \frac{1}{2}ab \sin C$$

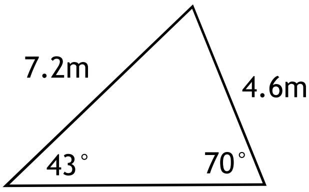
Example 80: Find the area of each triangle below.

a)



Equilateral triangle of side 4cm.

b)



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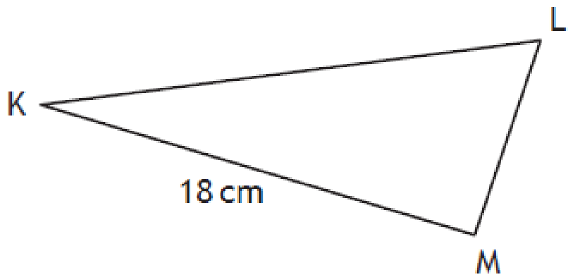
Past Paper Question: In triangle KLM:

KM = 18 centimetres

sin K = 0.4

sin L = 0.9

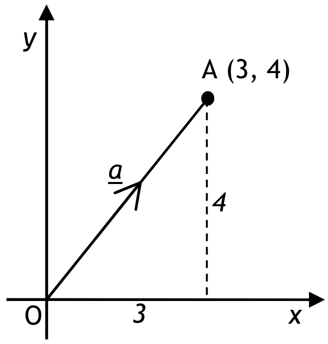
Calculate the length of LM. **(3 marks, NON CALC)**



Vectors (App)

A measurement which only describes size is called a **scalar** quantity, e.g. Glasgow is 11 miles from Coatbridge. A **vector** has **size and direction**, e.g. Glasgow is 11 miles West of Coatbridge.

The line joining the origin to the point A (3, 4) can be described as:



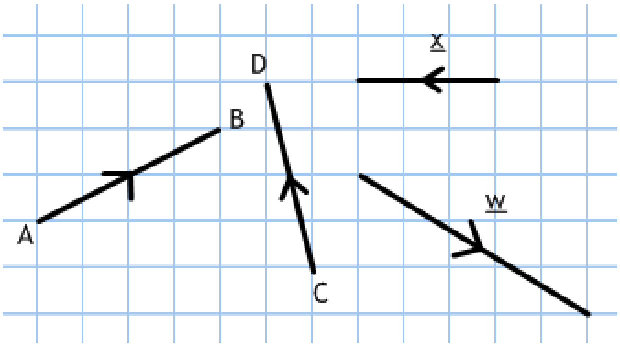
Line segment $\overrightarrow{OA}$

Position vector $\underline{a}$

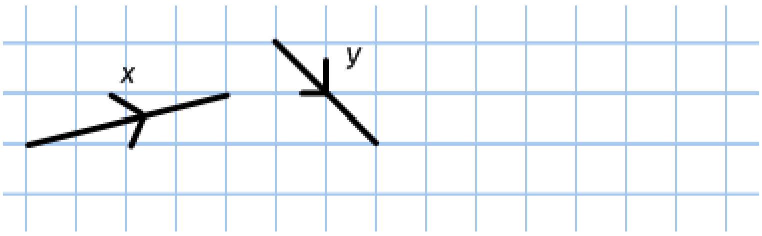
Components $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Note that $\overrightarrow{AO} = -\underline{a}$ and has components $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

Example 81: State the components of each line segment shown below.

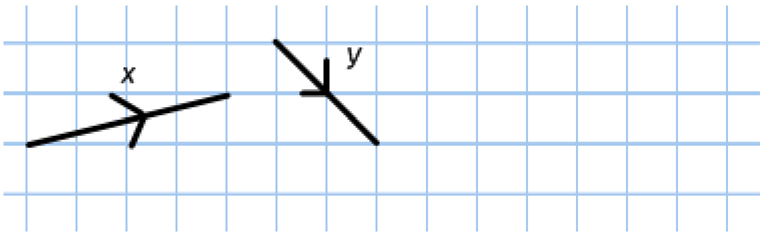


Two (or more) vectors can be added together to produce a **resultant vector.** Adding vectors can be thought of as travelling along the first vector, then going in a new set of directions at the end.



Example 82: Add $x + y$ to the grid and determine its components.

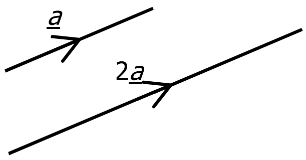
To **subtract** vectors, remember that $x - y = x + (-y)$, i.e. we can follow the same process as adding but we **go along the second vector in the wrong direction.**



Example 83: Add x - y to the grid and determine its components.

Examples 82 and 83 show that:

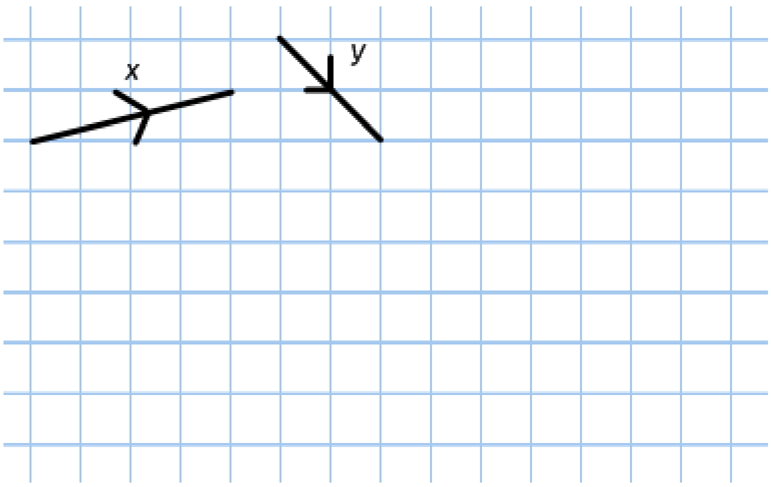
If $\underline{x} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} c \\ d \end{pmatrix}$, then $\underline{u} + \underline{v} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$ and $\underline{u} - \underline{v} = \begin{pmatrix} a-c \\ b-d \end{pmatrix}$



If we go along $\underline{a}$ twice, the resultant vector is $\underline{a} + \underline{a} = 2\underline{a}$. As we have not changed direction, it follows that $2\underline{a}$ must be **parallel** to $\underline{a}$.

If $\underline{x} = \begin{pmatrix} a \\ b \end{pmatrix}$ then $k\underline{x} = \begin{pmatrix} ka \\ kb \end{pmatrix}$

If $\underline{v} = k\underline{u}$, then $\underline{u}$ and $\underline{v}$ are parallel



Example 84: Add $2x + 3y$ to the grid and determine its components.

Example 85: $\underline{b} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

Find:

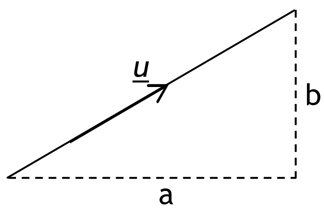
a) $2\underline{b} + \underline{c}$

b) $\underline{c} - \frac{1}{2}\underline{b}$

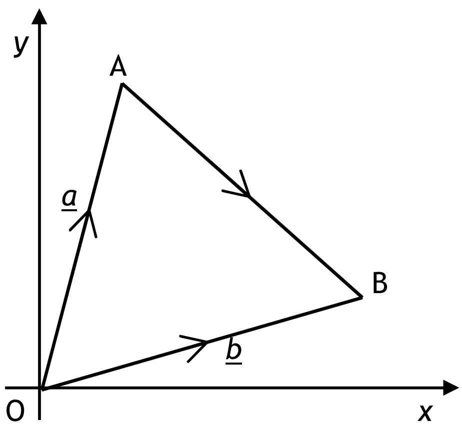
The length of a vector is called its **magnitude**, which can be found using Pythagoras' Theorem.

The magnitude of $\underline{u}$ is written as $|\underline{u}|$.

If $\underline{u} = \begin{pmatrix} a \\ b \end{pmatrix}$, then $|\underline{u}| = \sqrt{a^2 + b^2}$



Example 86: $\underline{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$. Find $|2\underline{b} + \underline{c}|$.



In the diagram opposite, $\underline{a}$ is the vector going from the origin to point A. in the diagram opposite. $\overrightarrow{AB}$ is the resultant vector of going

If A is the point (1, 5) then $\underline{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, etc.

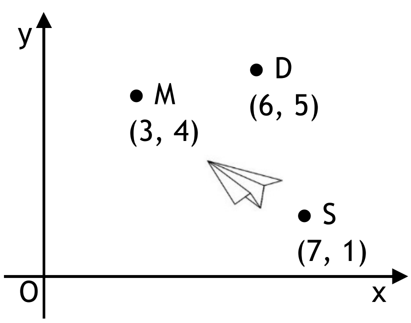
If the coordinates of A and B are given, we can find the components of $\overrightarrow{AB}$ by realising that to go from A to B we go along $\underline{a}$ in the opposite direction, followed by $\underline{b}$ in the correct direction.

So, $\overrightarrow{AB} = -\underline{a} + \underline{b}$, i.e.: $\boxed{\overrightarrow{\mathbf{AB}} = \underline{b} - \underline{a}}$

Example 87: G is the point (4, 2), H is (-5, -3). Find the components of $\overrightarrow{GH}$ and calculate $|\overrightarrow{GH}|$.

Example 88: Sean throws a paper aeroplane out of a window, aiming it at his friend David. However, a gust of wind blows it off course and it hits his Mum instead.

Relative to suitable axes, the positions of Sean, David and Sean's Mum are shown on the diagram.



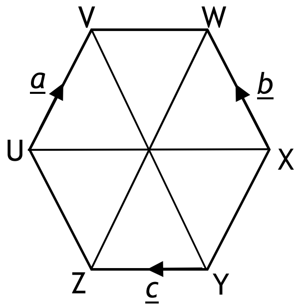
a) Find the components of the vector describing the gust of wind.

b) Find the distance travelled by the aeroplane.

When thinking about vectors, the **start** and **end** points are not always relevant. As long as two vectors are **parallel** and have the **same magnitude**, then those vectors are **equal.**

This is important to remember when asked to describe **vector journeys** around or across a shape.

Example 89: UVWXYZ is a regular hexagon. Vector $\overrightarrow{UV} = \underline{a}$, vector $\overrightarrow{XW} = \underline{b}$ and vector $\overrightarrow{YZ} = \underline{c}$.



Find, in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$:

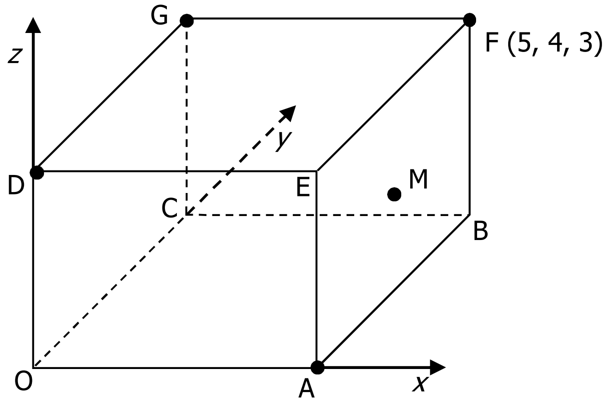
a) $\overrightarrow{YX}$

b) $\overrightarrow{XV}$

c) $\overrightarrow{YW}$

d) $\overrightarrow{ZX}$

The position of a point in 3-D space can be described if we add a third coordinate to indicate **height**.



Example 90: OABC DEFG is a cuboid, where F is the point (5, 4, 3). Write down the coordinates of the points:

a) A

b) D

c) G

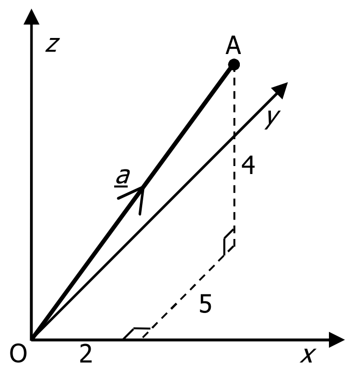
d) M, the centre of face ABFE

All rules for 2D vectors also apply to 3D vectors!

In the diagram opposite,

$$\overrightarrow{\mathbf{OA}} = \underline{a} = \begin{pmatrix} \mathbf{2} \\ \mathbf{5} \\ \mathbf{4} \end{pmatrix}$$

$$|\underline{a}| = \sqrt{x^2 + y^2 + z^2}$$



Example 91: $\underline{m} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ and $\underline{n} = \begin{pmatrix} -4 \\ 0 \\ 7 \end{pmatrix}$. Find:

a) $3\underline{m} + 2\underline{n}$

b) $|2\underline{m} - \underline{n}|$

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Example 92: A helicopter flies at a constant speed in a straight line.

A radar station defines its position at 0905 as (3, -7, 0.5). At 0920, its position is (-11, 5, 1).

Find the speed of the helicopter to the nearest km/hr, given that the coordinates are measured in km.

Past Paper Question 1: Find the resultant vector $2\underline{u} - \underline{v}$ when $\underline{u} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 0 \\ -4 \\ 7 \end{pmatrix}$.

Express your answer in component form. **(2 marks)**

Past Paper Question 2: The diagram shows a cube placed on top of a cuboid, relative to the coordinate axes.

Write down the coordinates of B and C. **(2 marks)**