

Coatbridge High School
Mathematics Department


Homework Booklet 2

## National 5 Maths

## Trigonometry

## Exercise 1 - Trig graphs

1. Each of the graphs below has an equation of the form $y=a \sin b x$ or $y=a \cos b x$. Write down the values of $a$ and $b$.

(c)

(g)


(b)
(d)
(f)
(h)




(j)

2. Each graph below is of the form $y=a \sin (x \pm b)$. Write down the equation of each graph.
(a)

(b)

(c)

(d)

3. Each graph below is of the form $y=a \cos (x \pm b)$. Write down the equation of each graph..

(b)

(c)

(d)

4. Make sketches of the following graphs for $0^{\circ} \leq x^{\circ} \leq 360^{\circ}$
(a) $y=6 \sin x$
(b) $y=4 \cos x$
(c) $y=3 \sin 2 x$
(d) $y=4 \sin 4 x$
(e) $y=2 \cos 3 x$
(f) $y=8 \cos 4 x$
(g) $y=2.5 \cos 3 x$
(h) $y=1.2 \sin 5 x$
(i) $y=4 \cos (x-45)$
(j) $y=2 \sin (x-30)$
(k) $y=5 \cos (x+20)$
(I) $y=3 \sin (x+60)$
5. Each graph below is of the form $y=\tan b x$. Write down the value of $b$.


6. The graphs below are of the form $y=\sin b x+c$ or $y=\cos b x+c$. In each case write down the values of $b$ and $c$.
(a)

(b)

(c)


(e) | 2 |
| :---: |
| 1 |

(d)


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7. Each graph below is of the form $y=a \sin b x+c$ or $y=a \cos b x+c$. For each question write down the values of $a, b$ and $c$.
(a)

(b)
(c) $4^{y}$

(d)


(g)


8. Each graph below is of the form $y=a \sin (x+b)$.

In each case write down the values of $a$ and $b$.
(a)


(c)

(d)

6. Each graph below is of the form $y=a \cos (x+b)$.

In each case write down the values of $a$ and $b$.
(a)

(b)

(c)


7. Make sketches of the following graphs for $0^{\circ} \leq x^{\circ} \leq 360^{\circ}$.
(a) $y=3 \cos x$
(b) $y=2 \sin x$
(c) $y=\tan x$
(d) $y=2 \sin 2 x$
(e) $y=4 \cos 2 x$
(f) $y=3 \sin 4 x$
(g) $y=5 \cos 3 x$
(h) $y=4 \sin 3 x$
(i) $y=\tan 2 x$
(j) $y=3 \cos (x-40)$
(k) $y=4 \sin (x+20)$
(I) $y=4 \sin (x-10)$

## Exercise 2 - Trig Equations

1. Solve the following equations for $0^{0} \leq x^{0} \leq 360^{\circ}$
(a) $2 \sin x^{0}-1=0$
(b) $4 \cos x^{0}-3=0$
(c) $5 \tan x^{0}-12=0$
(d) $3 \sin x^{0}+6=7$
(e) $4 \tan x^{0}-3=10$
(f) $6 \cos x^{0}-2=3$
(g) $3 \cos x^{\circ}+4=2$
(h) $7 \tan x^{0}+3=0$
(i) $6 \sin x^{0}+5=0$
(j) $10 \cos x^{0}+12=6$
(k) $4+3 \tan x^{0}=2$
(I) $8+7 \sin x^{\circ}=2$
(m) $7+9 \cos x^{0}=8$
2. The diagram shows the graph of $y=4 \sin x$. The line $y=3$ has been drawn on the graph.
Find the coordinates of $P$ and $Q$.

3. (a) Write down the equation of the graph opposite in the form $y=a \sin x$.
(b) The line $y=-1$ meets this graph at the points $P$ and $Q$. Find the coordinates of $P$ and $Q$.


## Exercise 3 - Trig Identities

1. Show that $\tan x \cos x=\sin x$
2. Show that $(\cos x+\sin x)^{2}=1+2 \sin x \cos x$
3. Show that $(\cos x+\sin x)(\cos x-\sin x)+2 \sin ^{2} x=1$
4. Show that $\sin ^{3} x+\sin x \cos ^{2} x=\sin x$
5. Show that $\cos ^{2} x \sin ^{2} x+\cos ^{4} x=\cos ^{2} x$
6. Show that $\tan ^{2} x \cos ^{2} x=1-\cos ^{2} x$

## Trig Rules

## Exercise 1 - Area of a triangle

1. A cross-section of a pyramid is shown. $P Q=Q R=10 \mathrm{~cm}$. Angle $P Q R=70^{\circ}$.
Calculate the area of this cross-section.

2. A piece of metal was cut in the shape of a triangle.
Calculate the area of this piece of metal.

3. A piece of plastic is cut out in the shape of an isosceles triangle.
Calculate the area of this piece of plastic.


## Exercise 2 - Sine rule

1. In the diagram opposite $A B$ represents a main road 28 kilometres long running due north.
$A C$ and $C B$ represent secondary roads. Calculate the size of angle BAC.

2. The diagram shows a triangular shaped garden.
A jogger runs from one corner of the garden to another, as shown.
Calculate how far he has run.

3. Three markers $P, Q$ and $R$ are placed at the corners of a triangular field. Calculate the size of angle RPQ.


4. Three oak trees have been surveyed and measurements taken as shown opposite.
Calculate the distance from the tree marked $R$ to the tree marked $Q$.

12.8 m


## Exercise 3-Cosine rule

1. A triangular field is shown.
$P Q=140$ metres, $Q R=120$ metres and angle $P Q R=132^{\circ}$. Calculate the length of PR.

2. Three holes are to be positioned and drilled as shown.
Calculate the size of angle $A B C$.

3. A cargo ship leaves Lowerford.

It travels 5 km to Marketville and then 6 km to Newtown delivering supplies.
The ship then returns to Lowerford, a distance of 8 km .
Calculate the size of angle $x^{0}$.

4. The first three holes on a golf course are shown in the diagram opposite.
Calculate the distance from the second to the third hole.


## Exercise 4 -Bearings

1. The sketch below shows the journey of a ship from a harbour.


The ship leaves the harbour and sails 30 km on a bearing of $040^{\circ}$ and then sails 50 km on a bearing of $100^{\circ}$. Calculate the distance the ship is from the harbour
2. A submarine leaves Faslane in Scotland and sails into the Atlantic Ocean on a bearing of $250^{\circ}$ for a distance of 120 km before changing course and sailing on a bearing of $190^{\circ}$ for 90 km . Calculate the distance the submarine is from Faslane.

3. A church is 600 metres due west of a flagpole. A statue is on a bearing of $160^{\circ}$ from the church and on a bearing of $220^{\circ}$ from the flagpole. Find the distance of the church from the statue.

4. A ship sails from harbour $H$ on a bearing of $084^{\circ}$ for 340 km until it reaches point $P$. It then sails on a bearing of $210^{\circ}$ for 160 km until it reaches point $Q$.
Calculate the distance between the point $Q$ and the harbour.
5. A surveyor is walking due west when he comes to a marsh. To avoid the marsh he turns at $P$ and walks for 60 m on a bearing of $215^{\circ}$ and then for 85 m on a bearing of $290^{\circ}$.
Calculate PR the direct distance across the marsh.

## Exercise 5 - mixed problems

1. Calculate $h$ in each of the following.
(a)

(h)

(b)

(i)


## Statistics

## Exercise 1 - boxplots

1. (a) Amanda takes the bus to work each day. Over a two-week period she records the number of minutes the bus is late each day. The results are shown below.

$$
\begin{array}{llllllllll}
5 & 6 & 15 & 0 & 6 & 11 & 2 & 8 & 9 & 7
\end{array}
$$

For the above data, find
(i) the median
(ii) the lower and upper quartiles
(iii)Construct a boxplot for this data.
(b) Amanda decides to take the train for the next two weeks and records the number of minutes the train is late each day.
The boxplot below was constructed for the new data.

(c) Compare the two boxplots and comment.
2. The pupils in a primary class record their shoe sizes as shown below.

$$
\begin{array}{rrrrrrrrrr}
8 & 7 & 6 & 5 & 6 & 5 & 7 & 11 & 7 & 7 \\
7 & 8 & 7 & 9 & 6 & 8 & 6 & 5 & 9 & 7
\end{array}
$$

(a) Construct a frequency table for the above data and add a column of cumulative frequencies.
(b) For this data, find (i) the median (ii) the lower and upper quartiles
(c) Construct a boxplot for this data.
3. (a) The marks of a group of pupils in a test are listed below.
$\begin{array}{llllllllllll}41 & 56 & 68 & 59 & 43 & 37 & 70 & 58 & 61 & 47 & 75 & 66\end{array}$

Calculate (i) the median (ii) the semi-interquartile range
(b) The teacher arranges supported study classes for the students before they sit their next test. In this test, the median is 67 and the semiinterquartile range is 7 .

Make two appropriate comments comparing the marks of the pupils in the first and second tests.

## Exercise 2-Standard deviation

## Mean and standard Deviation

1. Calculate the mean and standard deviation of
(a) $\begin{array}{llllll}14 & 15 & 18 & 20 & 23 & 18\end{array}$
(b) $\begin{array}{llllll}41 & 45 & 34 & 45 & 46 & 47\end{array}$
50
2. (a) The prices of a bag of sugar in 6 different shops are

$$
86 p \quad 88 p \quad 84 p \quad 79 p \quad 81 p \quad 86 p
$$

Calculate the mean and standard deviation of these prices.
(b) In 6 different shops the same bag of sugar has a mean price of 87 pence and a standard deviation of 5.2 pence. Make two comparisons between the prices in the two sets of shops.
3. A gardener grows tomatoes in his greenhouse.

The temperature of the greenhouse, in degrees Celsius, is recorded every day at noon for one week.

$$
\begin{array}{lllllll}
18 & 21 & 24 & 17 & 23 & 14 & 16
\end{array}
$$

Calculate the mean and standard deviation of these temperatures.

(b) For best growth the mean temperature should be ( $20 \pm 5)^{\circ} \mathrm{C}$ and the standard deviation should be less than $5^{\circ} \mathrm{C}$.

Are the conditions in the greenhouse likely to result in best growth?
4. Calculate the mean and standard deviation of 10 numbers where

$$
\Sigma x=180 \quad \text { and } \quad \Sigma x^{2}=3356
$$

5. The cost of the same kettle in 6 different stores is £42 £45 £39 £41 £46 £39
(a) Calculate the mean and standard deviation of these costs.
(b) Each store increases the cost of the kettle by $£ 2.50$.

Write down the mean and standard deviation of the costs now.

## Exercise 3-scattergraphs

1. The results for a group of students who sat tests in mathematics and physics are shown.

| Mathematics | 10 | 18 | 26 | 32 | 49 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Physics | 25 | 35 | 30 | 40 | 41 |

These marks are shown on the scattergraph below. A line of best fit has been drawn.

(a) Find the equation of the line of best fit.
(b) Another pupil scored 76 in the mathematics test. Use your equation to predict his physics mark.
2. The marks of a group of students in a class test ( $C$ ) and in the final exam (F) are shown in the scattergraph opposite. A line of best fit has been drawn.
David scored 20 in the class test and 35 in the final exam.
(a) Find the equation of the line of best fit.
(b) Use your answer to part (a) to predict the final exam mark for a student who scored a mark of 26 in the class test.


## Vectors

## Exercise 1

1. $\mathbf{p}=\binom{3}{2}$ and $\mathbf{q}=\binom{6}{-4}$. Sketch the vectors $\mathbf{p}$ and $\mathbf{q}$.
2. $\boldsymbol{a}=\binom{5}{-2}$ and $\mathbf{q}=\binom{7}{-4}$.

Sketch the vectors $a$ and $b$ and the resultant vector $a+b$.
3. $\boldsymbol{a}=\left(\begin{array}{l}2 \\ 2 \\ -1\end{array}\right)$ and $\boldsymbol{b}=\left(\begin{array}{l}6 \\ -8 \\ 0\end{array}\right)$. Find (a) $3 \boldsymbol{a}$ (b) $-\boldsymbol{b}$ (c) $2 \boldsymbol{a}-3 \mathrm{~b} \quad$ (d) $\frac{1}{2} \boldsymbol{b}$ (e) |b|
4. Find $\boldsymbol{p}$ and $\boldsymbol{q}$ in each equation below.
(a) $\binom{p}{q}+\binom{2}{-1}=\binom{6}{4}$
(b) $\binom{p}{q}-\binom{3}{-4}=\binom{8}{-4}$
5.

The diagram shows a regular hexagon.
(i) Write down another vector equal to
(a) $\overrightarrow{\mathrm{AB}}$
(b) $\overrightarrow{\mathrm{CD}}$
(ii) Find a vector equal to
(a) $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}$
(b) $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CD}}$
(c) $\overrightarrow{\mathrm{FE}}+\overrightarrow{\mathrm{ED}}$
(d) $\overrightarrow{\mathrm{FA}}+\overrightarrow{\mathrm{ED}}$
(e) $\overrightarrow{\mathrm{ED}}+\overrightarrow{\mathrm{DC}}+\overrightarrow{\mathrm{EF}}$
(f) $\overrightarrow{\mathrm{BC}}-\overrightarrow{\mathrm{DC}}$

6.
3. The diagram shows a cuboid ABCD EFGH.
(a) Express in terms of $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.
(i) $\overrightarrow{\mathrm{DC}}$
(ii) $\overrightarrow{\mathrm{ED}}$
(iii) $\overrightarrow{\mathrm{FH}}$
(iv) $\overrightarrow{\mathrm{HA}}$
(b) Express in terms of ABCD EFGH

(i) $\mathbf{u}+\mathbf{v}$
(ii) $\mathbf{u}-\mathbf{w}$
(iii) $\mathbf{w}-\mathbf{u}+\mathbf{v}$

