2018 Paper 1

① mid $= (1,2)$

$M_{med} = \frac{6-2}{3-1}$

$= \frac{4}{2}$

$= 2$

$y - b = m(x-a)$

$y - 2 = 2(x-1)$

$y - 2 = 2x - 2$

$\underline{y = 2x}$

② $y = \frac{1}{5}x - 4$

$\frac{1}{5}x = y + 4$

$x = 5(y+4)$

$\therefore \underline{g^{-1}(x) = 5(x+4)}$

③ $h'(x) = -3\sin(2x) \times 2$

$= -6\sin(2x)$

$h'\left(\frac{\pi}{6}\right) = -6\sin\left(\frac{2\pi}{6}\right)$

$= -6\sin\left(\frac{\pi}{3}\right)$

$= -6\left(\frac{\sqrt{3}}{2}\right)$

$= \underline{-3\sqrt{3}}$

④ $2g = -12$ $\quad 2f = -6$

$g = -6$ $\quad f = -3$

$\therefore$ centre $= (6,3)$

$M_R = \frac{3+5}{6-8}$

$= \frac{8}{-2}$

$= -4$

$\therefore M_T = \frac{1}{4}$

$(M_R M_T = -1)$

$y - b = m(x-a)$

$y + 5 = \frac{1}{4}(x-8)$

$4y + 20 = x - 8$

$\underline{4y = x - 28}$

⑤ $\begin{pmatrix} -3 \\ 4 \\ -7 \end{pmatrix}$ $\begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix}$ $\begin{pmatrix} 7 \\ 9 \\ 8 \end{pmatrix}$

A B C

$\underline{x}$ $A \to B = +8$ $\quad B \to C = +2$

$\therefore 8:2 = \underline{4:1}$

b) $\underline{y}$ $A \to C = +5$

$\therefore A \to B = +4, \quad B \to C = +1$

$\therefore t = 4 + 4$

$= \underline{\underline{8}}$

⑥ $\log_5 250 - \frac{1}{3}\log_5 8$

$= \log_5 250 - \log_5 8^{1/3}$

$= \log_5 250 - \log_5 2$

$= \log_5 125$

$= \log_5(5^3)$

$= \underline{\underline{3}}$

⑦ a) $(0,5)$

b) $\frac{dy}{dx} = 3x^2 - 6x + 2$

when $x = 0$, $m = 3(0)^2 - 6(0) + 2$

$= 2$

$\therefore \underline{y = 2x + 5}$

c) $y = y$

$x^3 - 3x^2 + 2x + 5 = 2x + 5$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0 \quad x = 3$

$\therefore x = 3 \quad y = 2(3) + 5$

$= 11$

$(3, 11)$

(8) $y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

$m = \sqrt{3} \quad \therefore \theta = \tan^{-1}\sqrt{3}$

$= 60^\circ$

(9) a) $\vec{BC} = \vec{BA} + \vec{AC}$

$= -\underline{t} + \underline{u}$

$= \underline{u} - \underline{t}$

b) $\vec{MD} = \vec{MB} + \vec{BC} + \vec{CD}$

$= -\frac{1}{2}\vec{BC} + \vec{AD} + \vec{BA}$

$= -\frac{1}{2}(\underline{u} - \underline{t}) + \underline{v} + (-\underline{t})$

$= -\frac{1}{2}\underline{u} + \frac{1}{2}\underline{t} + \underline{v} - \underline{t}$

$= \underline{v} - \frac{1}{2}\underline{u} - \frac{1}{2}\underline{t}$

(10) $y = \int 6x^2 - 3x + 4 \, dx$

$= 2x^3 - \frac{3x^2}{2} + 4x + C$

$\therefore 14 = 2(2)^3 - \frac{3(2)^2}{2} + 4(2) + C$

$14 = 16 - 6 + 8 + C$

$14 = 18 + C$

$C = -4$

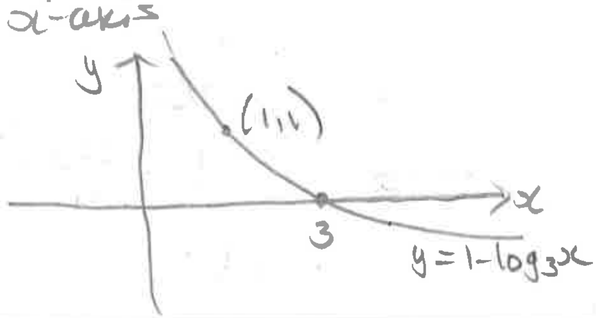
$\therefore y = 2x^3 - \frac{3x^2}{2} + 4x - 4$

(11) $y = 1 - \log_3 x$

a) $= -\log_3 x + 1$

Flip in x-axis

slide up 1



b) $y = y$

$\log_3 x = 1 - \log_3 x$

$2\log_3 x = 1$

$\log_3 x^2 = 1$

$x^2 = 3$

$x = \pm\sqrt{3}$

$\therefore x = \sqrt{3}$

(12) a) $2\underline{a} + \underline{b} = \begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$

b) $|2\underline{a} + \underline{b}| = 7$

$\sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$

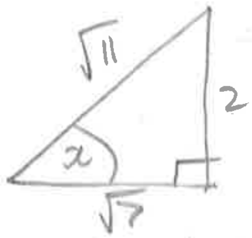
$36 + 9 + 16 + 8p + p^2 = 49$

$p^2 + 8p + 12 = 0$

$(p+2)(p+6) = 0$

$p = -2, \ p = -6$

(13) a)



(i) $\sin 2x = 2\sin x \cos x$

$= 2\left(\frac{2}{\sqrt{11}}\right)\left(\frac{\sqrt{7}}{\sqrt{11}}\right)$

$= \frac{4\sqrt{7}}{11}$

(ii) $\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{\sqrt{7}}{\sqrt{11}}\right)^2 - \left(\frac{2}{\sqrt{11}}\right)^2$

$= \frac{7}{11} - \frac{4}{11}$

$= \frac{3}{11}$

b) $\sin 3x$

$= \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= \left(\frac{4\sqrt{7}}{11} \times \frac{\sqrt{7}}{\sqrt{11}}\right) + \left(\frac{3}{11} \times \frac{2}{\sqrt{11}}\right)$

$= \frac{28}{11\sqrt{11}} + \frac{6}{11\sqrt{11}}$

$= \frac{34}{11\sqrt{11}}$

(14) $\int_{-4}^{9} (2x+9)^{-2/3}\, dx$

$= \left[\frac{(2x+9)^{1/3}}{\frac{1}{3} \times 2}\right]_{-4}^{9}$

$= \left[\frac{3}{2}\sqrt[3]{2x+9}\right]_{-4}^{9}$

$= \left(\frac{3}{2}\sqrt[3]{27}\right) - \left(\frac{3}{2}\sqrt[3]{1}\right)$

$= \left(\frac{3}{2} \times 3\right) - \left(\frac{3}{2} \times 1\right)$

$= \frac{9}{2} - \frac{1}{2}$

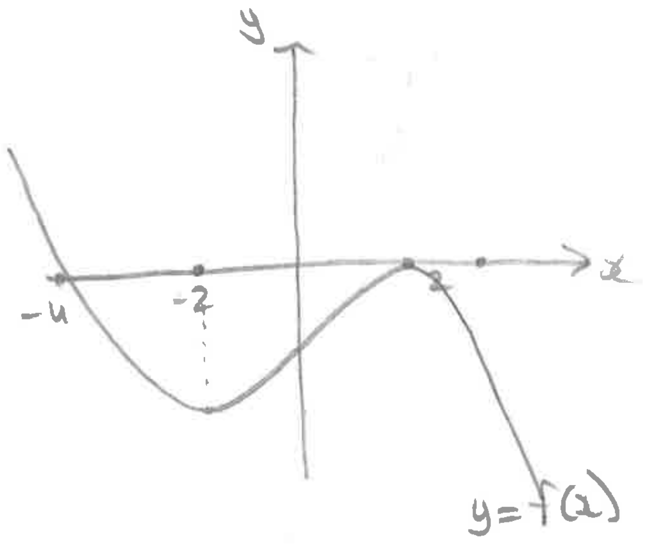
$= 4$

(15) $(x+4)$ is a factor $\rightarrow x = -4$ is a root.

$x = 2$ repeated $\rightarrow$ TP @ $x = 2$

$f'(-2) = 0 \rightarrow$ TP @ $x = -2$

$f'(x) > 0$ on y-axis $\rightarrow f(x)$ increasing @ $x = 0$



2018 Paper 2

(1) $A = \int_{-1}^{3} (3 + 2x - x^2)\, dx$

$= \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3}$

$= \left(3(3) + (3)^2 - \frac{3^3}{3}\right) - \left(3(-1) + (-1)^2 - \frac{(-1)^3}{3}\right)$

$= (9 + 9 - 9) - \left(-3 + 1 + \frac{1}{3}\right)$

$= 9 - \left(-\frac{9}{3} + \frac{3}{3} + \frac{1}{3}\right)$

$= 9 - \left(-\frac{5}{3}\right)$

$= \frac{32}{3} u^2$

(2) a) $\underline{u} \cdot \underline{v} = 7 + 32 - 15$

$= 24$

b) $|\underline{u}| = \sqrt{1 + 16 + 9}$

$= \sqrt{26}$

$|\underline{v}| = \sqrt{49 + 64 + 25}$

$= \sqrt{138}$

$\cos\theta = \frac{24}{\sqrt{26}\sqrt{138}}$

$\therefore \theta = \cos^{-1}\left(\frac{24}{\sqrt{26}\sqrt{138}}\right)$

$= 66.4^\circ$

③ $f'(x) = 3x^2 - 7$

$f'(2) = 3(2)^2 - 7$

$= 5$

$f'(2) > 0 \therefore f(x)$ is increasing @ $x = 2$.

④ $-3x^2 - 6x + 7$

$= -3[x^2 + 2x] + 7$

$= -3[(x+1)^2 - 1] + 7$

$= -3(x+1)^2 + 3 + 7$

$= -3(x+1)^2 + 10$

⑤ a) $M_{PQ} = \frac{4+2}{3-9}$ $\therefore M_{L_1} = 1$

$= \frac{6}{-6}$ $(M_{PQ}M_{L_1} = -1)$

$= -1$

Midpoint $= (6, 1)$

$\therefore y - b = m(x - a)$

$y - 1 = 1(x - 6)$

$y = x - 5$

b) $3y + x = 25$ $y = x - 5$

$x = 25 - 3y$ $x = y + 5$

$\therefore 25 - 3y = y + 5$

$20 = 4y$

$y = 5$

$x = y + 5$

$= 5 + 5$

$= 10$ $\therefore (10, 5)$

c) CP = radius

$\therefore r = \sqrt{(10-3)^2 + (5-4)^2}$

$= \sqrt{7^2 + 1^2}$

$= \sqrt{50}$

$\therefore (x - 10)^2 + (y - 5)^2 = 50$

⑥ a)

(i) $f(g(x)) = f(2x)$

$= 3 + \cos 2x$

(ii) $g(f(x)) = g(3 + \cos x)$

$= 6 + 2\cos x$

b) $f(g(x)) = g(f(x))$

$3 + \cos 2x = 6 + 2\cos x$

$\cos 2x - 2\cos x - 3 = 0$

$2\cos^2 x - 2\cos x - 4 = 0$

$2(\cos^2 x - \cos x - 2) = 0$

$2(\cos x - 2)(\cos x + 1) = 0$

$\cos x = 2$ NO SOLUTIONS

$\cos x = -1$

$x = \pi$

⑦ a)

(i)

2	2	-3	-3	2
		4	2	-2
	2	1	-1	$\boxed{0}$

$R = 0 \therefore (x - 2)$ is a factor

(ii) $(x-2)(2x^2 + x - 1)$

$= (x-2)(2x-1)(x+1)$

b) $u_6 = a(2a - 3) - 1$

$= 2a^2 - 3a - 1$

$u_7 = a(2a^2 - 3a - 1) - 1$

$= 2a^3 - 3a^2 - a - 1$ (as required)

c) $u_7 = u_5$

$2a^3 - 3a^2 - a - 1 = 2a - 3$

$2a^3 - 3a^2 - 3a + 2 = 0$

$(a-2)(2a-1)(a+1) = 0$

$\therefore a = 2 \quad a = \frac{1}{2} \quad a = -1$

limit exists $\therefore \underline{a = \frac{1}{2}}$

(ii) $L = \frac{-1}{1 - -\frac{1}{2}}$

$= \frac{-1}{\frac{1}{2}}$

$= \underline{\underline{-2}}$

(8)

a) $2\boxed{\cos x} - \sin x$

$= k\cos x \cos\alpha + k\sin x \sin\alpha$

$= k\cos\alpha \boxed{\cos x} + k\sin\alpha \sin x$

$\boxed{\begin{aligned} k\sin\alpha &= -1 \\ k\cos\alpha &= 2 \end{aligned}}$

$k^2 = 2^2 + (-1)^2$

$\underline{k = \sqrt{5}}$

$\tan\alpha = -\frac{1}{2}$

"$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$"

$\alpha =$ "$26.6°$"

$k\sin\alpha = \text{NEG}$

$k\cos\alpha = \text{POS}$

S | A ✓

✓T | C ✓

$\therefore \alpha$ in $\underline{\underline{Q4}}$

$\therefore \underline{2\cos x - \sin x = \sqrt{5}\cos(x - 333.4)°}$

b) (i) $6\cos x - 3\sin x$

$= 3(2\cos x - \sin x)$

$= 3\sqrt{5}\cos(x - 333.4)°$

$\therefore \underline{\text{Max} = 3\sqrt{5}}$

(ii) $3\sqrt{5}\cos(x - 333.4)° = 3\sqrt{5}$

$\cos(x - 333.4)° = 1$

$x - 333.4° = 0, 360$

$\underline{x = 333.4°}$

(9) $P = 2x + 128x^{-1}$

SP's where $\frac{dP}{dx} = 0$

$\therefore 2 - 128x^{-2} = 0$

$2 - \frac{128}{x^2} = 0$

$2 = \frac{128}{x^2}$

$x^2 = \frac{128}{2}$

$x^2 = 64$

$\underline{x = \pm 8}$

$\therefore \underline{\underline{x = 8}}$

x	$\to 8 \to$ (9)
$\frac{dP}{dx}$	$-\ 0\ +$
shape	\ — /

$\therefore$ Min @ $x = 8$

$\therefore P = 2(8) + \frac{128}{8}$

$= 16 + 16$

$= \underline{32 \text{ cm}}$

(10) For real, distinct roots,

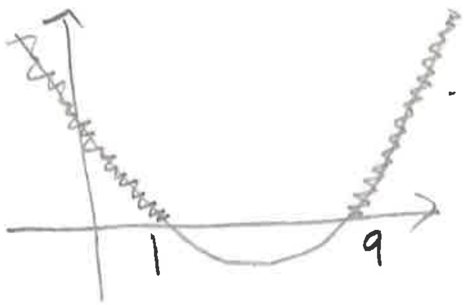
$b^2 - 4ac > 0$

$\therefore (m-3)^2 - 4(1)(m) > 0$

$m^2 - 6m + 9 - 4m > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$



$\therefore m < 1$

$m > 9$

(11) a) $50 = 100(1-e^{3k})$

$0.5 = 1-e^{3k}$

$e^{3k} = 1-0.5$

$e^{3k} = 0.5$

$3k = \ln 0.5$

$k = \frac{\ln 0.5}{3}$

$k = -0.231$ (3sf)

b) $P = 100(1-e^{-0.231\times 5})$

$= 100(0.6849426...)$

$= 68.49426...$

∴ 68.5% wait less than 5 minutes

∴ 31.5% wait 5 mins or more.

(12) a) (i) $(13, -4)$

(ii) sub $x=13$, $y=-4$ into C_2

∴ $13^2+(-4)^2+14(13)-22(-4)+c=0$

$169+16+182+88+c=0$

$455+c=0$

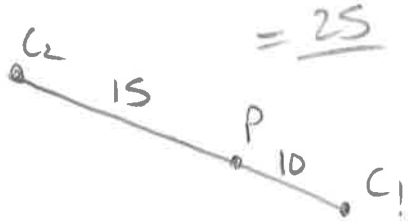
∴ $c=-455$

(as required).

b) $r_1 = 10$ $r_2 = \sqrt{7^2+(-11)^2+455}$

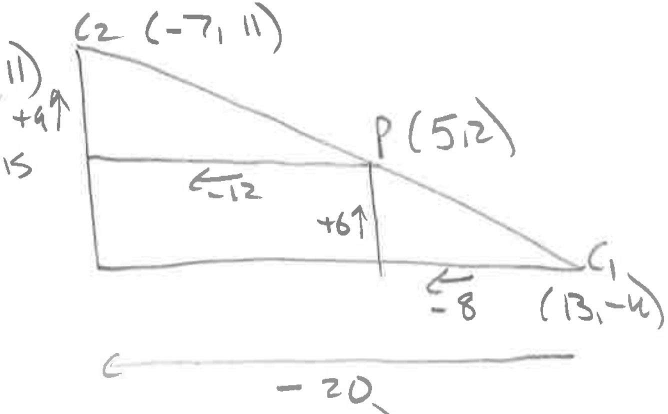
$= \sqrt{625}$

$= 25$



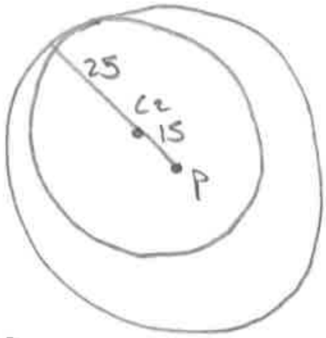
∴ 3:2 ratio.

(ii)



∴ $P = (5,2)$.

c) P is the centre.



∴ $r_{C_3} = 15+25$

$= 40$

∴ C_3 is $(x-5)^2+(y-2)^2=1600$