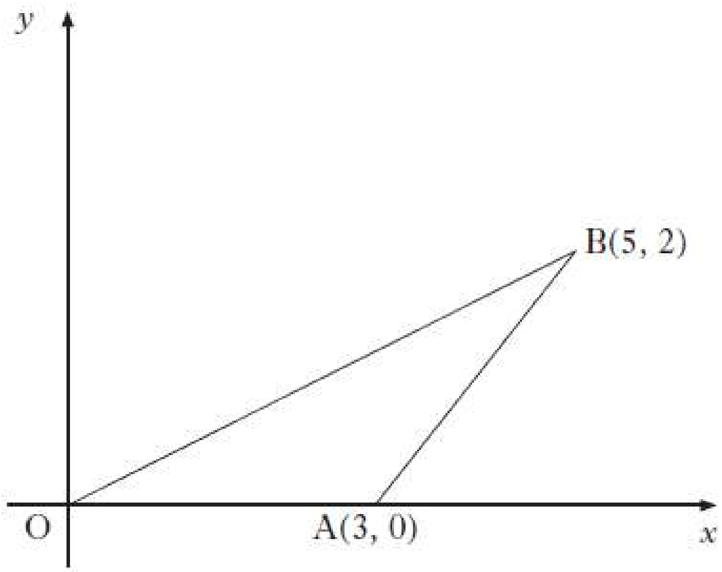
The Straight Line

2014 P2 Q1

A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



(*a*) Obtain the equation of the perpendicular bisector of AB. **4**

(*b*) The median from A has equation $y + 2x = 6$.

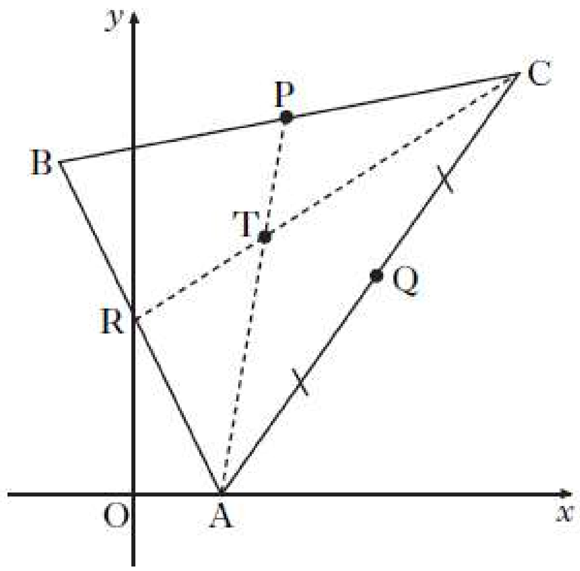
Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. **2**

2010 P1 Q21

Triangle ABC has vertices A(4, 0), B(−4, 16) and C(18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



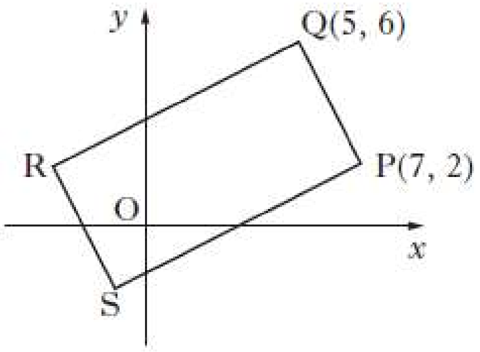
(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

(*c*) Find the ratio in which T divides BQ. **2**

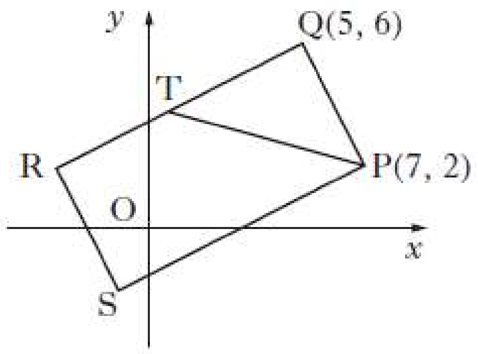
2013 P2 Q2

The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



(*a*) Find the equation of QR. **3**

(*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.



Find the coordinates of T. **3**

(*c*) Given that T is the midpoint of QR, find the coordinates of R and S. **3**

2012 P2 Q23

(*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). **4**

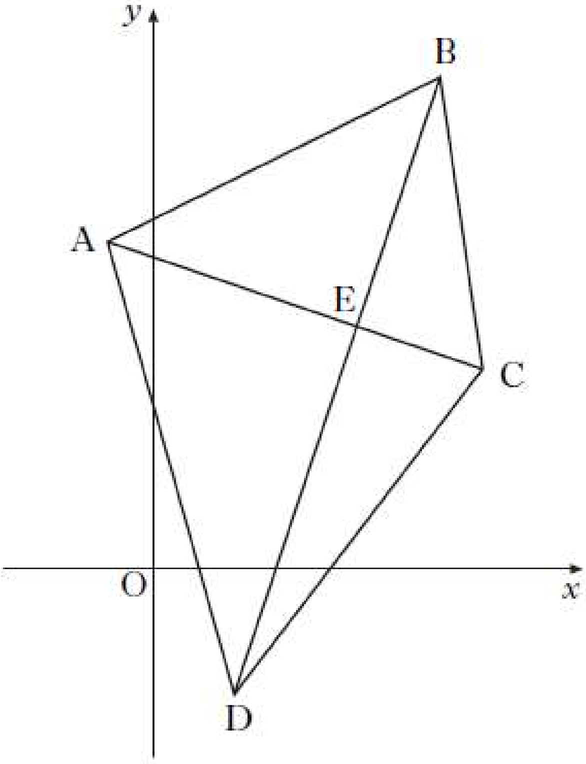
(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). **2**

(*c*) Find the point of intersection of ℓ_1 and ℓ_2. **3**

(*d*) Hence find the shortest distance between PQ and ℓ_2. **2**

2011 P2 Q22

A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.



(*a*) Find the equation of diagonal BD. **2**

(*b*) The equation of diagonal AC is $x + 3y = 23$.

Find the coordinates of E, the point of intersection of the diagonals. **3**

(*c*) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

Sets & Functions

2014 P2 Q3

Functions f and g are defined on suitable domains by

$$f(x) = x(x - 1) + q \text{ and } g(x) = x + 3.$$

(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. **4**

2011 P2 Q2

Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

Recurrence Relations

2013 P2 Q1

The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of m and c. **4**

2011 P2 Q3

(*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

Write down the values of u_1 and u_2. **1**

(*b*) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q. **3**

(*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

Differentiation

2014 P1 Q21

A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

2014 P2 Q2

A curve has equation $y = x^4 - 2x^3 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

2014 P2 Q8

Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(*c*) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

2011 P1 Q22

A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. **2**

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$. **3**

2012 P2 Q3

A function f is defined on the domain $0 \leq x \leq 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. **7**

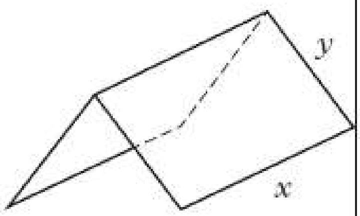
2013 P2 Q7

A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,m^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

3

(*b*) These rods cost £8·25 per metre.

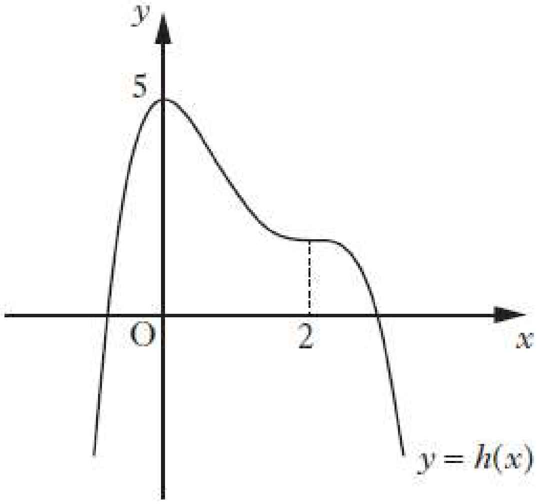
To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. **7**

2012 P2 Q4

The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(a) $y = h'(x)$; **3**

(b) $y = 2 - h'(x)$. **3**

2011 P1 Q22

(a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(b) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

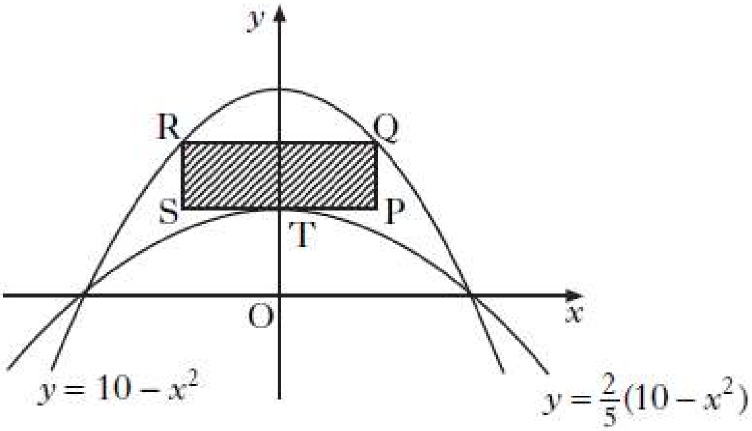
Find the coordinates of G. **5**

(d) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

2010 P2 Q5

The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ **3**

(*b*) Find the maximum area of this rectangle. **6**

Integration

2014 P2 Q5

Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\, dx = 2$, find the value of t. **5**

2013 P2 Q6

Given that $\int_0^a 5\sin 3x\, dx = \frac{10}{3}$, $0 \le a < \pi$,

calculate the value of a. **5**

2014 P2 Q7

Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,m^2$ of land.

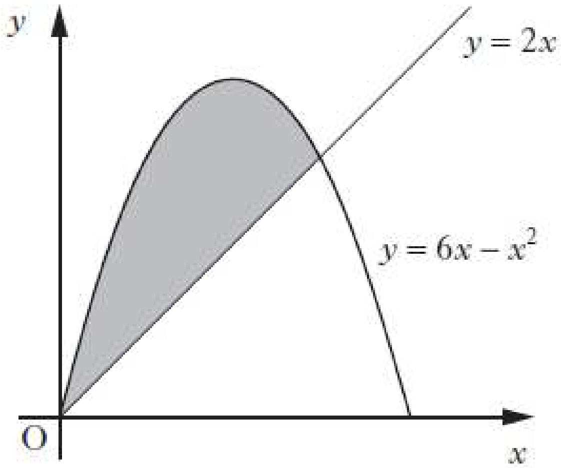


Diagram 1

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

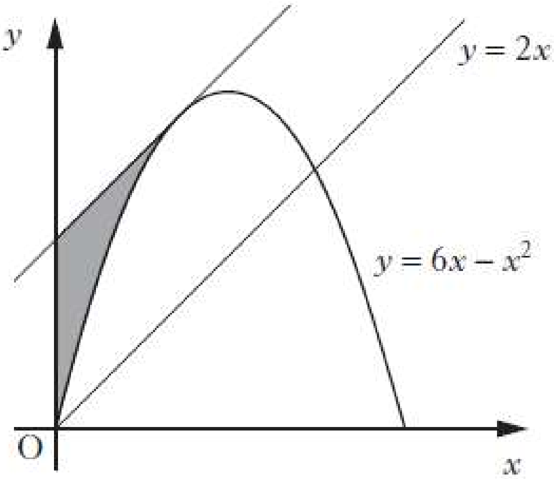


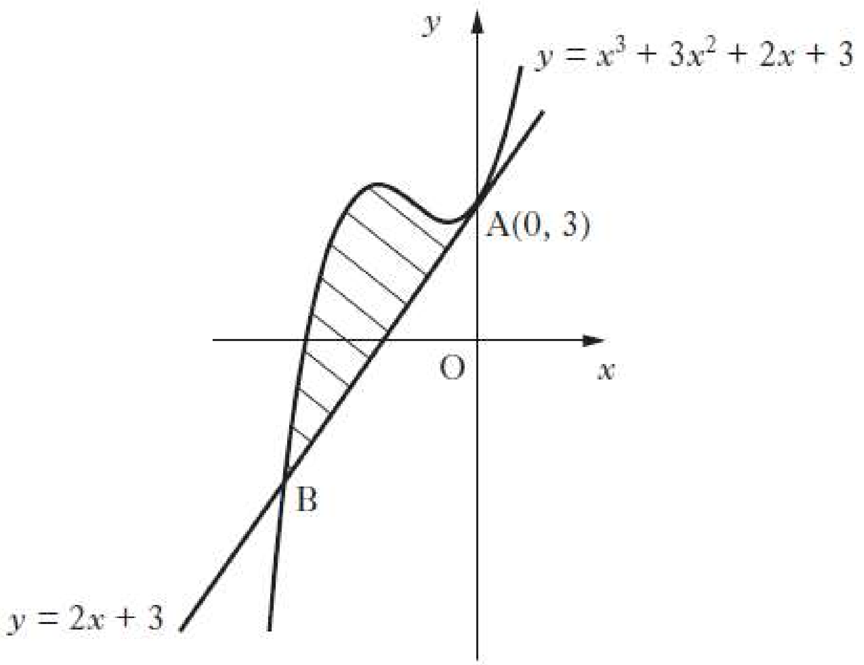
Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park. **5**

2013 P2 Q4

The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



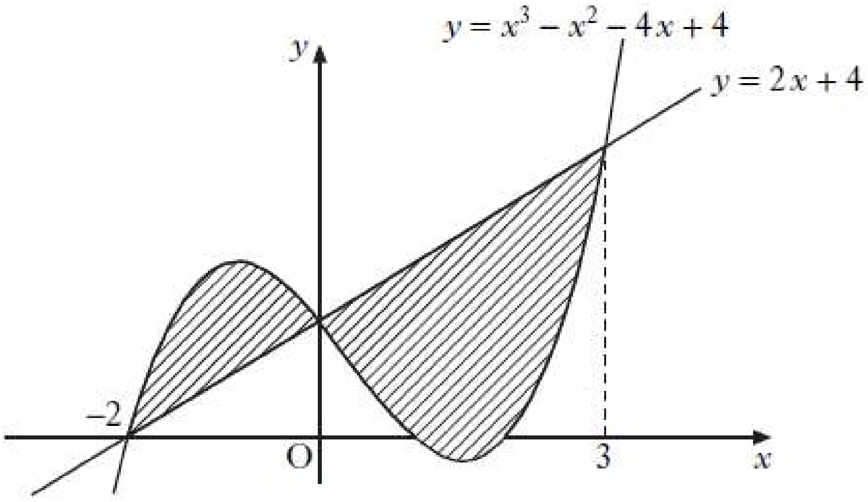
The line meets the curve again at B.

Show that B is the point (–3, –3) and find the area enclosed by the line and the curve. **6**

2011 P2 Q4

The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).



Calculate the total shaded area. **10**

2010 P2 Q6

(*a*) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

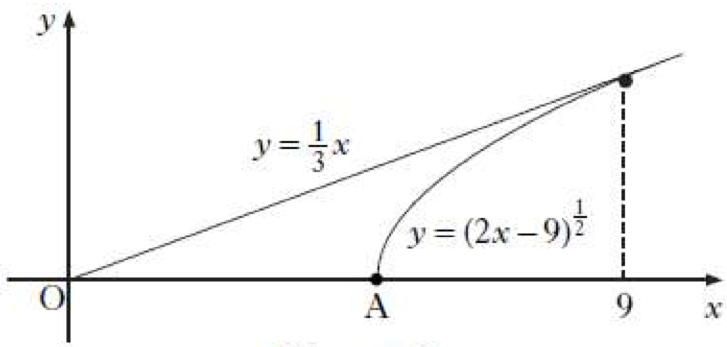


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

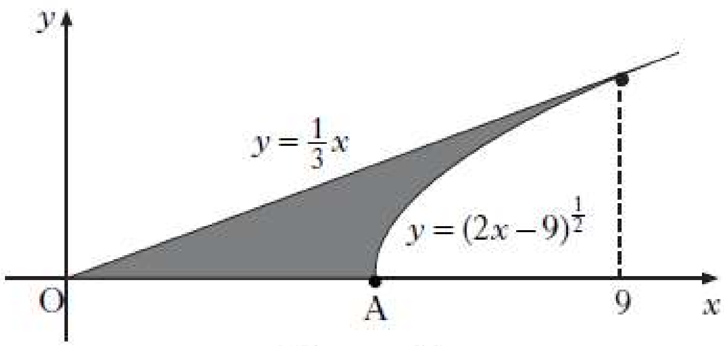


Diagram 2

Quadratic Functions

2013 P1 Q 21

Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

2012 P2 Q1

Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **3**

Polynomials

2014 P1 Q22

For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

2013 P2 Q3 (Part b Differentiation)

(*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

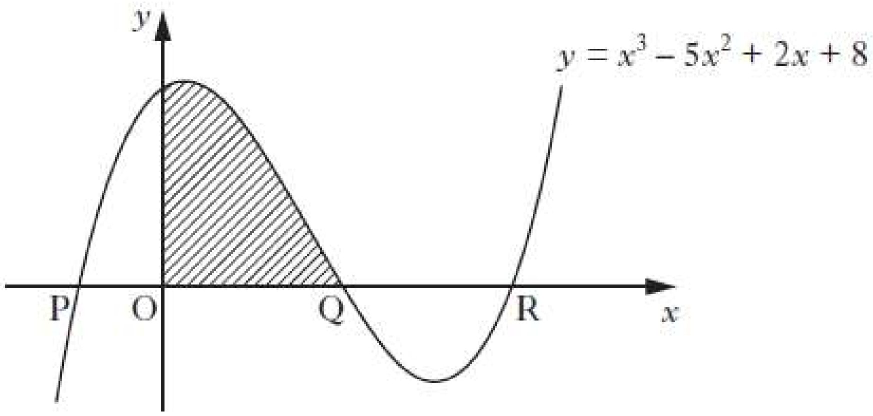
2012 P1 Q21 (part b Integration)

(*a*) (i) Show that $(x-4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area. **6**

The Circle

2014 P1 Q 23

(*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

(*c*) A second circle C_2 passes through P, Q and T.

Find the equation of C_2. **3**

2013 P2 Q22

A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(*a*) Write down the centre and calculate the radius of C_1. **2**

(*b*) The point P(3, 2) lies on the circle C_1.

Find the equation of the tangent at P. **3**

(*c*) A second circle C_2 has centre $(10, -1)$. The radius of C_2 is half of the radius of C_1.

Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. **3**

(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. **4**

2012 P2 Q2

(*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

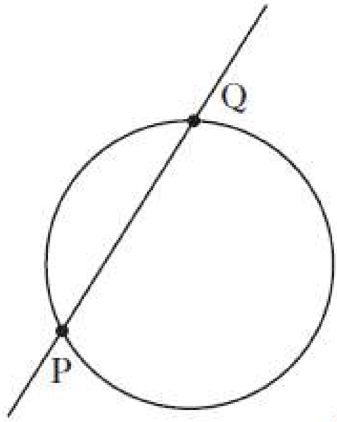


Diagram 1

Find the coordinates of P and Q. **6**

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.

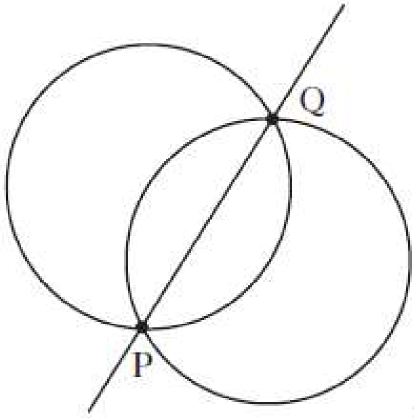


Diagram 2

6

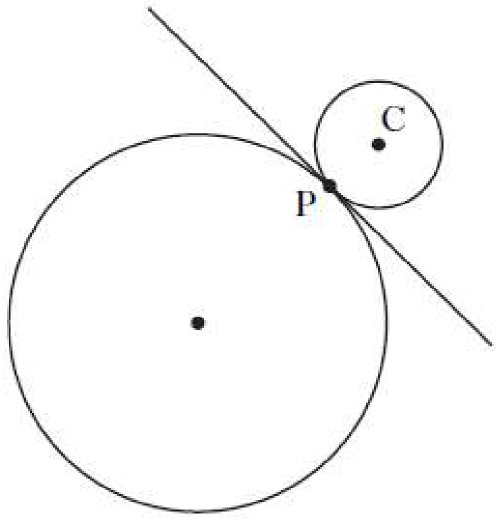
Determine the equation of this second circle.

2010 P2 Q3

(*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

2014 P2 Q 8

Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p. **5**

2011 P2 Q7

Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

The circles have no points of contact.

What is the range of values of p? **9**

Trigonometry

2014 P2 Q6

Solve the equation

$$\sin x - 2\cos 2x = 1 \qquad \text{for } 0 \le x < 2\pi.$$ **5**

2013 P2 Q8

Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \le x < 2\pi$$ **6**

2012 P2 Q6

For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.

Find the value(s) of x. **7**

2011 P2 Q23

(*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \le x < 360$. **5**

(*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \le x < 360$. **2**

2010 P2 Q4

Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \le x < 2\pi$. **5**

2010 P1 Q23

(*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

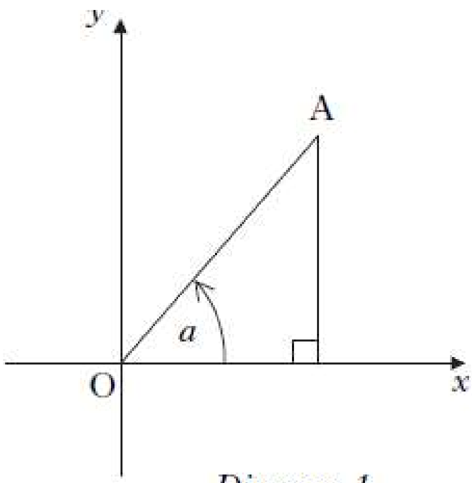


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

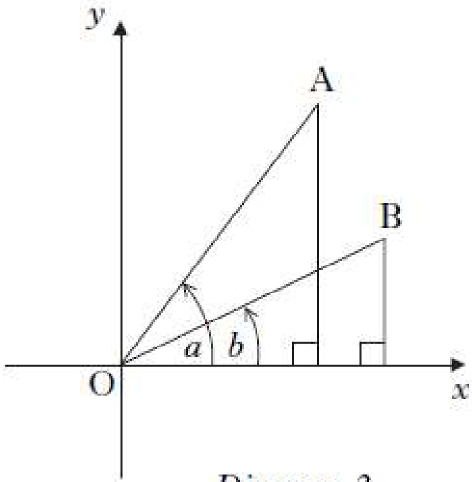


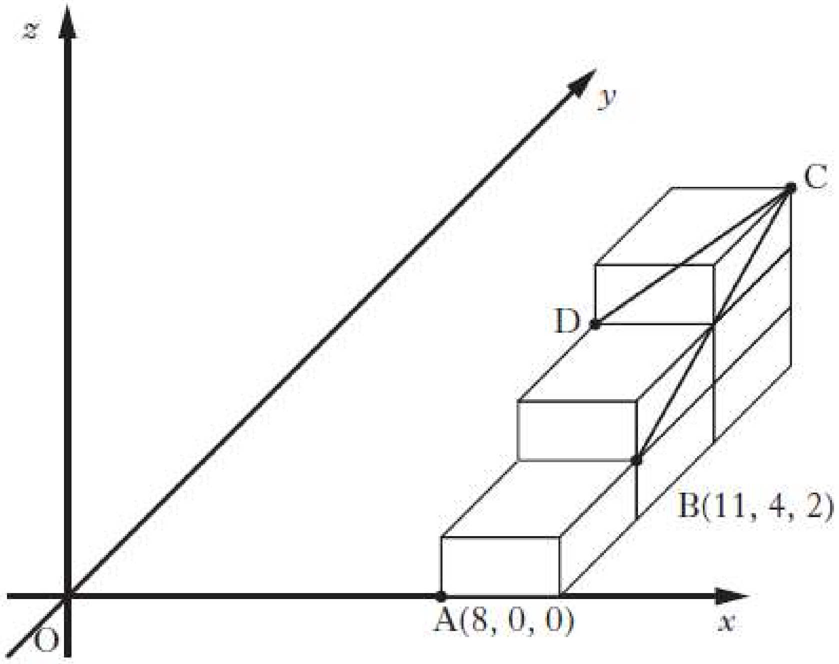
Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

2014 P2 Q4

Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. **2**

(*c*) Find the size of the angle BCD. **5**

2013 P1 Q24

(*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB. **4**

(*b*) The point C lies on the *x*-axis.

If TB and TC are perpendicular, find the coordinates of C. **5**

2012 P2 Q5

A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, k, 0).

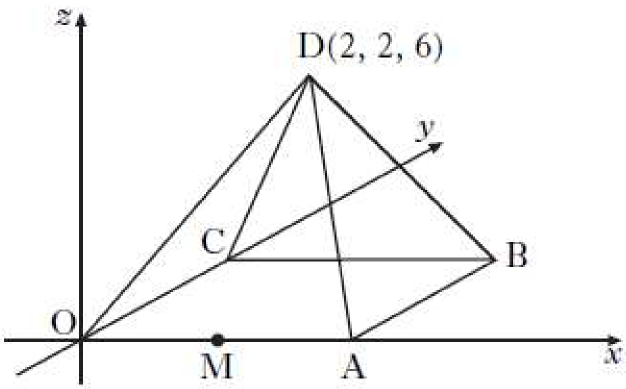
(a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$. **7**

(b) If angle ABC = 30°, find the possible values of k. **5**

2011 P2 Q1

D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(a) State the coordinates of B. **1**

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(c) Find the size of angle BDM. **5**

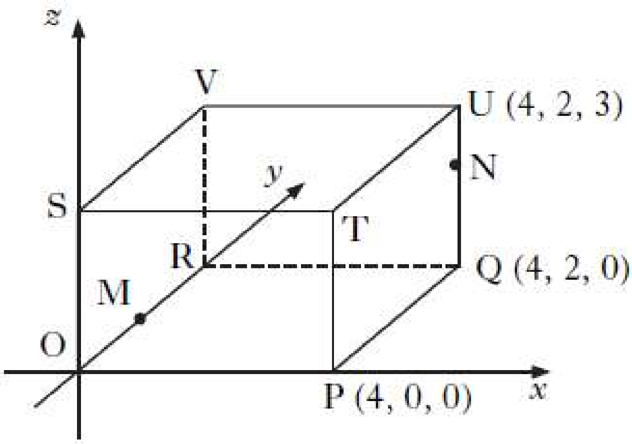
2010 P2 Q1

The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

(*c*) Calculate the size of angle MVN. **5**

The Wave Function

2013 P1 Q23

(*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k \sin(x - a)°$, where $k > 0$ and $0 \le a < 360$.

Calculate the values of k and a. **4**

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \le x < 360$. **2**

2012 P1 Q 22

(*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a < 2\pi$.

Calculate the values of k and a. **4**

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \le x \le 2\pi$. **3**

2011 P2 Q6 (Part b Integration)

(*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \le a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \le t \le 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\, dx = 3.$$ **7**

2010 P2 Q2

(*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \le a < 360$.

Calculate the values of k and a. **4**

(*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$.

(ii) Determine the values of x, in the interval $0 \le x < 360$, at which these maximum and minimum values occur. **3**

Logarithms and Exponentials

2013 P2 Q5

Solve the equation

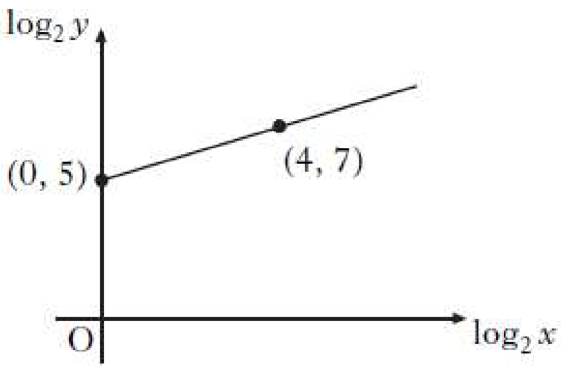
$$\log_5(3-2x)+\log_5(2+x)=1,\ \text{where } x \text{ is a real number.}$$ **4**

2011 P2 Q4

Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.

Find the values of k and n. **5**

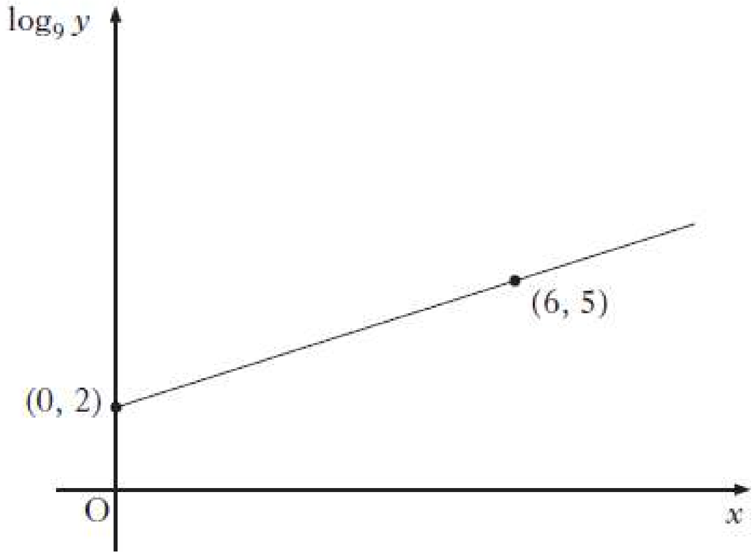


2014 P1 Q 24

Two variables, x and y, are related by the equation

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a. **5**

2013 P2 Q9

The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

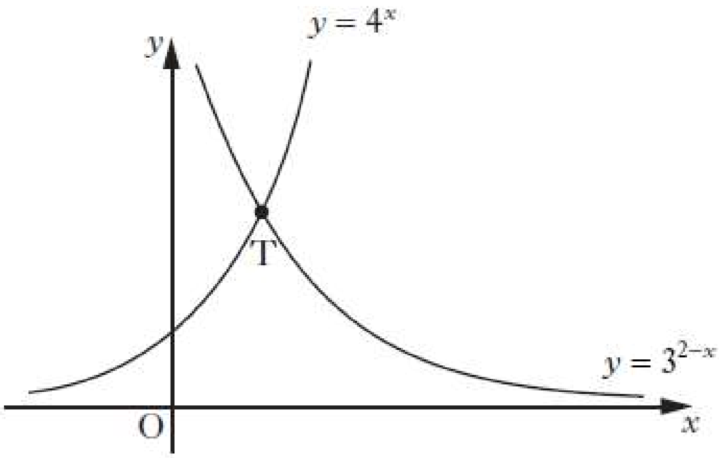
(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

2012 P2 Q7

The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\dfrac{\log_a p}{\log_a q}$, for all $a > 1$. **6**

(*b*) Calculate the y – coordinate of T. **2**

2010 P2 Q7

(*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3**

(*b*) Solve $\log_3 x + \log_9 x = 12$. **3**