2017 Paper 1

(1) a) $g(0) = 2\cos 0$
$= 2$
$f(1) = 5(2)$
$= \underline{10}$

b) $g(f(x)) = g(5x)$
$= \underline{2\cos 5x}$

(2) centre $= (4, 3)$
$m_R = \frac{3-1}{4+2}$ $\therefore m_T = -3$ $(m_R m_T = -1)$
$= \frac{2}{6}$
$= \frac{1}{3}$
$y - 1 = -3(x+2)$
$y - 1 = -3x - 6$
$\underline{y = -3x - 5}$

(3) $\frac{dy}{dx} = 12(4x-1)^{11} \times 4$
$= \underline{48(4x-1)^{11}}$

(4) For equal roots,
$b^2 - 4ac = 0$
$(4)^2 - 4(1)(k-5) = 0$
$16 - 4k + 20 = 0$
$-4k + 36 = 0$
$\underline{k = 9}$

(5) a) $\underline{u}.\underline{v} = 15 - 8 - 6$
$= \underline{1}$

b) $|\underline{u}| = \sqrt{5^2 + 1^2 + (-1)^2}$
$= \sqrt{27}$

$\underline{u}.\underline{w} = \sqrt{27}\sqrt{3}\cos\frac{u}{3}$
$= \sqrt{81}\left(\frac{1}{2}\right)$
$= \frac{9}{2}$

(6) $y = x^3 + 7$
$x^3 = y - 7$
$x = \sqrt[3]{y-7}$
$\therefore \underline{h^{-1}(x) = \sqrt[3]{x-7}}$

(7) Midpoint $= \left(\frac{-3+7}{2}, \frac{5+9}{2}\right)$
$= (2, 7)$

$m_{CM} = \frac{7-11}{2-2}$ $\therefore$ line is vertical
$= \frac{-4}{0}$ $\therefore \underline{x = 2}$
$= \underline{\text{undefined}}$

(8) $d(t) = \frac{1}{2}t^{-1}$
$d'(t) = -\frac{1}{2}t^{-2}$
$= -\frac{1}{2t^2}$

$\therefore d'(5) = -\frac{1}{2(5)^2}$
$= \underline{-\frac{1}{50}}$

(9) a) $u_2 = mu_1 + 6$
$13 = 25m + 6$
$7 = 25m$
$\underline{m = \frac{1}{4}}$

b) (i) As $-1 < \frac{1}{4} < 1$

(ii) $L = \frac{6}{1 - \frac{1}{4}}$
$= \frac{6}{\frac{3}{4}}$
$= \underline{8}$

(10) a) TOP − (BOTTOM)
$= x^3 - 4x^2 + 3x + 1 - (x^2 - 3x + 1)$
$= x^3 - 4x^2 + 3x + 1 - x^2 + 3x - 1$
$= x^3 - 5x^2 + 6x$

$\therefore A = \int_0^2 (x^3 - 5x^2 + 6x)\,dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_0^2$

$= \left(\frac{16}{4} - \frac{40}{3} + 12\right) - (0)$

$= 16 - \frac{40}{3}$

$= \frac{48}{3} - \frac{40}{3}$

$= \frac{8}{3}u^2$

b) TOP − (BOTTOM)

$= 1 - x - (x^2 - 3x + 1)$

$= 1 - x - x^2 + 3x - 1$

$= 2x - x^2$

$\therefore A = \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2$

$= \left(4 - \frac{8}{3}\right) - (0)$

$= \frac{4}{3}u^2$

$\frac{4}{3} \div \frac{8}{3} = \frac{1}{2}$ $\therefore \frac{1}{2}$ under the line.

(11) $3y - 2x = 4$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$ $\therefore m = \frac{2}{3}$

$M = \frac{a-2}{5+7}$

$= \frac{a-2}{12}$

$\therefore \frac{a-2}{12} = \frac{2}{3}$

$a - 2 = \frac{24}{3}$

$a = 8 + 2$

$a = 10$

(12) $\log_a 36 - \log_a 4 = \frac{1}{2}$

$\log_a 9 = \frac{1}{2}$

$9 = a^{1/2}$

$a = 81$

(13) $\int (5 - 4x)^{-1/2}\,dx$

$= \frac{(5-4x)^{1/2}}{\frac{1}{2}(-4)} + C$

$= -\frac{1}{2}(5-4x)^{1/2} + C$

(14) $\sqrt{3}\sin x - \cos x$

a) $= k(\sin x \cos\alpha - \cos x \sin\alpha)$

$= k\cos\alpha \sin x - k\sin\alpha \cos x$

$-k\sin\alpha = -1$

$k\sin\alpha = 1$

$k\cos\alpha = \sqrt{3}$

$b^2 = 1^2 + \sqrt{3}^2$

$b^2 = 4$

$k = 2$

$\tan\alpha = \frac{1}{\sqrt{3}}$

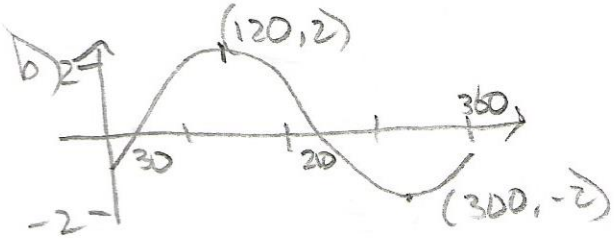
$\alpha = 30°$

S | A ✓✓

T | C ✓ $k\sin\alpha = +$, $k\cos\alpha = +$

$\therefore$ Q1

$\therefore 2\sin(x - 30)°$



(15) a) Sigma, Sup $\therefore a = -5$, $b = 3$

b) 1, 6, 3, 2 $\therefore 7$

c) $F'(1) = h$

$\therefore h'(6)$

$\therefore h'(8)$

(by sym

2017 Paper 2

(1) a) Mid $= \left(\frac{3+9}{2}, \frac{0-2}{2}\right)$
$= (6, -1)$

$m_{BC} = \frac{0+2}{3-9}$ $\therefore m_{PB} = 3$ $(m_{BC} \times m_{PB} = -1)$
$= \frac{2}{-6}$
$= -\frac{1}{3}$

$y - b = m(x-a)$
$y+1 = 3(x-6)$
$y+1 = 3x-18$
$y = 3x-19$

b) $m = \tan\theta$ $y-b = m(x-a)$
$= \tan 45^\circ$ $y-0 = 1(x-3)$
$= 1$ $y = x-3$.

c) $y = 3x-19$ and $y = x-3$
$\therefore 3x-19 = x-3$
$2x = 16$
$x = 8$ $\therefore (8,5)$
$y = 8-3$
$= 5$

(2) a)

1	2	−5	1	2
		2	−3	−2
	2	−3	−2	0

$R = 0$ $\therefore (x-1)$ is a factor
$x = 1$ is a root

b) $2x^3 - 5x^2 + x + 2 = 0$
$(x-1)(2x^2 - 3x - 2) = 0$
$(x-1)(2x+1)(x-2) = 0$
$x = 1$ $x = -\frac{1}{2}$ $x = 2$

(3) $(x-2)^2 + (y-1)^2 = 25$
$(x-2)^2 + (3x-1)^2 = 25$
$x^2 - 4x + 4 + 9x^2 - 6x + 1 - 25 = 0$
$10x^2 - 10x - 20 = 0$
$10(x^2 - x - 2) = 0$
$10(x-2)(x+1) = 0$

$x = 2$	$x = -1$
$y = 6$	$y = -3$
$\therefore (2,6)$	$(-1,-3)$

(4) a) $3x^2 + 24x + 50$
$= 3[x^2 + 8x] + 50$
$= 3[(x+4)^2 - 16] + 50$
$= 3(x+4)^2 - 48 + 50$
$= 3(x+4)^2 + 2$

b) $f'(x) = 3x^2 + 24x + 50$

c) The minimum value of
$f'(x) = 2$ (as $f'(x) = 3(x+4)^2 + 2$)
$\therefore f'(x)$ is always positive
$\therefore f(x)$ strictly increasing for all x.

(5) a) $\vec{PQ} = \vec{PR} + \vec{RQ}$
$= -3\underline{i} - 4\underline{j} + 5\underline{k}$

b) $\vec{PS} = \vec{PR} + \frac{2}{3}\vec{RQ}$
$= \begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -8 \\ -6 \\ 2 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$
$= \underline{i} - \underline{j} + 4\underline{k}$ (as required)

c) $\vec{PQ} \cdot \vec{PS} = -3 + 4 + 20$

$= \underline{21}$

$|\vec{PQ}| = \sqrt{9 + 16 + 25}$

$= \sqrt{50}$

$|\vec{PS}| = \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$\therefore \cos QPS = \frac{21}{\sqrt{50}\sqrt{18}}$

$QPS = \cos^{-1}\left(\frac{21}{\sqrt{900}}\right)$

$\underline{QPS = 45.6^\circ}$

(6) $5\sin x - 4 = 2\cos 2x$

$5\sin x - 4 = 2(1 - 2\sin^2 x)$

$5\sin x - 4 = 2 - 4\sin^2 x$

$4\sin^2 x + 5\sin x - 6 = 0$

$4s^2 + 5s - 6$

$= 4s^2 + 8s - 3s - 6$

$= 4s(s+2) - 3(s+2)$

$= (s+2)(4s-3)$

24

1 24

2 12

3 8

4 6

$\therefore (\sin x + 2)(4\sin x - 3) = 0$

$\sin x + 2 = 0$

$\sin x = -2$

NO SOLUTIONS

$4\sin x - 3 = 0$

$\sin x = \frac{3}{4}$

$x = 48.6^\circ, 131.4^\circ$

$\therefore \underline{x = 0.85, 2.29}$

(7) a) $y = 6x - 2x^{3/2}$

$\frac{dy}{dx} = 6 - 3x^{1/2}$

SP's @ $\frac{dy}{dx} = 0$

$\therefore 6 - 3\sqrt{x} = 0$

$3\sqrt{x} = 6$

$\sqrt{x} = 2$

$\underline{x = 4}$

b) when $x = 1$, $y = 6 - 2(1)$

$= \underline{4}$

$x = 9$ $y = 54 - 2\sqrt{}$

$= 54 - 54$

$= \underline{0}$

$\therefore$ Max = 4, Min = 0

(8) a) $u_1 = 5k - 20$

$u_2 = k(5k - 20) - 20$

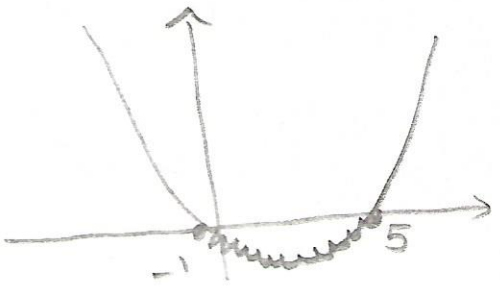
$= \underline{5k^2 - 20k - 20}$

b) $5k^2 - 20k - 20 < 5$

$5k^2 - 20k - 25 < 0$

$5(k^2 - 4k - 5) < 0$

$5(k-5)(k+1) < 0$



$-1 < k < 5$

(9) $(0,3)$ $(-12,0)$

$$M = \frac{3-0}{0+12} = \frac{3}{12} = \frac{1}{4}$$

$\therefore \log_2 y = \frac{1}{4}\log_2 x + 3$

$\log_2 y = \log_2 x^{1/4} + \log_2 8$

$\log_2 y = \log_2 8x^{1/4}$

$\therefore y = 8x^{1/4}$

$\therefore \underline{k = 8 \quad n = \frac{1}{4}}$

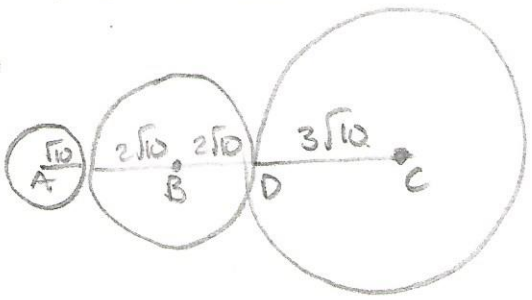
(10) a) $M_{AB} = \frac{1+2}{2+7} = \frac{3}{9} = \frac{1}{3}$ $\quad M_{BC} = \frac{6-1}{17-2} = \frac{5}{15} = \frac{1}{3}$

$M_{AB} = M_{BC}$ $\therefore$ parallel lines

B is a common point

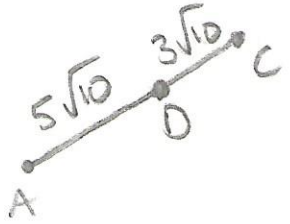
$\therefore$ collinear.

b)



radius circle D $= 2(\sqrt{10}) + 2(2\sqrt{10})$

$= 6\sqrt{10}$



D divides AC in a 5:3 ratio.

$\therefore \underline{d} = \frac{1}{8}\left[5\underline{c} + 3\underline{a}\right]$

$= \frac{1}{8}\left[\begin{pmatrix}85\\30\end{pmatrix} + \begin{pmatrix}-21\\-6\end{pmatrix}\right]$

$= \frac{1}{8}\left[\begin{pmatrix}64\\24\end{pmatrix}\right]$

$= \begin{pmatrix}8\\3\end{pmatrix}$

$\therefore D = (8,3)$

$\therefore (x-8)^2 + (y-3)^2 = (6\sqrt{10})^2$

$\underline{(x-8)^2 + (y-3)^2 = 360}$

(11) a) $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x$

$= \frac{2\sin x \cos x}{2\cos x} - \sin x \cos^2 x$

$= \sin x - \sin x \cos^2 x$

$= \sin x (1 - \cos^2 x)$

$= \sin x (\sin^2 x)$

$= \underline{\sin^3 x \text{ (as required)}}$

b) $\frac{d}{dx}\left(\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x\right)$

$= \frac{d}{dx}(\sin^3 x)$

$= \frac{d}{dx}(\sin x)^3$

$= 3(\sin x)^2 \times \cos x$

$= \underline{3\sin^2 x \cos x}$