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Mathematics Paper 2

Amended Marking Instructions

FRIDAY 5 MAY

Strictly Confidential
These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.

Version 4
29/05/2017

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

(a) Marks for each candidate response must <u>always</u> be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.

(d) Credit must be assigned in accordance with the specific assessment guidelines.

(e) One mark is available for each •. There are no half marks.

(f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

(g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

(h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6\times6=12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment **(j)**.

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
Eased as no longer a solution of a quadratic equation so mark is not awarded.	$x=1$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(k) **Horizontal/vertical marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

  $2x^4+4x^3+6x^2+4x+x^3+2x^2+3x+2$ written as $2x^4+5x^3+8x^2+7x+2$ gains full credit

- Repeated error within a question, but not between questions or papers

(o) In any 'Show that…' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.

(p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

(q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

(r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking Information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :

- If the candidate has written in any unexpected areas of their answer booklet
- If the script is legible and that it does not require to be re-scanned
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate
- If the candidate has continued an answer to a question at the back or in a different location in the booklet
- The presence of any non-script related objects.

No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).

Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.

Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.

Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking annotations such as tick/cross or the SEEN annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.

Link tool: The Link tool allows you to link pages/additional objects to a particular question item on a response.

In "Full Response View":

- Check which question the candidate's answer relates to
- Click on the question in the marks display panel
- On the left hand side, select the Link Page check box beneath the thumbnail for the page.
- Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed.

The following is a list of exceptions which can be raised for this assessment. Please ensure that you are familiar with these in order that you select the appropriate exception for the issue encountered, take the Marker Action advised and note that all exceptions must be raised in RM Assessor **before** the response is submitted.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed **do not** belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

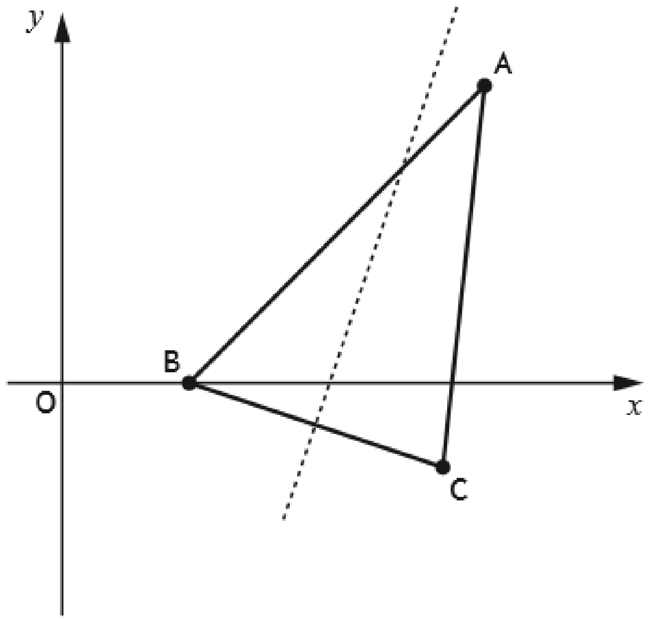
Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking **OR** If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or if coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations

Annotation	Annotation Name	Instructions on use of annotation
	Tick	A tick should be placed on the script at the point where a mark is awarded (or at the end of that line of working).
	Tick 1	A tick 1 should be used to indicate “correct” working where a mark is awarded as a result of follow through from an error.
	Tick 2	A tick 2 should be used to indicate correct working which is irrelevant or insufficient to award any marks. This should also be used for working which has been eased.
	Cross	A cross is used to indicate where a mark has not been awarded.
	Omission	A “roof” should be used to show that something is missing, such as part of a solution or a crucial step in the working.

One of the above annotations must appear for each available mark e.g. a question worth 4 marks must have 4 of the above annotations.

	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
	Horizontal wavy line	A tilde should be used to indicate a minor error which is not being penalised, e.g. bad form (bad form only becomes bad form if subsequent working is correct).
	Highlight	This is used to highlight or underline an error.

Question	Generic Scheme	Illustrative Scheme	Max Mark
1. (a)	Triangle ABC is shown in the diagram below. The coordinates of B are (3,0) and the coordinates of C are (9,–2). The broken line is the perpendicular bisector of BC.  Find the equation of the perpendicular bisector of BC.		
	$\bullet^1$ find mid-point of BC	$\bullet^1$ $(6,-1)$	
	$\bullet^2$ calculate gradient of BC	$\bullet^2$ $-\frac{2}{6}$	
	$\bullet^3$ use property of perpendicular lines	$\bullet^3$ 3	
	$\bullet^4$ determine equation of line in a simplified form	$\bullet^4$ $y = 3x - 19$	4

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient **and** a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be awarded.
3. At $\bullet^4$, accept $3x - y - 19 = 0$, $3x - y = 19$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
1.	(b)	The line AB makes an angle of 45° with the positive direction of the x-axis. Find the equation of AB.		
		•5 use $m = \tan\theta$ •6 determine equation of AB	•5 1 •6 $y = x - 3$	2
Notes:				
4. At •6, accept $y - x + 3 = 0$, $y - x = -3$ or any other rearrangement of the equation where the constant terms have been simplified.				
Commonly Observed Responses:				

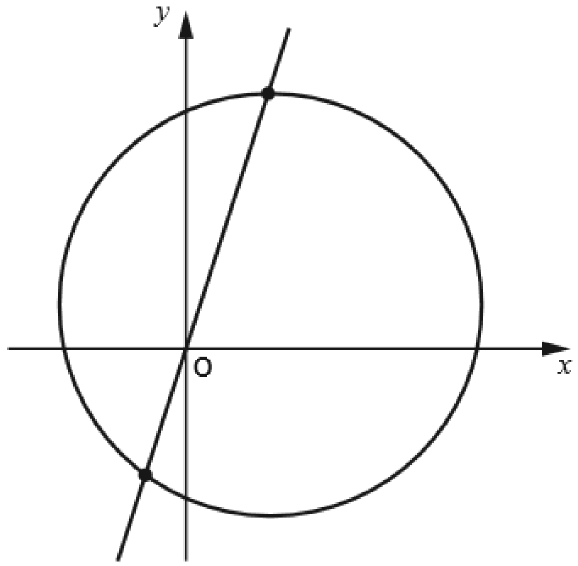
Question		Generic Scheme	Illustrative Scheme	Max Mark
1.	(c)	Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.		
		•7 find x or y coordinate •8 find remaining coordinate	•7 $x = 8$ or $y = 5$ •8 $y = 5$ or $x = 8$	2
Notes:				
Commonly Observed Responses:				

Question	Generic Scheme	Illustrative Scheme	Max Mark
2. (a)	Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$.		
	Method 1 •1 know to use $x=1$ in synthetic division •2 complete division, interpret result and state conclusion	**Method 1** •1 $\begin{array}{c\|cccc} 1 & 2 & -5 & 1 & 2 \\ \hline & 2 & & & \end{array}$ •2 $\begin{array}{c\|cccc} 1 & 2 & -5 & 1 & 2 \\ & & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & 0 \end{array}$ Remainder $=0$ $\therefore (x-1)$ is a factor	2
	Method 2 •1 know to substitute $x=1$ •2 complete evaluation, interpret result and state conclusion	**Method 2** •1 $2(1)^3-5(1)^2+(1)+2$ •2 $=0$ $\therefore (x-1)$ is a factor	2
	Method 3 •1 start long division and find leading term in quotient •2 complete division, interpret result and state conclusion	**Method 3** •1 $\begin{array}{r} 2x^2 \\ (x-1)\overline{)\,2x^3-5x^2+x+2} \end{array}$ •2 $\begin{array}{r} 2x^2-3x-2 \\ (x-1)\overline{)\,2x^3-5x^2+x+2} \\ \underline{2x^3-2x^2} \\ -3x^2+x \\ \underline{-3x^2+3x} \\ -2x+2 \\ \underline{-2x+2} \\ 0 \end{array}$ remainder $=0$ $\therefore$ $(x-1)$ is a factor	2

Notes:
1. Communication at $\bullet^2$ must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before $\bullet^2$ can be awarded. 2. Accept any of the following for $\bullet^2$: • '$f(1)=0$ so $(x-1)$ is a factor' • 'since remainder = 0, it is a factor' • the 0 from any method linked to the word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$' 3. Do not accept any of the following for $\bullet^2$: • double underlining the zero or boxing the zero without comment • '$x=-1$ is a factor', '$(x+1)$ is a factor', '$(x+1)$ is a root', '$x=1$ is a root', '$(x-1)$ is a root' '$x=-1$ is a root'. • the word 'factor' only with no link
Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
2. (b)	Hence, or otherwise, solve $f(x)=0$.		
	$\bullet^3$ state quadratic factor $\bullet^4$ find remaining factors $\bullet^5$ state solution	$\bullet^3$ $2x^2-3x-2$ $\bullet^4$ $(2x+1)$ and $(x-2)$ $\bullet^5$ $x=-\frac{1}{2}, 1, 2$	3

Notes:
4. The appearance of "$=0$" is not required for $\bullet^5$ to be awarded. 5. Candidates who identify a different initial factor and subsequent quadratic factor can gain all available marks. 6. $\bullet^5$ is only available as a result of a valid strategy at $\bullet^3$ and $\bullet^4$. 7. Accept $\left(-\frac{1}{2},0\right)$, $(1,0)$, $(2,0)$ for $\bullet^5$.
Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3.	The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.  Find the coordinates of the points of intersection.		
	•[1] substitute for y	•[1] $(x-2)^2+(3x-1)^2=25$ or $x^2-4x+4+(3x)^2-2(3x)+1=25$	
	•[2] express in standard quadratic form	•[2] $10x^2-10x-20=0$	
	•[3] factorise	•[3] $10(x-2)(x+1)=0$	
	•[4] find x coordinates	•[4] •[5] •[4] $x=2$ $x=-1$	
	•[5] find y coordinates	•[5] $y=6$ $y=-3$	5

Notes:

1. At •[3] the quadratic must lead to two distinct real roots for •[4] and •[5] to be available.
2. •[2] is only available if '$=0$' appears at •[2] or •[3] stage.
3. If a candidate arrives at an equation which is not a quadratic at •[2] stage, then •[3], •[4] and •[5] are not available
4. At •[3] do not penalise candidates who fail to extract the common factor or who have divided the quadratic equation by 10.
5. •[3] is available for substituting correctly into the quadratic formula.
6. •[4] and •[5] may be marked either horizontally or vertically.
7. For candidates who identify **both** solutions by inspection, full marks may be awarded provided they justify that their points lie on **both** the line and the circle. Candidates who identify **both** solutions, but justify only one gain 2 out of 5.

Commonly Observed Responses:	
Candidate A $(x-2)^2+(3x-1)^2=25$ •¹ ✓ $10x^2-10x=20$ •² ✗ $10x(x-1)=20$ •³ $\boxed{✓2}$ $x=2 \quad x=3$ •⁴ ✗ $y=6 \quad y=9$ •⁵ $\boxed{✓2}$	**Candidate B** Candidates who substitute into the circle equation only •¹ ✓ •² ✓ •³ ✓ •⁴ ✓ Sub $x=2$ Sub $x=-1$ $y^2-2y-24=0$ $y^2-2y-15=0$ $(y-6)(y+4)=0$ $(y+3)(y-5)=0$ $y=6$ or $\cancel{y=-4}$ $y=-3$ or $\cancel{y=5}$ $(2,6)$ $(-1,-3)$ •⁵ ✗

Question		Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)	Express $3x^2+24x+50$ in the form $a(x+b)^2+c$.		
		Method 1 •¹ identify common factor •² complete the square •³ process for c and write in required form	**Method 1** •¹ $3(x^2+8x$...... stated or implied by •² •² $3(x+4)^2$...... •³ $3(x+4)^2+2$	**3**
		Method 2 •¹ expand completed square form •² equate coefficients •³ process for b and c and write in required form	**Method 2** •¹ $ax^2+2abx+ab^2+c$ •² $a=3,\ 2ab=24,\ ab^2+c=50$ •³ $3(x+4)^2+2$	**3**
Notes:				
1. $3(x+4)^2+2$ with no working gains •¹ and •² only; however, see Candidate G. 2. •³ is only available for a calculation involving both multiplication and subtraction of integers.				

Commonly Observed Responses:	
Candidate A $3\left(x^2+8x+\frac{50}{3}\right)$ •1 ✓ $3\left(x^2+8x+16-16+\frac{50}{3}\right)$ •2 ^ further working is required	**Candidate B** $3x^2+24x+50=3(x+8)^2-64+50$ •1 ✗ •2 ✗ $=3(x+8)^2-14$ •3 $\boxed{\checkmark 2}$
Candidate C $ax^2+2abx+ab^2+c$ •1 ✓ $a=3,\ \ 2ab=24,\ \ b^2+c=50$ •2 ✗ $a=3,\ \ b=4,\ \ c=34$ $3(x+4)^2+34$ •3 $\boxed{\checkmark 1}$	**Candidate D** $3\left(\left(x^2+24x\right)+50\right)$ •1 ✗ $3\left((x+12)^2-144\right)+50$ •2 $\boxed{\checkmark 1}$ $3(x+12)^2-382$ •3 $\boxed{\checkmark 1}$
Candidate E $a(x+b)^2+c=ax^2+2abx+ab^2+c$ •1 ✓ $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •2 ✓ $b=4,\ \ c=2$ •3 ✓ •3 is awarded as all working relates to completed square form	**Candidate F** $ax^2+2abx+ab^2+c$ •1 ✓ $a=3,\ \ 2ab=24,\ \ ab^2+c=50$ •2 ✓ $b=4,\ \ c=2$ •3 ✗ •3 is lost as no reference is made to completed square form
Candidate G $3(x+4)^2+2$ Check: $3\left(x^2+8x+16\right)+2$ $=3x^2+24x+48+2$ $=3x^2+24x+50$ **Award 3/3**	**Candidate H** $3x^2+24x+50$ $=3(x+4)^2-16+50$ •1 ✓ •2 ✓ $=3(x+4)^2+34$ •3 ✗

Question	Generic Scheme	Illustrative Scheme	Max Mark
4. (b)	Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$.		
	•4 differentiate two terms •5 complete differentiation	•4 $3x^2+24x....$ •5 $....+50$	2
Notes:			
3. •4 is awarded for any two of the following three terms: $3x^2$, $+24x$, $+50$			
Commonly Observed Responses:			

<table>
<tr><th>Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td>4. (c)</td><td colspan="3">Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x.</td></tr>
<tr><td></td><td>Method 1

•6 link with (a) and identify sign of $(x+4)^2$

•7 communicate reason

Method 2

•6 identify minimum value of $f'(x)$

•7 communicate reason</td><td>Method 1

•6 $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \forall\ x$

•7 $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing

Method 2

•6 eg minimum value =2 or annotated sketch

•7 $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing</td><td>2</td></tr>
</table>

Notes:

4. Do not penalise $(x+4)^2>0$ or the omission of $f'(x)$ at •6 in Method 1.
5. Responses in part (c) must be consistent with working in parts (a) and (b) for •6 and •7 to be available.
6. Where erroneous working leads to a candidate considering a function which is not always strictly increasing, only •6 is available.
7. At •6 communication should be explicitly in terms of the given function. Do not accept statements such as "(something) $^2 \geq 0$", "something squared ≥ 0". However, •7 is still available.

Commonly Observed Responses:

<table>
<tr><td>Candidate I

$f'(x)=3(x+4)^2+2$

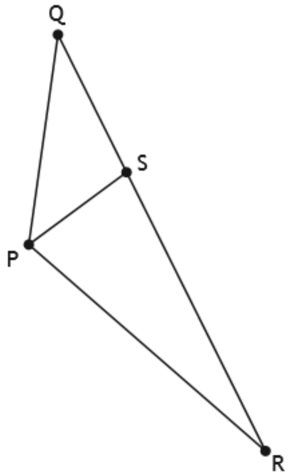
$3(x+4)^2+2>0 \Rightarrow$ strictly increasing.
Award 1 out of 2</td><td>Candidate J

Since $3x^2+24x+50=3(x+4)^2+\frac{166}{50}$

and $(x+4)^2$ is >0 for all x then

$3(x+4)^2+\frac{166}{50}>0$ for all x.

Hence the curve is strictly increasing for all values of x. •6 ✓ •7 $\boxed{✓1}$</td></tr>
</table>

Question	Generic Scheme	Illustrative Scheme	Max Mark
5. (a)	In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.  Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.		
	$\bullet^1$ identify pathway $\bullet^2$ state $\overrightarrow{PQ}$	$\bullet^1$ $\overrightarrow{PR} + \overrightarrow{RQ}$ stated or implied by $\bullet^2$ $\bullet^2$ $-3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$	2

Notes:

1. Award $\bullet^1$ $(9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) + (-12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k})$.
2. Candidates who choose to work with column vectors and leave their answer in the form $\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ cannot gain $\bullet^2$.
3. $\bullet^2$ is not available for simply adding or subtracting vectors within an invalid strategy.
4. Where candidates choose specific points consistent with the given vectors, only $\bullet^1$ and $\bullet^4$ are available. However, should the statement 'without loss of generality' precede the selected points then marks $\bullet^1$, $\bullet^2$, $\bullet^3$ and $\bullet^4$ are all available.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
5.	(b)	The point S divides QR in the ratio 1:2. Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$.		
		•³ interpret ratio	•³ $\frac{2}{3}$ or $\frac{1}{3}$	
		•⁴ identify pathway and demonstrate result	•⁴ $\overrightarrow{PR} + \frac{2}{3}\overrightarrow{RQ}$ or $\overrightarrow{PQ} + \frac{1}{3}\overrightarrow{QR}$ leading to $\mathbf{i} - \mathbf{j} + 4\mathbf{k}$	2

Notes:

5. This is a 'show that' question. Candidates who choose to work with column vectors must write their final answer in the required form to gain •⁴. $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ does not gain •⁴.
6. Beware of candidates who fudge their working between •³ and •⁴.

Commonly Observed Responses:

Candidate A - legitimate use of the section formula	Candidate B - BEWARE - treating P as the origin
$\overrightarrow{PS} = \dfrac{n\overrightarrow{PQ} + m\overrightarrow{PR}}{m+n}$ $\overrightarrow{PS} = \dfrac{2\overrightarrow{PQ} + \overrightarrow{PR}}{3}$ •³ ✓ $\overrightarrow{PS} = \dfrac{2\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}}{3} + \dfrac{\begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix}}{3}$ $= \begin{pmatrix} -2 \\ -8/3 \\ 10/3 \end{pmatrix} + \begin{pmatrix} 3 \\ 5/3 \\ 2/3 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ •⁴ ✓	$2\overrightarrow{QS} = \overrightarrow{SR}$ $3\mathbf{s} = 2\mathbf{q} + \mathbf{r}$ •³ ✓ $3\mathbf{s} = 2\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 9 \\ 5 \\ 2 \end{pmatrix}$ $\mathbf{s} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ •⁴ ✗

Question	Generic Scheme	Illustrative Scheme	Max Mark
5. (c)	Hence, find the size of angle QPS.		
	Method 1	**Method 1**	
	$\bullet^5$ evaluate $\overrightarrow{PQ}.\overrightarrow{PS}$	$\bullet^5$ $\overrightarrow{PQ}.\overrightarrow{PS}=21$	
	$\bullet^6$ evaluate $\lvert\overrightarrow{PQ}\rvert$	$\bullet^6$ $\lvert\overrightarrow{PQ}\rvert=\sqrt{50}$	
	$\bullet^7$ evaluate $\lvert\overrightarrow{PS}\rvert$	$\bullet^7$ $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$	
	$\bullet^8$ use scalar product	$\bullet^8$ $\cos QPS=\dfrac{21}{\sqrt{50}\times\sqrt{18}}$	
	$\bullet^9$ calculate angle	$\bullet^9$ $45\cdot6°$ or $0\cdot795$ radians	5
	Method 2	**Method 2**	
	$\bullet^5$ evaluate $\lvert\overrightarrow{QS}\rvert$	$\bullet^5$ $\lvert\overrightarrow{QS}\rvert=\sqrt{26}$	
	$\bullet^6$ evaluate $\lvert\overrightarrow{PQ}\rvert$	$\bullet^6$ $\lvert\overrightarrow{PQ}\rvert=\sqrt{50}$	
	$\bullet^7$ evaluate $\lvert\overrightarrow{PS}\rvert$	$\bullet^7$ $\lvert\overrightarrow{PS}\rvert=\sqrt{18}$	
	$\bullet^8$ use cosine rule	$\bullet^8$ $\cos QPS=\dfrac{\left(\sqrt{50}\right)^2+\left(\sqrt{18}\right)^2-\left(\sqrt{26}\right)^2}{2\times\sqrt{50}\times\sqrt{18}}$	
	$\bullet^9$ calculate angle	$\bullet^9$ $45\cdot6°$ or $0\cdot795$ radians	5

Notes:

7. For candidates who use $\overrightarrow{PS}$ **not equal to** $\mathbf{i}-\mathbf{j}+4\mathbf{k}$ $\bullet^5$ is not available in Method 1 or $\bullet^7$ in Method 2.
8. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. However, $\sqrt{1^2-1^2+4^2}$ leading to $\sqrt{16}$ indicates an invalid method for calculating the magnitude. No mark can be awarded for any magnitude arrived at using an invalid method.
9. $\bullet^8$ is not available to candidates who simply state the formula $\cos\theta=\dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$.

However, $\cos\theta=\dfrac{\overrightarrow{PQ}.\overrightarrow{PS}}{\lvert\overrightarrow{PQ}\rvert\times\lvert\overrightarrow{PS}\rvert}$ or $\cos\theta=\dfrac{21}{\sqrt{50}\times\sqrt{18}}$ is acceptable. Similarly for Method 2.

10. Accept answers which round to $46°$ or $0\cdot8$ radians.
11. Do not penalise the omission or incorrect use of units.
12. $\bullet^9$ is only available as a result of using a valid strategy.
13. $\bullet^9$ is only available for a single angle.
14. For a correct answer with no working award 0/5.

<table>
<tr><th colspan="2">Commonly Observed Responses:</th></tr>
<tr>
<td>Candidate C - Calculating wrong angle

$\overrightarrow{QP}.\overrightarrow{QS} = 29$ •5 ✗
$\left|\overrightarrow{QP}\right| = \sqrt{50}$ •6 $\boxed{\checkmark 1}$
$\left|\overrightarrow{QS}\right| = \sqrt{26}$ •7 $\boxed{\checkmark 1}$
$\cos P\hat{Q}S = \dfrac{29}{\sqrt{50} \times \sqrt{26}}$ •8 $\boxed{\checkmark 1}$
$P\hat{Q}S = 36 \cdot 5$ •9 ✗ strategy incomplete

For candidates who continue, and use the angle found to evaluate the required angle, then all marks are available.</td>
<td>Candidate D- Calculating wrong angle

$\overrightarrow{PS}.\overrightarrow{QP} = -21$ •5 ✗
$\left|\overrightarrow{QP}\right| = \sqrt{50}$ •6 ✓
$\left|\overrightarrow{PS}\right| = \sqrt{18}$ •7 ✓
$\cos\theta = \dfrac{-21}{\sqrt{50} \times \sqrt{18}}$ •8 $\boxed{\checkmark 1}$
$\theta = 134 \cdot 4$ •9 ✗ strategy incomplete

For candidates who continue, and use the angle found to evaluate the required angle, then all marks are available.</td>
</tr>
<tr>
<td>Candidate E
From (a) $\overrightarrow{PQ} = -21\mathbf{i} - 14\mathbf{j} + \mathbf{k}$

$\overrightarrow{PQ}.\overrightarrow{PS} = -3$ •5 $\boxed{\checkmark 1}$
$\left|\overrightarrow{PQ}\right| = \sqrt{638}$ •6 $\boxed{\checkmark 1}$
$\left|\overrightarrow{PS}\right| = \sqrt{18}$ •7 ✓
$\cos Q\hat{P}S = \dfrac{-3}{\sqrt{638} \times \sqrt{18}}$ •8 $\boxed{\checkmark 1}$
$Q\hat{P}S = 91 \cdot 6$ •9 $\boxed{\checkmark 1}$</td>
<td>Candidate F
From (a) $\overrightarrow{PQ} = 21\mathbf{i} + 14\mathbf{j} - \mathbf{k}$

$\overrightarrow{PQ}.\overrightarrow{PS} = 3$ •5 $\boxed{\checkmark 1}$
$\left|\overrightarrow{PQ}\right| = \sqrt{638}$ •6 $\boxed{\checkmark 1}$
$\left|\overrightarrow{PS}\right| = \sqrt{18}$ •7 ✓
$\cos Q\hat{P}S = \dfrac{3}{\sqrt{638} \times \sqrt{18}}$ •8 $\boxed{\checkmark 1}$
$Q\hat{P}S = 88 \cdot 4$ •9 $\boxed{\checkmark 1}$</td>
</tr>
<tr>
<td>Candidate G

From (b) $\overrightarrow{PS} = -4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

$\overrightarrow{PQ}.\overrightarrow{PS} = 3$ •5 ✗
$\left|\overrightarrow{PQ}\right| = \sqrt{50}$ •6 ✓
$\left|\overrightarrow{PS}\right| = \sqrt{26}$ •7 $\boxed{\checkmark 1}$
$\cos Q\hat{P}S = \dfrac{3}{\sqrt{50} \times \sqrt{26}}$ •8 $\boxed{\checkmark 1}$
$Q\hat{P}S = 85 \cdot 2$ •9 $\boxed{\checkmark 1}$</td>
<td></td>
</tr>
</table>

Question	Generic Scheme	Illustrative Scheme	Max Mark
6.	Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$.		
	•¹ substitute appropriate double angle formula	•¹ $5\sin x - 4 = 2\left(1 - 2\sin^2 x\right)$	
	•² express in standard quadratic form	•² $4\sin^2 x + 5\sin x - 6 = 0$	
	•³ factorise	•³ $(4\sin x - 3)(\sin x + 2)$	
	•⁴ solve for $\sin x°$	•⁴ $\sin x = \frac{3}{4}$, (•⁴) $\sin x = -2$ (•⁵)	
	•⁵ solve for x	•⁵ $x = 0\cdot848,\ 2\cdot29$, $\cancel{\sin x = -2}$	5

Notes:

1. •¹ is not available for simply stating $\cos 2x = 1 - 2\sin^2 x$ with no further working.
2. In the event of $\cos^2 x° - \sin^2 x°$ or $2\cos^2 x° - 1$ being substituted for $\cos 2x$, •¹ cannot be awarded until the equation reduces to a quadratic in $\sin x°$.
3. Substituting $1 - 2\sin^2 A$ or $1 - 2\sin^2 \alpha$ for $\cos 2x$ at •¹ stage should be treated as bad form provided the equation is written in terms of x at •² stage. Otherwise, •¹ is not available.
4. '$= 0$' must appear by •³ stage for •² to be awarded. However, for candidates using the quadratic formula to solve the equation, '$= 0$' must appear at •² stage for •² to be awarded.
5. $5\sin x + 4\sin^2 x - 6 = 0$ does not gain •² unless •³ is awarded.
6. $\sin x = \dfrac{-5 \pm \sqrt{121}}{8}$ gains •³.
7. Candidates may express the equation obtained at •² in the form $4s^2 + 5s - 6 = 0$ or $4x^2 + 5x - 6 = 0$. In these cases, award •³ for $(4s - 3)(s + 2) = 0$ or $(4x - 3)(x + 2) = 0$. However, •⁴ is only available if $\sin x$ appears explicitly at this stage.
8. •⁴ and •⁵ are only available as a consequence of solving a quadratic equation.
9. •³, •⁴ and •⁵ are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •⁵ is not available to candidates who work in degrees and do not convert their solutions into radian measure.
11. Accept answers which round to $0\cdot85$ and $2\cdot3$ at •⁵ e.g. $\dfrac{49\pi}{180}, \dfrac{131\pi}{180}$.
12. Answers written as decimals should be rounded to no fewer than 2 significant figures.
13. Do not penalise additional solutions at •⁵.

Commonly Observed Responses:	
Candidate A $\bullet^1$ ✓ $\bullet^2$ ✓ $(4s-3)(s+2)=0$ $\bullet^3$ ✓ $s=\frac{3}{4},\ \ s=-2$ $\bullet^4$ ✗ $x=0\cdot848,\ 2\cdot29$ $\bullet^5$ ✓	**Candidate B** $\bullet^1$ ✓ $4\sin^2 x+5\sin x-6=0$ $\bullet^2$ ✓ $9\sin x-6=0$ $\bullet^3$ ✗ $\sin x=\frac{2}{3}$ $\bullet^4$ $\boxed{✓2}$ $x=0\cdot730,\ 2\cdot41$ $\bullet^5$ $\boxed{✓2}$
Candidate C $5\sin x-4=2\left(1-2\sin^2 x\right)$ $\bullet^1$ ✓ $4\sin^2 x+5\sin x=6$ $\bullet^2$ $\boxed{✓2}$ $\sin x\left(4\sin x+5\right)=6$ $\bullet^3$ $\boxed{✓2}$ $\sin x=6,\ \ 4\sin x+5=6$ $\bullet^4$ ✗ no solution, $\sin x=\frac{1}{4}$ $x=0\cdot253,\ 2\cdot89$ $\bullet^5$ ✗	**Candidate D** $5\sin x-4=2\left(1-2\sin^2 x\right)$ $\bullet^1$ ✓ $4\sin^2 x+5\sin x-6=0$ $\bullet^2$ ✓ $4\sin^2 x+5\sin x=6$ $\sin x\left(4\sin x+5\right)=6$ $\bullet^3$ $\boxed{✓2}$ $\sin x=6,\ \ 4\sin x+5=6$ $\bullet^4$ ✗ no solution, $\sin x=\frac{1}{4}$ $x=0\cdot253,\ 2\cdot89$ $\bullet^5$ ✗
Candidate E - reading $\cos 2x$ as $\cos^2 x$ $5\sin x-4=2\cos^2 x$ $\bullet^1$ ✗ $5\sin x-4=2\left(1-\sin^2 x\right)$ $2\sin^2 x+5\sin x-6=0$ $\bullet^2$ $\boxed{✓1}$ $\sin x=\frac{-5\pm\sqrt{73}}{4}$ $\bullet^3$ $\boxed{✓1}$ $\sin x=0\cdot886,\ \ \cancel{\sin x=-3\cdot386}$ $\bullet^4$ $\boxed{✓1}$ $x=1\cdot08,\ 2\cdot05$ $\bullet^5$ $\boxed{✓1}$	

Question		Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)	Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$.		
		•¹ write in differentiable form	•¹ $\ldots - 2x^{\frac{3}{2}}$ **stated or implied**	
		•² differentiate one term	•² $\frac{dy}{dx} = 6\ldots$ or $\frac{dy}{dx} = \ldots - 3x^{\frac{1}{2}}\ldots$	
		•³ complete differentiation and equate to zero	•³ $\ldots - 3x^{\frac{1}{2}} = 0$ or $6\ldots = 0$	
		•⁴ solve for x	•⁴ $x = 4$	4

Notes:

1. For candidates who do not differentiate a term involving a fractional index, either •² or •³ is available but not both.
2. •⁴ is available only as a consequence of solving an equation involving a fractional power of x.
3. For candidates who integrate one or other of the terms •⁴ is unavailable.

Commonly Observed Responses:

Candidate A - differentiating incorrectly		Candidate B - integrating the second term	
$y = 6x - 2x^{\frac{3}{2}}$	•¹ ✓	$y = 6x - 2x^{\frac{3}{2}}$	•¹ ✓
$\frac{dy}{dx} = 6 - 3x^{\frac{5}{2}}$	•² ✓	$\frac{dy}{dx} = 6 - \frac{4}{5}x^{\frac{5}{2}}$	•² ✓
$6 - 3x^{\frac{5}{2}} = 0$	•³ ✗	$6 - \frac{4}{5}x^{\frac{5}{2}} = 0$	•³ ✗
$x = 1{\cdot}32$	•⁴ $\boxed{✓1}$	$x = 2{\cdot}24$	•⁴ ✗

Question		Generic Scheme	Illustrative Scheme	Max Mark
7.	(b)	Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$.		
		•5 evaluate y at stationary point •6 consider value of y at end points •7 state greatest and least values	•5 8 •6 4 and 0 •7 greatest 8, least 0 **stated explicitly**	3

Notes:

4. The only valid approach to finding the stationary point is via differentiation. A numerical approach can only gain •6.
5. •7 is not available to candidates who do not consider both end points.
6. Vertical marking is not applicable to •6 and •7.
7. Ignore any nature table which may appear in a candidate's solution; however, the appearance of $(4,8)$ at a nature table is sufficient for •5.
8. Greatest $(4,8)$; least $(9,0)$ does not gain •7.
9. •5 and •7 are not available for evaluating y at a value of x, obtained at •4 stage, which lies outwith the interval $1 \le x \le 9$.
10. For candidates who **only** evaluate the derivative, •5,•6 and •7are not available.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
8.	(a)	Sequences may be generated by recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$. Show that $u_2 = 5k^2 - 20k - 20$.		
		•1 find expression for u_1 •2 find expression for u_2 and express in the correct form	•1 $5k - 20$ •2 $u_2 = k(5k-20) - 20$ leading to $u_2 = 5k^2 - 20k - 20$	2

Notes:

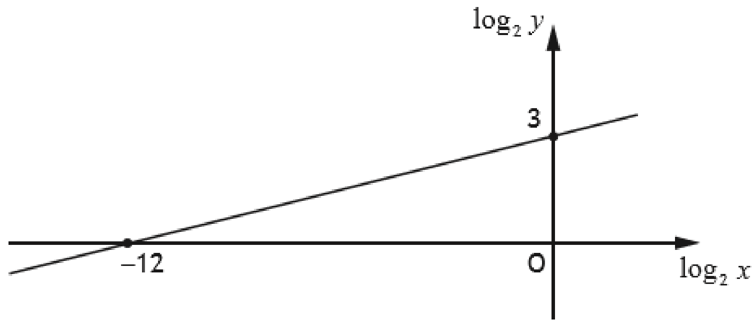
Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
8.	(b)	Determine the range of values of k for which $u_2 < u_0$.		
		•3 interpret information	•3 $5k^2 - 20k - 20 < 5$	
		•4 express inequality in standard quadratic form	•4 $5k^2 - 20k - 25 < 0$	
		•5 determine zeros of quadratic expression	•5 $-1, 5$	
		•6 state range with justification	•6 $-1 < k < 5$ with eg sketch or table of signs	4

Notes:

1. Candidates who work with an equation from the outset lose •3 and •4. However, •5 and •6 are still available.
2. At •5 do not penalise candidates who fail to extract the common factor or who have divided the quadratic inequation by 5.
3. •4 and •5 are only available to candidates who arrive at a quadratic expression at •3.
4. At •6 accept "$k > -1$ and $k < 5$" or "$k > -1$, $k < 5$" together with the required justification.
5. For a trial and error approach award 0/4.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
9.	Two variables, x and y, are connected by the equation $y = kx^n$. The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.  Find the values of k and n.		
	Method 1 •¹ state linear equation •² introduce logs •³ use laws of logs •⁴ use laws of logs •⁵ state k and n	**Method 1** •¹ $\log_2 y = \frac{1}{4}\log_2 x + 3$ •² $\log_2 y = \frac{1}{4}\log_2 x + 3\log_2 2$ •³ $\log_2 y = \log_2 x^{\frac{1}{4}} + \log_2 2^3$ •⁴ $\log_2 y = \log_2 2^3 x^{\frac{1}{4}}$ •⁵ $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	5
	Method 2 •¹ state linear equation •² use laws of logs •³ use laws of logs •⁴ use laws of logs •⁵ state k and n	**Method 2** •¹ $\log_2 y = \frac{1}{4}\log_2 x + 3$ •² $\log_2 y = \log_2 x^{\frac{1}{4}} + 3$ •³ $\log_2 \frac{y}{x^{\frac{1}{4}}} = 3$ •⁴ $\frac{y}{x^{\frac{1}{4}}} = 2^3$ •⁵ $k = 8,\ n = \frac{1}{4}$ or $y = 8x^{\frac{1}{4}}$	5

	Method 3 $\bullet^1$ introduce logs to $y = kx^n$ $\bullet^2$ use laws of logs $\bullet^3$ interpret intercept $\bullet^4$ use laws of logs $\bullet^5$ interpret gradient	**Method 3** **The equations at $\bullet^1$, $\bullet^2$ and $\bullet^3$ must be stated explicitly.** $\bullet^1$ $\log_2 y = \log_2 kx^n$ $\bullet^2$ $\log_2 y = n\log_2 x + \log_2 k$ $\bullet^3$ $\log_2 k = 3$ $\bullet^4$ $k = 8$ $\bullet^5$ $n = \frac{1}{4}$	**5**
	Method 4 $\bullet^1$ interpret point on log graph $\bullet^2$ convert from log to exp. form $\bullet^3$ interpret point and convert $\bullet^4$ substitute into $y = kx^n$ and evaluate k $\bullet^5$ substitute other point into $y = kx^n$ and evaluate n	**Method 4** $\bullet^1$ $\log_2 x = -12$ **and** $\log_2 y = 0$ $\bullet^2$ $x = 2^{-12}$ **and** $y = 2^0$ $\bullet^3$ $\log_2 x = 0,\ \log_2 y = 3$ $x = 1,\ y = 2^3$ $\bullet^4$ $2^3 = k \times 1^n \Rightarrow k = 8$ $\bullet^5$ $2^0 = 2^3 \times 2^{-12n}$ $\Rightarrow 3 - 12n = 0$ $\Rightarrow n = \frac{1}{4}$	**5**

Notes:

1. Markers must not pick and choose between methods. Identify the method which best matches the candidates approach.
2. Treat the omission of base 2 as bad form at $\bullet^1$ and $\bullet^3$ in Method 1, at $\bullet^1$ and $\bullet^2$ for Method 2 and Method 3, and at $\bullet^1$ in Method 4.
3. '$m = \frac{1}{4}$' or 'gradient $= \frac{1}{4}$' does not gain $\bullet^5$ in Method 3.
4. Accept 8 in lieu of 2^3 throughout.
5. In Method 4 candidates may use $(0,3)$ for $\bullet^1$ and $\bullet^2$ followed by $(-12,0)$ for $\bullet^3$.

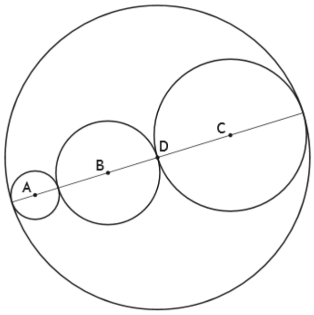
Commonly Observed Responses:	
Candidate A With no working. Method 3: $k=8$ •4 ✓ $n=\frac{1}{4}$ •5 ✓ **Award 2/5**	**Candidate B** With no working. Method 3: $n=8$ •4 ✗ $k=\frac{1}{4}$ •5 ✗ **Award 0/5**
Candidate C Method 3: $\log_2 k=3$ •3 ✓ $k=8$ •4 ✓ $n=\frac{1}{4}$ •5 ✓ **Award 3/5**	**Candidate D** Method 2: $\log_2 y=\frac{1}{4}\log_2 x+3$ •1 ✓ $\log_2 y=\log_2 x^{\frac{1}{4}}+3$ •2 ✓ $y=x^{\frac{1}{4}}+3$ •3 ✗ •4 ✗ $k=1,\ n=\frac{1}{4}$ •5 ✗ **Award 2/5**
Candidate E Method 2: $y=\frac{1}{4}x+3$ $\log_2 y=\frac{1}{4}\log_2 x+3$ •1 ✓ $\log_2 y=\log_2 x^{\frac{1}{4}}+3$ •2 ✓ $\frac{y}{x^{\frac{1}{4}}}=3$ •3 ^ •4 ✗ $y=3x^{\frac{1}{4}}$ •5 $\boxed{✓1}$ **Award 3/5**	

Question	Generic Scheme	Illustrative Scheme	Max Mark
10. (a)	Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear.		
	Method 1 •¹ calculate m_{AB} •² calculate m_{BC} •³ interpret result and state conclusion	**Method 1** •¹ $m_{AB}=\frac{3}{9}=\frac{1}{3}$ see Note 1 •² $m_{BC}=\frac{5}{15}=\frac{1}{3}$ •³ $\ldots\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 2 •¹ calculate an appropriate vector e.g. $\overrightarrow{AB}$ •² calculate a second vector e.g. $\overrightarrow{BC}$ and compare •³ interpret result and state conclusion	**Method 2** •¹ $\overrightarrow{AB}=\begin{pmatrix}9\\3\end{pmatrix}$ see Note 1 •² $\overrightarrow{BC}=\begin{pmatrix}15\\5\end{pmatrix}\therefore\ \overrightarrow{AB}=\frac{3}{5}\overrightarrow{BC}$ •³ $\ldots\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 3 •¹ calculate m_{AB} •² find equation of line and substitute point •³ communication	**Method 3** •¹ $m_{AB}=\frac{3}{9}=\frac{1}{3}$ •² eg, $y-1=\frac{1}{3}(x-2)$ leading to $6-1=\frac{1}{3}(17-2)$ •³ since C lies on line A, B and C are collinear	

Notes:

1. At •¹ and •² stage, candidates may calculate the gradients/vectors using any pair of points.
2. •³ can only be awarded if a candidate has stated "parallel", "common point" and "collinear".
3. Candidates who state "points A, B and C are parallel" or "m_{AB} and m_{BC} are parallel" do not gain •³.

Commonly Observed Responses:		
Candidate A	**Candidate B**	**Candidate C**
$m_{AB} = \frac{3}{9} = \frac{1}{3}$ $\bullet^1$ ✓ $m_{BC} = \frac{5}{15}$ $\bullet^2$ ^ $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. $\bullet^3$ $\boxed{\checkmark 1}$	$\begin{pmatrix}9\\3\end{pmatrix}$ $\bullet^1$ ✓ $\begin{pmatrix}15\\5\end{pmatrix}$ $\therefore\ \overrightarrow{AB} = \frac{5}{3}\overrightarrow{BC}$ $\bullet^2$ ✗ $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. $\bullet^3$ $\boxed{\checkmark 1}$	$\overrightarrow{AB} = \begin{pmatrix}9\\3\end{pmatrix}$ $\bullet^1$ ✓ $\overrightarrow{BC} = \begin{pmatrix}15\\5\end{pmatrix} = 5\begin{pmatrix}3\\1\end{pmatrix}$ and $\begin{pmatrix}9\\3\end{pmatrix} = 3\begin{pmatrix}3\\1\end{pmatrix}$ $\bullet^2$ ✓ $\therefore\ \overrightarrow{AB} = \frac{5}{3}\overrightarrow{BC}$ ignore working subsequent to correct statement at $\bullet^2$. $\Rightarrow$ AB and BC are parallel, B is a common point, hence A, B and C are collinear. $\bullet^3$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
10. (b)	Three circles with centres A, B and C are drawn inside a circle with centre D as shown.  The circles with centres A, B and C have radii r_A, r_B and r_C respectively. • $r_A = \sqrt{10}$ • $r_B = 2r_A$ • $r_C = r_A + r_B$ Determine the equation of the circle with centre D.		
	•4 find radius	•4 $6\sqrt{10}$	
	•5 determine an appropriate ratio	•5 e.g. 2:3 or $\frac{2}{5}$ (using B and C) or 3:5 or $\frac{8}{5}$ (using A and C)	
	•6 find centre	•6 $(8,3)$	
	•7 state equation of circle	•7 $(x-8)^2+(y-3)^2=360$	4

Notes:

4. Where the correct centre appears without working •5 is lost, •6 is awarded and •7 is still available. Where an incorrect centre or radius **from working** then •7 is available. However, if an incorrect centre or an incorrect radius appears ex nihilo •7 is not available.
5. Do not accept $\left(6\sqrt{10}\right)^2$ for •7.

Commonly Observed Responses:

Candidate D		Candidate E	
Radius $=6\sqrt{10}$	•4 ✓	Radius $=3\sqrt{10}$	•4 ✗
Interprets D as midpoint of BC	•5 ✗	Interprets D as midpoint of AC	•5 ✗
Centre D is $(9\cdot5,\ 3\cdot5)$	•6 ✓2	Centre D is $(5,\ 2)$	•6 ✓2
$(x-9\cdot5)^2+(y-3\cdot5)^2=360$	•7 ✓1	$(x-5)^2+(y-2)^2=90$	•7 ✓1
Candidate F		**Candidate G**	
Radius $=\sqrt{10}$	•4 ✗	Radius $=6\sqrt{10}$	•4 ✓
Interprets D as midpoint of AC	•5 ✗	$\frac{CD}{BD}=\frac{3}{2}$ or simply $\frac{3}{2}$	•5 ✓
Centre D is $(5,\ 2)$	•6 ✓2	Centre D is $(11,\ 4)$	•6 ✗
$(x-5)^2+(y-2)^2=10$	•7 ✓2	$(x-11)^2+(y-4)^2=360$	•7 ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
11. (a)	Show that $\frac{\sin 2x}{2\cos x} - \sin x\cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$.		
	Method 1 •1 substitute for $\sin 2x$ •2 simplify and factorise •3 substitute for $1-\cos^2 x$ and simplify	**Method 1** •1 $\frac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •2 $\sin x\left(1-\cos^2 x\right)$ •3 $\sin x \times \sin^2 x$ leading to $\sin^3 x$	3
	Method 2 •1 substitute for $\sin 2x$ •2 simplify and substitute for $\cos^2 x$ •3 expand and simplify	**Method 2** •1 $\frac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •2 $\sin x - \sin x\left(1-\sin^2 x\right)$ •3 $\sin x - \sin x + \sin^3 x$ leading to $\sin^3 x$	3

Notes:

1. •1 is not available to candidates who simply quote $\sin 2x = 2\sin x\cos x$ without substituting into the expression given on the LHS. See Candidate B
2. In method 2 where candidates attempt •1 and •2 in the same line of working •1 may still be awarded if there is an error at •2.
3. •3 is not available to candidates who work throughout with A in place of x.
4. Treat multiple attempts which are not scored out as different strategies, and apply General Marking Principle (r).
5. On the appearance of LHS $= 0$, the first available mark is lost; however, any further marks are still available.

Commonly Observed Responses:

Candidate A	Candidate B
$\frac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x = \sin^3 x$ •1 ✓ $\sin x - \sin x\cos^2 x = \sin^3 x$ •2 ^ $1-\cos^2 x = \sin^2 x$ •3 ✗ $\sin^2 x = \sin^2 x$ In proving the identity, candidates must work with both sides independently. i.e. in each line of working the LHS must be equivalent to the line above.	$\text{LHS} = \frac{\sin 2x}{2\cos x} - \sin x\cos^2 x$ $\frac{\sin 2x}{2\cos x} = \frac{2\sin x\cos x}{2\cos x}$ $= \sin x$ $\sin x - \sin x\cos^2 x$ •1 ✓ $\sin x\left(1-\cos^2 x\right)$ •2 ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
11. (b)	Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$.		
	•⁴ know to differentiate $\sin^3 x$ •⁵ start to differentiate •⁶ complete differentiation	•⁴ $\dfrac{d}{dx}(\sin^3 x)$ •⁵ $3\sin^2 x ....$ •⁶ $... \times \cos x$	**3**
Notes:			
Commonly Observed Responses:			

[END OF MARKING INSTRUCTIONS]