

National Qualifications 2017

X747/76/11

Mathematics Paper 1 (Non-Calculator)

Amended Marking Instructions

FRIDAY, 5 MAY

Strictly Confidential

These instructions are **strictly confidential** and, in common with the scripts you will view and mark, they must never form the subject of remark of any kind, except to Scottish Qualifications Authority staff.



General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

- (a) Marks for each candidate response must <u>always</u> be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.
- (b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.
- (c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.
- (d) Credit must be assigned in accordance with the specific assessment guidelines.
- (e) One mark is available for each •. There are no half marks.
- (f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.
- (g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.
- (h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.
- (i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6 \times 6 = 12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment (j).

(j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg



(k) Horizontal/vertical marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

Horizontal:
$${}^{6}x = 2$$
 and $x = -4$
 ${}^{6}y = 5$ and $y = -7$
 ${}^{6}y = 5$ and $y = -7$
 ${}^{6}x = 2$ and $y = 5$
 ${}^{6}x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

(I) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

 $\frac{15}{12} \text{ must be simplified to } \frac{5}{4} \text{ or } 1\frac{1}{4} \qquad \frac{43}{1} \text{ must be simplified to } 43$ $\frac{15}{0\cdot 3} \text{ must be simplified to } 50 \qquad \frac{\frac{4}{5}}{3} \text{ must be simplified to } \frac{4}{15}$ $\sqrt{64} \text{ must be simplified to } 8^*$

*The square root of perfect squares up to and including 100 must be known.

(m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

- (n) Unless specifically mentioned in the marking instructions, the following should not be penalised:
 - Working subsequent to a correct answer
 - Correct working in the wrong part of a question
 - Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
 - Omission of units
 - Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3+2x^2+3x+2)(2x+1)$ written as $(x^3+2x^2+3x+2)\times 2x+1$

 $2x^4 + 4x^3 + 6x^2 + 4x + x^3 + 2x^2 + 3x + 2$ written as $2x^4 + 5x^3 + 8x^2 + 7x + 2$ gains full credit

- Repeated error within a question, but not between questions or papers
- (o) In any 'Show that...' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.
- (p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.
- (q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.
- (r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Key E-marking Information

Response Overview: Before you start marking you must check every page of the candidate's response. This is to identify :				
- If the candidate has written in any unexpected areas of their answer booklet				
- If the script is legible and that it does not require to be re-scanned				
- If there is an additional answer booklet/answer sheet, you need to check that it belongs to the same candidate				
 If the candidate has continued an answer to a question at the back or in a different location in the booklet 				
- The presence of any non-script related objects.				
No Response (NR): Where a candidate has not attempted to answer a question use No Response (NR).				
Candidates are advised in the 'Your Exams' booklet to cross out any rough work when they have made a final copy. However, crossed-out work must be marked if the candidate has not made a second attempt to answer the question. Where a second attempt has been made, the crossed-out answers should be ignored.				
Zero marks should only be applied when a candidate has attempted the question/item and their response does not attract any marks.				
Additional Objects: Where a candidate has used an additional answer sheet this is known as an additional object. When you open a response that contains an additional object, a popup message will advise you of this. You are required to add a minimum of one annotation on every additional page to confirm that you have viewed it. You can use any of the normal marking appotations such as tick/cross				
or the SEEN annotation to confirm that you have viewed the page. You will not be able to submit a script with an additional object, until every additional page contains an annotation.				
Link tool : The Link tool allows you to link pages/additional objects to a particular question item on a response.				
 In "Full Response View": Check which question the candidate's answer relates to Click on the question in the marks display panel On the left hand side, select the Link Page check box beneath the thumbnail for the page. Once all questions have been linked, click 'Structured Response View' to start marking. When you select a linked question item in the mark input panel, the linked page(s) are displayed. 				

The following is a list of exceptions which can be raised for this assessment. Please ensure that you are familiar with these in order that you select the appropriate exception for the issue encountered, take the Marker Action advised and note that all exceptions must be raised in RM Assessor **before** the response is submitted.

Exception	Description	Marker Action
Image Rescan request	You should raise this exception when you are unable to mark the candidate's response because the image you are viewing is of poor quality and you believe a rescan would improve the quality of the image, therefore allowing you to mark the response. Some examples of this include scan lines, folded pages or image skew.	If image is to be rescanned RM will remove the script from your work list. RM will inform you of this. No further action is required from you. If RM do not think that a rescan will improve the image then you should raise the script as an Undecipherable exception.
Offensive Content	You should raise this exception when the candidate's response contains offensive, obscene or frivolous material. Examples of this include vulgarity, racism, discrimination or swearing.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner
Incorrect Question Paper	You should raise this exception when the image you are viewing does not correspond to the paper you are marking.	Raise script as an exception. Do not mark the image until SQA have contacted you and provided advice.
Undecipherable	You should raise this exception when you are unable to mark the candidate's response because the response cannot be read and you do not believe that a re-scan will improve the situation because the problem is with the writing and not the image. Some examples of this include poor handwriting and overwriting the original response.	Raise script as an exception to alert SQA staff. SQA will contact you to advise further action and when to close the exception.
Answer Outside of Guidance	You should raise this exception when you are unable to mark because the Marking Instructions do not cover this candidate's response.	Act on advice from Team Leader.
Concatenated Script Exception	You should raise this exception when the additional object(s) ie pages or scripts displayed do not belong to the candidate you are marking. You need not use this exception if the additional objects are transcriptions or additional pages submitted for the candidate.	Raise script as an exception. You can mark the correct script then review the marks once the erroneous script has been removed. SQA will contact you and advise of any actions and when to close the exception.

Exception	Description	Marker Action
Non-Script Object	You should raise this exception when the additional object displayed does not relate to the script you are marking OR If you think that there is a piece of the candidate's submission missing eg because the script you are marking contains only responses to diagrams or tables and you suspect there should be a further script or word processed response or the response on the last page ends abruptly.	Raise script as an exception. Write a short report to advise the issue and continue to mark. SQA will contact you and advise of any actions and when to close the exception.
Candidate Welfare Concern	You should raise this exception when you have concerns about the candidate's well-being or welfare when marking any examination script or if coursework and there is no tick on the flyleaf to identify these issues are being or have been addressed by the centre.	Telephone the Child Welfare Contact on 0345 213 6587 as early as possible on the same or next working day for further instruction. Click on the Candidate Welfare Concern button and complete marking the script and submit the mark as normal.
Malpractice	You should raise this exception when you suspect wrong doing by the candidate. Examples of this include plagiarism or collusion.	Raise this exception and enter a short report in the comments box. You should then mark the script and submit in the normal manner

Annotations			
Annotation	Annotation	Instructions on use of annotation	
	Name		
	Tick	A tick should be placed on the script at the point where a mark is	
		awarded (or at the end of that line of working).	
	Tick 1	A tick 1 should be used to indicate "correct" working where a mark	
		is awarded as a result of follow through from an error.	
		A tick 2 should be used to indicate correct working which is irrelevant	
✓ 2	Tick 2	or insufficient to award any marks. This should also be used for	
		working which has been eased.	
*	Cross	A cross is used to indicate where a mark has not been awarded.	
	0000		
	Omission	A "roof" should be used to show that something is missing, such as	
	UTITISSION	part of a solution or a crucial step in the working.	

One of the above annotations must appear for each available mark e.g. a question worth 4 marks must have 4 of the above annotations.

SEEN	SEEN	This annotation should be used by the marker on a blank page to show that they have viewed this page and confirm it contains no candidate response.
	Horizontal wavy line	A tilde should be used to indicate a minor error which is not being penalised, e.g. bad form (bad form only becomes bad form if subsequent working is correct).
	Highlight	This is used to highlight or underline an error.

Specific Marking Instructions for each question

Question	Generic Scheme	Illustrative Scheme	Max Mark
1. (a)	Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$.		
	Evaluate $f(g(0))$.		
	• ¹ evaluate expression • ¹ 10		1
Notes:			
Commonly Obs	served Responses:		

Question	Generic Scheme	Illustrative Scheme	Max Mark
1. (b)	Find an expression for $g(f(x))$.		
	• ² interpret notation	$\bullet^2 g(5x)$	
	• ³ state expression for $g(f(x))$	$\bullet^3 2\cos 5x$	2
Notes:			
 For 2cos5x without working, award both •² and •³. Candidates who interpret the composite function as either g(x)×f(x) or g(x)+f(x) do not gain any marks. g(f(x))=10cos x award •². However, 10cos x with no working does not gain any marks. g(f(x)) leading to 2cos(5x) followed by incorrect 'simplification' of the function award •² and •³. 			
Commonly Obs	served Responses:		
$\begin{bmatrix} \text{Candidate A} \\ g(f(x)) = 2 \cos \alpha \\ = 10 \cos \alpha \end{bmatrix}$	s(5x) = 2 = 2 = 3 s(x) = 2 = 2 = 3		

Question	Generic Scheme Illustrative Scheme			
2.	The point $P(-2, 1)$ lies on the circ	le $x^2 + y^2 - 8x - 6y - 15 = 0$.		
	Find the equation of the tangent t	o the circle at P.		
	• ¹ state coordinates of centre	• ¹ (4, 3)		
	• ² find gradient of radius	• ² $\frac{1}{3}$		
	• ³ state perpendicular gradient	• ³ -3		
	• ⁴ determine equation of tangent	etermine equation of tangent $e^4 y = -3x - 5$		
Notes:				
1. Accept $\frac{2}{6}$ for \bullet^2 .				
2. The perpendicular gradient must be simplified at \bullet^3 or \bullet^4 stage for \bullet^3 to be available. 3. \bullet^4 is only available as a consequence of trying to find and use a perpendicular gradient. 4. At \bullet^4 , accept $y+3x+5=0$, $y+3x=-5$ or any other rearrangement of the equation where				

the constant terms have been simplified. Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
3.	Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$.		
	• ¹ start to differentiate	• ¹ $12(4x-1)^{11}$	
	• ² complete differentiation	• ² ×4	2
Notes:			
1. \bullet^2 is awarded for correct application of the chain rule.			
Commonly Obs	served Responses:		
Candidate A	C	andidate B	
$\frac{dy}{dx} = 12(4x-1)^{11} \times 4 \bullet^{1} \checkmark \bullet^{2} \checkmark$ $\frac{dy}{dx} = 36(4x-1)^{11}$		$\frac{dy}{dx} = 36(4x - 1)^{11} \bullet^1 \times \bullet^2 \times$	
Working subse	quent to a correct answer:		
General Marki	ng Principle (n)		

Question	Generi	ic Scheme Illustrative Scheme			Max Mark
4.	Find the value of	k for which the equati	on $x^2 + 4x + (k)$	(x-5)=0 has equal roots.	
	Me ¹ use the discr	thod 1 Timinant	• $4^2 - 4 \times 12^{-1}$	Method 1 $\times (k-5)$	
	• ² apply conditi	on and simplify	• ² 36-4 $k =$	0 or $36 = 4k$	
	• ³ determine th	e value of <i>k</i>	• ³ $k = 9$		3
	Method 2 • ¹ communicate and express in factorised form $x^{2} + 4x + (k-5) = (x)$		Method 2 ots $(k-5) = (x+2)^2$		
	• ² expand and compare		• ² $x^2 + 4x +$	4 leading to $k-5=4$	
	\bullet^3 determine the value of k		• ³ $k=9$		
Notes: 1. At the • ¹ star is bracketed 2. In Method 1 • ³ is unavail	age, treat 4 ² – 4 I in their next lin if candidates use able.	$\times 1 \times k - 5$ as bad form e of working. See C e any condition othe	m only if the andidates A r than 'discr	candidate treats ' $k-5$ and B. iminant = 0 ' then \bullet^2 is lo	' as if it ost and
Candidate A	served Response	Candidate B			
$4^2 - 4 \times 1 \times k - 5$	• ¹ ✓	$4^2 - 4 \times 1 \times k - 5$	• ¹ x		
36 - 4k = 0	• ² √	11 - 4k = 0	● ² ✓ 1		
<i>k</i> = 9	• ³ •	$k = \frac{11}{4} \qquad \bullet^3 \checkmark 1$			

Question	Generic Scheme	Illustrative Scheme	Max Mark
5. (a)	Vectors u and v are $\begin{pmatrix} 5\\1\\-1 \end{pmatrix}$ and $\begin{pmatrix} 3\\-8\\-8\\6 \end{pmatrix}$	respectively.	
	Evaluate u.v .		
	• ¹ evaluate scalar product	• ¹ 1	1
Notes:			
Commonly Obs	served Responses:		

Question	Generic Scheme	Illustrative Scheme	Max Mark
5. (b)	$\frac{\pi}{3}$		
	Vector w makes an angle of $\frac{\pi}{3}$ wi	th u and $ \mathbf{w} = \sqrt{3}$.	
	Calculate u.w .		
	•² calculate u	• ² \sqrt{27}	
	• ³ use scalar product	$\bullet^3 \sqrt{27} \times \sqrt{3} \times \cos \frac{\pi}{3}$	
	• ⁴ evaluate u .w	• $\frac{9}{2}$ or 4.5	3
Notes:			
1. Candidates	who treat negative signs with a lack	of rigour and arrive at $\sqrt{27}$ gain \bullet^2 .	
2. Surds must	be fully simplified for • to be award served Responses:	ea	
Commonly OD.			

Question	Generic Scheme	Illustrative Scheme	Max Mark	
6.	A function, h, is defined by $h(x) =$	$x^3 + 7$, where $x \in \mathbb{R}$.		
	Determine an expression for $h^{-1}(x)$.			
	Method 1	Method 1		
	• ¹ equate composite function to <i>x</i>	• ¹ $h(h^{-1}(x)) = x$		
	• ² write $h(h^{-1}(x))$ in terms of $h^{-1}(x)$	• ² $(h^{-1}(x))^3 + 7 = x$		
	• ³ state inverse function	• ³ $h^{-1}(x) = \sqrt[3]{x-7}$ or $h^{-1}(x) = (x-7)^{\frac{1}{3}}$		
	Nothod 2	$\frac{h^{2}(x) = (x - 7)^{3}}{4}$	3	
	• ¹ write as $y = x^3 + 7$ and start to rearrange	• $y - 7 = x^3$		
	• ² complete rearrangement	• ² $x = \sqrt[3]{y-7}$		
	• ³ state inverse function	• ³ $h^{-1}(x) = \sqrt[3]{x-7}$ or		
		$h^{-1}(x) = (x-7)^3$	3	
	Method 3	Method 3		
	• ¹ interchange variables	• ¹ $x = y^3 + 7$		
	• ² complete rearrangement	• ² $y = \sqrt[3]{x-7}$		
	• ³ state inverse function	• ³ $h^{-1}(x) = \sqrt[3]{x-7}$ or		
Mataa		$h^{-1}(x) = (x-7)^3$	3	
Notes:				
1. $y = \sqrt[3]{x-7} \left(\text{ or } y = (x-7)^{\overline{3}} \right)$ does not gain • ³ .				
2. At \bullet^3 stage,	2. At • ³ stage, accept h^{-1} expressed in terms of any dummy variable e.g. $h^{-1}(y) = \sqrt[3]{y-7}$.			
3. $h^{-1}(x) = \sqrt[3]{x}$	$\overline{-7}$ or $h^{-1}(x) = (x-7)^{\frac{1}{3}}$ with no wor	king gains 3/3.		

Commonly Observed Responses:	
Candidate A	
$x \to x^3 \to x^3 + 7 = h$ ^3 $\to +7$ $\therefore -7 \to \sqrt[3]{}$	(x) $\bullet^1 \checkmark$ awarded for knowing to perform the inverse operations in reverse order
$\sqrt[3]{x-7}$	• ² ✓
$h^{-1}(x) = \sqrt[3]{x-7}$	•3√
Candidate B - BEWARE	Candidate C
$h'(x) = \dots \bullet^3 *$	$h^{-1}(x) = \sqrt[3]{x} - 7 \cdot \cdot^{3}x$ With no working 0/3

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Question	Generic Scheme	Illustrative Scheme	Max Mark		
7.	A (-3, 5), B (7, 9) and C (2, 11) are t	he vertices of a triangle.			
	Find the equation of the median t	hrough C.			
	 ¹ find midpoint of AB ² demonstrate the line is 	• ¹ (2,7) • ² m_{median} undefined			
	 •³ state equation 	• ³ $x = 2$	3		
Notes:					
1. $m_{median} = \frac{\pm 4}{0}a$	alone is not sufficient to gain \bullet^2 . Ca	ndidates must use either 'vertical' or	ſ		
'undefined'	. However \bullet^3 is still available.				
2. ' $m_{median} = \frac{4}{0}$	× ' ' $m_{median} = \frac{4}{0}$ impossible' ' $m_{median} = \frac{4}{0}$	1 infinite' are not acceptable for	• ² .		
However, if still availab	However, if these are followed by either 'vertical' or 'undefined' then award \bullet^2 , and \bullet^3 is still available.				
3. ' $m_{median} = \frac{4}{0}$	3. ' $m_{median} = \frac{4}{0} = 0$ undefined' ' $m_{median} = \frac{1}{0}$ undefined' are not acceptable for \bullet^2 .				
4. • ³ is not available as a consequence of using a numeric gradient; however, see notes 5 and 6. 5. For candidates who find an incorrect midpoint (a,b) , using the coordinates of A and B and					
find the 'median' through C without any further errors award 1/3. However, if $a = 2$, then both \bullet^2 and \bullet^3 are available.					
6. For candidates who find $15y = 2x + 121$ (median through B) or $3y = 2x + 21$ (median through A) award 1/3.					

Commonly Observed Responses:						
Candidate A		Candidate B		Candidate C		
(2,7) • ¹	✓	(2,7)	● ¹ ✓	(2,7)	● ¹ ✓	
$m = \frac{4}{0}$		$m = \frac{4}{0}$		$m = \frac{4}{0}$	• ² ^	
$=0$ undefined \bullet^2	x	=0	• ² *	x - 7 = 4(x - 2)		
x=2 • ³	√ 1	<i>y</i> = 7	● ³ √ 2	$y - 7 = \frac{1}{0}(x - 2)$		
				0 = 4x - 8		
				x = 2	• ³ ¥	
Candidate D		Candidate E				
(2,7) •1.	✓	(2,7)	● ¹ ✓			
Median passes through and $(2,11)$ • ²	(2,7)	Both coordinates have value $2 \Rightarrow$ vertical l	ine $e^2 \checkmark$			
x=2 • ³	₃ ✓1	<i>x</i> = 2	•3√			

Question	Generi	ic Scheme		Illust	trative Scheme	Max Mark
8.	Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$.			when $t = 5$.		
	• ¹ write in diffe	erentiable form		• $\frac{1}{2}t^{-1}$		
	• ² differentiate			• ² $-\frac{1}{2}t^{-2}$		
	• ³ evaluate derivative			• $^{3} -\frac{1}{50}$		3
Notes:						
 Candidates •² is only av 	who arrive at an ailable for differ	expression cont entiating a tern	tainir n con	ng more than Itaining a neg	one term at \bullet^1 award C gative power of t .	/3.
Commonly Obs	served Response	s:				
Candidate A		Candidate B			Candidate C	
$2t^{-1}$ • $-2t^{-2}$ • 2	¹ x 2 √1	$2t^{-1} \qquad \bullet^{1} \mathbf{x}$ $-2t^{-2} \qquad \bullet^{2} \mathbf{\checkmark}$	1		$-\frac{1}{2}t^{-2} \bullet^{1} \checkmark \text{ implied } t$ $-\frac{1}{2} \bullet^{3} \checkmark$	oy ● ² ✓
$\left -\frac{2}{25} \right $	3 ✓1	$-\frac{1}{50}$ $\bullet^3 \times$			50 •••	

Candidate D	Candidate E	Candidate F Bad form of chain	Candidate G
$(2t)^{-1}$ $\bullet^1 \checkmark$	$(2t)^{-1}$ • ¹ \checkmark	rule	$2t^{-1}$ • ¹ ×
$-(2t)^{-2}$ • ² *	$-(2t)^{-2}$ • ² ×	2t ⁻¹ ● ¹ ✓	$-2t^{-2} \times 2$ $\bullet^2 \times$
$-\frac{1}{100}$ \bullet^{3}	$-\frac{2}{25}$ $\bullet^3 x$	$-2t^{-2} \times 2 \qquad \bullet^2 \checkmark$	$-\frac{4}{25}$ $\bullet^{3}\sqrt{1}$
		$-\frac{1}{50}$ • ³	

Question	Generic Scheme	Illustrative Scheme	Max Mark			
9. (a)	A sequence is generated by the recurre	nce relation $u_{n+1} = mu_n + 6$ where m is a constant	nt.			
	Given $u_1 = 28$ and $u_2 = 13$, find the value	ue of <i>m</i> .				
	• ¹ interpret information	• ¹ $13 = 28m + 6$ stated explicitly				
	\bullet^2 state the value of <i>m</i>	or in a rearranged form				
		• ² $m = \frac{1}{4}$ or $m = 0.25$				
			2			
Notes:						
1. Stating 'm	1. Stating ' $m = \frac{1}{4}$ ' or simply writing ' $\frac{1}{4}$ ' with no other working gains only \bullet^2 .					
Commonly Obs	served Responses:					
Candidate A		Candidate B				
$13 = 28u_n + 6$	• ¹ ×	28 = 13m + 6 • ¹ x				
$u_n = \frac{1}{4}$	• ² <u>√</u> 1	$m = \frac{22}{13} \qquad \qquad \bullet^2 \checkmark 1$				

Quest	tion	Generic Scheme	Illustrative Scheme	Max Mark	
9.(b) ((i)	Explain why this sequence approaches a limit as $n \rightarrow \infty$.			
		• ³ communicate condition for limit to exist	• ³ a limit exists as the recurrence relation is linear and $-1 < \frac{1}{2} < 1$		
			4	1	
Notes:					
2. For •	• ³ acce	pt:			
a	any of	$-1 < \frac{1}{4} < 1$ or $\left \frac{1}{4}\right < 1$ or $0 < \frac{1}{4}$	<1with no further comment;		
0	or state	ments such as:			
	$\frac{1}{4}$ lies	between -1 and 1" or " $\frac{1}{4}$ is	a proper fraction "		
3. \bullet^3 is	not ava	ailable for:			
-	$-1 \le \frac{1}{4} \le 1$ or $\frac{1}{4} < 1$				
0	or statements such as:				
	"It is between -1 and 1." or " $\frac{1}{4}$ is a fraction."				
4. Cand	4. Candidates who state $-1 < m < 1$ can only gain \bullet^3 if it is explicitly stated				
that	that $m = \frac{1}{4}$ in part (a).				
5. Do not accept ' $-1 < a < 1$ ' for \bullet^3 .					
Commonly Observed Responses:					
Candida	te C	C	andidate D		
(a) <i>r</i> (b) -	$m = \frac{1}{4}$ $-1 < m < \infty$		a) $\frac{1}{4}$ • ¹ \checkmark • ² b) -1 < m < 1 • ³ \checkmark	√	

C	uestic	n	Generic Scheme Illustrative Scheme		Max Mark
9.	(b)	(ii)	Calculate this limit.		
			• ⁴ know how to calculate limit	• $\frac{6}{1-\frac{1}{4}}$ or $L = \frac{1}{4}L + 6$	
			 ⁵ calculate limit 	• ⁵ 8	2
Not	es:				
6.	Do not	acce	ept $L = \frac{b}{1 - a}$ with no further workin	g for ∙ ⁴ .	
7.	● ⁴ and	● ⁵ ar	e not available to candidates who c	onjecture that $L = 8$ following the	
	calcul	ation	of further terms in the sequence.		
8.	For L	=8 v	with no working, award $0/2$.		
9.	For ca answe	ndida r in p	ates who use a value of m appearing part (a) \bullet^4 and \bullet^5 are not available.	g ex nihilo or which is inconsistent wi	th their
Cor	nmonl	y Obs	served Responses:		
Car	ndidate	e E - 1	no valid limit		
(a)	m = 4		• ¹ x		
(b)	$L = \frac{\epsilon}{1 - \epsilon}$	5	● ⁴ <mark>√1</mark>		
	L = -2		•5 🗴		

Question	Generic Scheme	Illustrative Scheme	Max Mark
10. (a)	Two curves with equations $y = x^3 - 4x^2$ in the diagram.	$x^{2} + 3x + 1$ and $y = x^{2} - 3x + 1$ intersect as	s shown
	y $y = x^3$	$-4x^2 + 3x + 1$ $y = x^2 - 3x + 1$	
	Calculate the shaded area.	Method 1	
	• ¹ know to integrate between appropriate limits	$\bullet^1 \int_0^2 \dots dx$	
	• ² use "upper - lower"	• ² $\int_{0}^{2} \left(\left(x^{3} - 4x^{2} + 3x + 1 \right) - \left(x^{2} - 3x + 1 \right) \right)$	
	• ³ integrate	• $\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2$	
	• ⁴ substitute limits	• $\left(\frac{2^4}{4} - \frac{5 \times 2^3}{3} + 3 \times 2^2\right) - (0)$	
	• ⁵ evaluate area	• ⁵ $\frac{8}{3}$	5

		Method 2
		Method 2
	 ¹ know to integrate between appropriate limits for both integrals 	• $\int_{0}^{2} \dots dx$ and $\int_{0}^{2} \dots dx$
	• ² integrate both functions	• ² $\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2} + x$ and $\frac{x^3}{3} - \frac{3x^2}{2} + x$
	• ³ substitute limits into both functions	• ³ $\left(\frac{2^4}{4} - \frac{4(2^3)}{3} + \frac{3(2^2)}{2} + 2\right) - 0$ and $\left(\frac{2^3}{3} - \frac{3(2^2)}{2} + 2\right) - 0$
	• ⁴ evaluation of both functions	• ⁴ $\frac{4}{3}$ and $\frac{-4}{3}$
	• ⁵ evidence of subtracting areas	• ⁵ $\frac{4}{3} - \frac{-4}{3} = \frac{8}{3}$
Notes:		
 ●¹ is n Treat obtair 	not available to candidates who omit ' dx '. the absence of brackets at \bullet^2 stage as bac ned at \bullet^3 . See Candidates A and B.	I form only if the correct integral is
3. Where unava	e a candidate differentiates one or more t ailable.	erms at \bullet^3 , then \bullet^3 , \bullet^4 and \bullet^5 are
4. Accep	ot unsimplified expressions at \bullet^3 e.g. $\frac{x^4}{2}$ –	$\frac{4x^3}{3} + \frac{3x^2}{3} + x - \frac{x^3}{3} + \frac{3x^2}{3} - x$.
5 Do no	$\frac{1}{4}$	3 2 3 2
J. Do not penalise the inclusion of $+c$. 6. Condidatos who substituto limits without integrating do not gain $a^3 a^4$ or a^5		

- 6. Candidates who substitute limits without integrating do not gain \bullet^3 , \bullet^4 or \bullet^5 .
- 7. \bullet^4 is only available if there is evidence that the lower limit '0' has been considered.
- 8. Do not penalise errors in substitution of x = 0 at \bullet^3 .



Question	Generic Scheme	Illustrative Scheme	Max Mark		
10. (b)	The line passing through the points of intersection of the curves has equation $y = 1 - x$				
	y y y = $x^3 - 4x^2 + 3x + 1$ y = $x^2 - 3x + 1$ y = $1 - x$				
	Determine the fraction of the shaded a	rea which lies below the line $y = 1 - x$.			
	• ⁶ use "line - quadratic"	Method 1 • ⁶ $\int ((1-x)-(x^2-3x+1)) dx$			
	• ⁷ integrate	• ⁷ $-\frac{x^3}{3} + x^2$			
	 ⁸ substitute limits and evaluate integral 	• ⁸ $\left(-\frac{2^3}{3}+2^2\right)-(0)=\frac{4}{3}$			
	• ⁹ state fraction	• $9 \frac{1}{2}$			
	• ⁶ use "cubic - <i>line</i> "	Method 2 • $^{6}\int ((x^{3}-4x^{2}+3x+1)-(1-x))dx$			
	• ⁷ integrate	• ⁷ $\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2$			
	 ⁸ substitute limits and evaluate integral 	$\bullet^{8}\left(\frac{2^{4}}{4}-4\times\frac{2^{3}}{3}+2\times2^{2}\right)-(0)=\frac{4}{3}$			
	• ⁹ state fraction	• 9 $\frac{1}{2}$	4		

		Method 3
	• ⁶ integrate line	$\bullet^{6} \int (1-x) dx = \begin{bmatrix} \frac{2}{x-\frac{x}{2}} \end{bmatrix}_{0}^{2}$
	• ⁷ substitute limits and evaluate integral	$\bullet^7 \left(2 - \frac{2^2}{2}\right) - (0) = 0$
	• ⁸ evidence of subtracting integrals	• $^{8}0 - \left(-\frac{4}{3}\right) = \frac{4}{3} \text{ or } \frac{4}{3} - 0$
	• ⁹ state fraction	• $9 \frac{1}{2}$
Notes:		
IMPORTANT: candidate ha time for the	Notes prefixed by *** may be sub as been penalised for the error in (a same error in (b).	ject to General Marking Principle (n). If a) then they must not be penalised a second
0 *** e ⁶ is r	at available to candidates who omit	(dr)
7. ● IS I 10 In Metho	not available to callulates wild office	u_{Λ} . f brackets at \bullet^6 stage as had form only if the
correct i	integral is obtained at \bullet^7	brackets at • stage as bad form only if the
11. Candidat	tes who have an incorrect expression	to integrate at the \bullet^3 and \bullet^7 stage due solely
to the al	osence of brackets lose \bullet^2 , but are av	varded \bullet^6 .
12. Where a	candidate differentiates one or more	e terms at \bullet^7 , then \bullet^7 , \bullet^8 and \bullet^9 are
unavaila	ble.	· · · · · · · · · · · · · · · · · · ·
*** In cas	ses where Note 3 has applied in part	(a), •' is lost but •° and •' are available.
13. In Metho	ods 1 and 2 only, accept unsimplified	expressions at • ⁷ e.g. $x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{3x^2}{2} - x$
14. Do not p	enalise the inclusion of ' $+c$ '.	
15. *** • ⁸ in lower lin	Methods 1 and 2 and \bullet^7 in method 3 nit '0' has been considered.	is only available if there is evidence that the
16 At the -9	stage the fraction must be consisted	but with the answers at a^5 and a^8 for a^9 to be

16. At the •⁹ stage, the fraction must be consistent with the answers at •⁵ and •⁸ for •⁹ to be awarded.

17. Do not penalise errors in substitution of x = 0 at \bullet^8 in Method 1 & 2 or \bullet^7 in Method 3. Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
11.	A and B are the points (–7, 2) and (5, <i>a</i>).	
	AB is parallel to the line with equation $3y - 2x = 4$.		
	Determine the value of <i>a</i> .		
		Method 1	
	 ¹ determine the gradient of given line or of AB 	•1 $\frac{2}{3}$ or $\frac{a-2}{12}$	
	• ² determine the other gradient	• ² $\frac{a-2}{12}$ or $\frac{2}{3}$	
	• ³ find a	• ³ 10	
	• ¹ determine the gradient of given line	Method 2 •1 $\frac{2}{3}$ stated or implied by • ²	
	• ² equation of line and substitute	• ² $y-2 = \frac{2}{3}(x+7)$ $a-2 = \frac{2}{3}(5+7)$	
	• ³ solve for a	• ³ 10	3
Notes:			
	served Responses.		

Candidate A - using simultaneous equations	Candidate B		Candidate C - Metho	od 2
$m_{\text{line}} = \frac{2}{3} \qquad \bullet^{1} \checkmark$ $3y = 2x + 20$ $3y = 2x - 10 + 3a \qquad \bullet^{2} \checkmark$ $0 = 0 + 30 - 3a$ $3a = 30$ $a = 10 \qquad \bullet^{3} \checkmark$	$m_{AB} = \frac{a-2}{12}$ $\frac{a-2}{12} = -2$ $a = -22$	•1 \checkmark •2 x •3 \checkmark 1	• ¹ $y-2 = \frac{2}{3}(x+7)$ $3y = 2x+20$ $3y = 2 \times 5+20$ $3y = 30$ $y = 10$ No mention of a	• ² ✓ • ³ ^

Question	Generi	ic Scheme	Illus	trative Scheme	2	Max Mark
12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of <i>a</i> .						
	• ¹ use laws of l	ogs	• ¹ $\log_a 9$			
	• ² write in exp	onential form	• ² $a^{\frac{1}{2}} = 9$			
	• ³ solve for a		• ³ 81			3
Notes:						
1. $\frac{36}{4}$ must be 2. Accept log 9	e simplified at \bullet^1 9 at \bullet^1 .	or \bullet^2 stage for \bullet^1 to	be awarded.			
$3 \bullet^2$ may be in	nolied by \bullet^3					
Commonly Ob	sorved Personse					
Candidate A	served Kesponse	Candidate B		Candidate C		
\log_a 144	• ¹ ×	$\log_a 32$	• ¹ 🗴	$\log_a 9$	● ¹ ✓	
$a^{\frac{1}{2}} = 144$	● ² ✓1	$a^{\frac{1}{2}} = 32$	● ² <mark>√1</mark>	$a = 9^{\frac{1}{2}}$	• ² ¥	
<i>a</i> = 12	• ³ ¥		• ³ ^	<i>a</i> = 3	• ³ √ 2	
Candidate D						
$2\log_a 36 - 2\log_a 36$	$g_a 4 = 1$					
$\log_a \mathbf{36^2} - \log_a$	$4^2 = 1 \bullet^1 \checkmark$					
$\log_a \frac{36^2}{4^2} = 1$						
$\log_a 81 = 1$	• ² ✓					
a = 81	● ³ ✓					

Question	Generic Scheme	Illustrative Scheme	Max Mark
13.	Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}} dx, \ x < \frac{5}{4}.$		
	• ¹ write in integrable form	• $(5-4x)^{-\frac{1}{2}}$	
	• ² start to integrate	• ² $\frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}}$	
	• ³ process coefficient of x	• ³ × $\frac{1}{(-4)}$	
	 ⁴ complete integration ar simplify 	$\int e^{4} -\frac{1}{2} (5-4x)^{\frac{1}{2}} + c$	4
Notes:			
 For candid form awar If candidat bracket no '+c' is rec 	lates who differentiate throughout lates who 'integrate the denomina d 0/4. tes start to integrate individual te o further marks are available. quired for• ⁴ .	t, only • ¹ is available. Itor' without attempting to write in int rms within the bracket or attempt to e	egrable xpand a
Condidate A	selved Responses.	Candidata B	
Candidate A			
$(5-4x)^{-\frac{1}{2}}$	•1 🗸	$(5-4x)^{\frac{1}{2}}$ $\bullet^1 *$	
$\frac{\left(5-4x\right)^{\frac{1}{2}}}{\frac{1}{2}}$	● ² ✓ ● ³ ^	$\frac{\left(5-4x\right)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{\left(-4\right)} \qquad \qquad \bullet^{2} \checkmark 1 \bullet^{3}$	✓
$2(5-4x)^{\frac{1}{2}}+c$	•4 12	$-\frac{(5-4x)^{\frac{3}{2}}}{6}+c$ • ⁴ \checkmark 1	

Candidate C		Candidate D	
Differentiate in part:		Differentiate in part:	
$(5-4x)^{-\frac{1}{2}}$	● ¹ ✓	$(5-4x)^{-\frac{1}{2}}$	•1 🗸
$-\frac{1}{2}(5-4x)^{-\frac{3}{2}}\times\frac{1}{(-4)}$	• ² × • ³ ✓	$\frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2}} \times (-4)$	• ² 🖌 • ³ 🗴
$\frac{1}{8}(5-4x)^{-\frac{3}{2}}+c$	• ⁴ <u>√1</u>	$-8(5-4x)^{\frac{1}{2}}+c$	• ⁴ 1

Question	Generic Scheme	Illustrative Scheme	Max Mark	
14. (a)	Express $\sqrt{3} \sin x^{\circ} - \cos x^{\circ}$ in the form $k \sin(x-a)^{\circ}$, where $k > 0$ and $0 < a < 360$.			
	• ¹ use compound angle formula	• ¹ $k \sin x^{\circ} \cos a^{\circ} - k \cos x^{\circ} \sin a^{\circ}$ stated explicitly		
	• ² compare coefficients	• ² $k \cos a^\circ = \sqrt{3}, k \sin a^\circ = 1$ stated explicitly		
	• ³ process for k	• ³ $k = 2$		
	• ⁴ process for <i>a</i> and express in required form	• ⁴ $2\sin(x-30)^{\circ}$	4	
Notes:				

1. Accept $k(\sin x^{\circ} \cos a^{\circ} - \cos x^{\circ} \sin a^{\circ})$ for \bullet^{1} . Treat $k \sin x^{\circ} \cos a^{\circ} - \cos x^{\circ} \sin a^{\circ}$ as bad form only if the equations at the \bullet^{2} stage both contain k.

- 2. Do not penalise the omission of degree signs.
- 3. $2\sin x^{\circ}\cos a^{\circ} 2\cos x^{\circ}\sin a^{\circ}$ or $2(\sin x^{\circ}\cos a^{\circ} \cos x^{\circ}\sin a^{\circ})$ is acceptable for \bullet^{1} and \bullet^{3} .
- 4. In the calculation of k = 2, do not penalise the appearance of -1.
- 5. Accept $k \cos a^\circ = \sqrt{3}$, $-k \sin a^\circ = -1$ for \bullet^2 .
- 6. •² is not available for $k \cos x^\circ = \sqrt{3}$, $k \sin x^\circ = 1$, however, •⁴ is still available.
- 7. •³ is only available for a single value of k, k > 0.
- 8. •³ is not available to candidates who work with $\sqrt{4}$ throughout parts (a) and (b) without simplifying at any stage.
- 9. •⁴ is not available for a value of a given in radians.
- 10. Candidates may use any form of the wave equation for \bullet^1 , \bullet^2 and \bullet^3 , however, \bullet^4 is only available if the value of a is interpreted in the form $k \sin(x-a)^\circ$
- 11. Evidence for \bullet^4 may only appear as a label on the graph in part (b).

Commonly Observe	Commonly Observed Responses:					
Responses with mis	sing inform	ation in working:				
Candidate A		Candidate B				
	• ¹ ^	$k\sin x\cos a - k\cos x\sin a \bullet^1 \checkmark$				
$2\cos a = \sqrt{3}$ $2\sin a = 1$ $\tan a = \frac{1}{\sqrt{3}}, a = 30$ $2\sin(x - 30)^{\circ}$	• ² • • ³ •	$\cos a = \sqrt{3}$ $\sin a = 1$ $\tan a = \frac{1}{\sqrt{3}}$ Not consistent with equations at • ² .				
		$2\sin(x-30)^{\circ}$ • ³ \checkmark • ⁴ \bigstar				

Responses with the correct expansion of $k\sin(x-a)^\circ$ but errors for either \bullet^2 or \bullet^4 .				
Candidate C	Candidate D	Candidate E		
$k\cos a = \sqrt{3}, k\sin a = 1$ $\bullet^2 \checkmark$	$k\cos a = 1, k\sin a = \sqrt{3} \bullet^2 *$	$k\cos a = \sqrt{3}, k\sin a = -1 \bullet^2 \mathbf{x}$		
$\tan a = \sqrt{3}$ • ⁴ ×	$\tan a = \sqrt{3}$	1 320		
a = 60	a = 60	$\tan a = -\frac{1}{\sqrt{3}}, \ a = 330$		
	$2\sin(x-60)^{\circ}$ • ⁴ \checkmark 1	· ·		
		$2\sin(x-330)^{\circ}$ • ⁴ \checkmark 1		
Responses with the incorrect	abelling; $k \sin A \cos B - k \cos A \sin B$ from formula list.			
Candidate F	Candidate G	Candidate H		
$k \sin A \cos B - k \cos A \sin B \bullet^{1} \mathbf{x}$	$k \sin A \cos B - k \cos A \sin B \bullet^1 x$	$k \sin A \cos B - k \cos A \sin B \bullet^1 x$		
$k\cos a = \sqrt{3}$	$k\cos x = \sqrt{3}$	$k\cos \mathbf{B} = \sqrt{3}$		
$k\sin a = 1$ $\bullet^2 \checkmark$	$k \sin x = 1$ $\bullet^2 x$	$k\sin \mathbf{B} = 1$ $\bullet^2 \mathbf{x}$		
$\tan a = \frac{1}{\sqrt{3}}, \ a = 30$	$\tan x = \frac{1}{\sqrt{3}}, \ x = 30$	$\tan B = \frac{1}{\sqrt{3}}, B = 30$		
$2\sin(x-30)^\circ$ $\bullet^3\checkmark \bullet^4\checkmark$	$2\sin(x-30)^\circ$ $\bullet^3\checkmark\bullet^4\checkmark1$	$2\sin(x-30)^\circ$ $\bullet^3\checkmark \bullet^4\checkmark 1$		

Question	Generic Scheme	Illustrative Scheme	Max Mark	
14. (b)	Hence, or otherwise, sketch the graph with equation $y = \sqrt{3} \sin x^{\circ} - \cos x^{\circ}$, $0 \le x \le 360$.			
	Use the diagram provided in the answer booklet.			
	• ⁵ roots identifiable from graph	• ⁵ 30 and 210		
	• ⁶ coordinates of both turning points identifiable from graph	• ⁶ (120, 2) and (300, -2)		
	• ⁷ y-intercept and value of y at $x = 360$ identifiable from graph	•7 -1	3	
Notes:				
 12. •⁵, •⁶and •⁷ are only available for attempting to draw a "sine" graph with a period of 360° 13. Ignore any part of a graph drawn outwith 0 ≤ x ≤ 360. 14. Vertical marking is not applicable to •⁵ and •⁶. 15. Candidates sketch arrived at in (b) must be consistent with the equation obtained in (a) see also candidates I and J. 16. For any incorrect horizontal translation of the graph of the wave function arrived at in part(a) only •⁶ is available. 				
Commonly Ubs	served Responses:	Candidata I		
Candidate I		Candidate J		
(a) $2\sin(x-30)$ correct equation		(a) $2\sin(x+30)$ incorrect equation		
(b) Incorrect translation: Sketch of $2\sin(x+30)$		(b) Sketch of $2\sin(x+30)$		
Only • ⁶ is availa	able	All 3 marks are available		



Question	Generic Scheme	Illustrative Scheme	Max Mark		
15. (b)	It is known that $\int_{1}^{3} f(x) dx = 4$.				
	Determine the value of $\int_6^8 h(x) dx$.				
	• ³ state value of integral	• ³ 10	1		
Notes:					
 Candidates answer at (b) must be consistent with the value of b obtained in (a). In parts (b) and (c), candidates who have 10 and -6 accompanied by working, the working must be checked to ensure that no errors have occurred prior to the correct answer appearing. 					
Commonly Observed Responses:					
Candidate A From (a) $a = -3 \bullet^1 x$					
$b = 5 \bullet^2 \mathbf{x}$					
$\int h(x) dx = 14 \bullet^3 \checkmark_1$					

Question	Generic Scheme		Illustrative Scheme	Max Mark		
15. (c)	Given $f'(1) = 6$, state the value of $h'(8)$.					
	 state value of derivative 	• ⁴	-6	1		
Notes:						
Commonly Observed Responses:						

[END OF MARKING INSTRUCTIONS]