HIGHER 2016 – Paper 1

(1) $y = -4x + 7 \quad m = -4$

$y - b = m(x - a)$

$y - 3 = -4(x + 2)$

$y = -4x - 8 + 3$

$\underline{y = -4x - 5}$

(2) $y = 12x^3 + 8x^{1/2}$

$\frac{dy}{dx} = 36x^2 + 4x^{-1/2}$

$= 36x^2 + \frac{4}{\sqrt{x}}$

(3) a) $u_4 = \frac{1}{3}u_3 + 10$

$= \frac{1}{3}(6) + 10$

$= \underline{12}$

b) $-1 < \frac{1}{3} < 1$

c) $L = \frac{10}{1 - \frac{1}{3}}$

$= \frac{10}{\frac{2}{3}}$

$= \frac{30}{2}$

$= \underline{\underline{15}}$

(4) $C = \left(\frac{-7+1}{2}, \frac{3+5}{2}\right)$

$= (-3, 4)$

$r = \sqrt{(-3-1)^2 + (4-5)^2}$

$= \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{17}$

$\therefore (x+3)^2 + (y-4)^2 = 17$

(5) $\int 8\cos(4x+1)\,dx$

$= \frac{8\sin(4x+1)}{4} + C$

$= \underline{2\sin(4x+1) + C}$

(6) a) $y = 3x + 5$

$3x = y - 5$

$x = \frac{y-5}{3}$

$\therefore f^{-1}(x) = \frac{x-5}{3}$

b) If $g(2) = 7$ then

$g^{-1}(7) = 2$

(7) a) $\overrightarrow{FH} = \overrightarrow{FG} + \overrightarrow{GH}$

$= \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

b) $\overrightarrow{FE} = \overrightarrow{FH} + \overrightarrow{HE}$

$= \overrightarrow{FH} - \overrightarrow{EH}$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix}}}$

(8) $x^2 + (3x-5)^2 + 2x - 4(3x-5) - 5 = 0$

$x^2 + 9x^2 - 30x + 25 + 2x - 12x + 20 - 5 = 0$

$10x^2 - 40x + 40 = 0$

$10(x^2 - 4x + 4) = 0 \qquad b^2 - 4ac$

$= (-4)^2 - 4(1)(4)$

$= 16 - 16$

$\therefore$ line is a tangent.

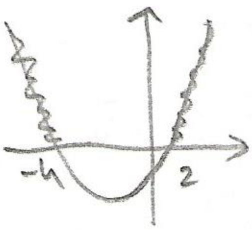
$$10(x^2-4x+4)=0$$
$$10(x-2)^2=0$$
$$x=2$$
$$y=3(2)-5$$
$$=1$$
$$\therefore \text{P of C} = (2,1)$$

(9) a) SP's where $f'(x)=0$

$$\therefore 3x^2+6x-24=0$$
$$3(x^2+2x-8)=0$$
$$3(x+4)(x-2)=0$$
$$x=-4 \quad x=2$$

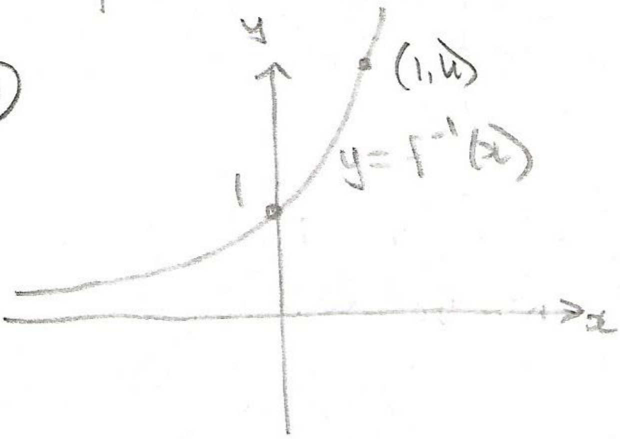
b) Increasing where

$$3x^2+6x-24>0$$

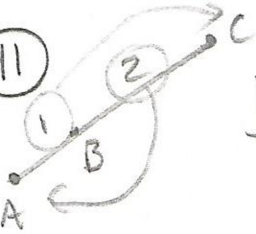


$$\therefore x<-4$$
$$x>2$$

(10)



(11)



$$\underline{b} = \frac{1}{3}\left[2\underline{a}+\underline{c}\right]$$
$$= \frac{1}{3}\left[\begin{pmatrix}2\\6\\-4\end{pmatrix}+\begin{pmatrix}4\\-3\\4\end{pmatrix}\right]$$
$$= \frac{1}{3}\begin{pmatrix}6\\3\\0\end{pmatrix}$$
$$= \begin{pmatrix}2\\1\\0\end{pmatrix}$$
$$\therefore B=(2,1,0)$$

b) $\vec{AC} = \underline{c}-\underline{a}$

$$= \begin{pmatrix}4\\-3\\4\end{pmatrix}-\begin{pmatrix}1\\3\\-2\end{pmatrix}$$
$$= \begin{pmatrix}3\\-6\\6\end{pmatrix}$$
$$|\vec{AC}| = \sqrt{3^2+(-6)^2+6^2}$$
$$= \sqrt{9+36+36}$$
$$= \sqrt{81}$$
$$= 9$$
$$\therefore k = \frac{1}{9}$$

(12) a) $f(g(x))$

$$= f(3-x)$$
$$= 2(3-x)^2-4(3-x)+5$$
$$= 2(9-6x+x^2)-12+4x+5$$
$$= 18-12x+2x^2-7+4x$$
$$= 2x^2-8x+11$$

(as required)

b) $h(x) = 2x^2-8x+11$

$$= 2\left[x^2-4x\right]+11$$
$$= 2\left[(x-2)^2-4\right]+11$$
$$= 2(x-2)^2-8+11$$
$$= 2(x-2)^2+3$$

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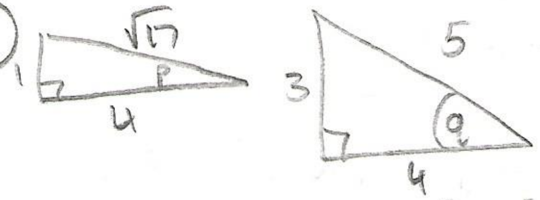


$$\cos(q-p) = \cos q \cos p + \sin q \sin p$$
$$= \left(\frac{4}{5} \times \frac{4}{\sqrt{17}}\right) + \left(\frac{3}{5} \times \frac{1}{\sqrt{17}}\right)$$
$$= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}}$$
$$= \frac{19}{5\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$$
$$= \frac{19\sqrt{17}}{5(17)}$$
$$= \frac{19\sqrt{17}}{85} \text{ (as required)}$$

14 a) $\log_5 25 = 2$

b) $\log_4 x + \log_4 (x-6) = \log_5 25$

$\log_4 x(x-6) = 2$

$x(x-6) = 4^2$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ ~~$x = -2$~~

15 a) $y = k(x+5)^2(x-4)$

$9 = k(1+5)^2(1-4)$

$9 = k(36)(-3)$

$9 = -108k$

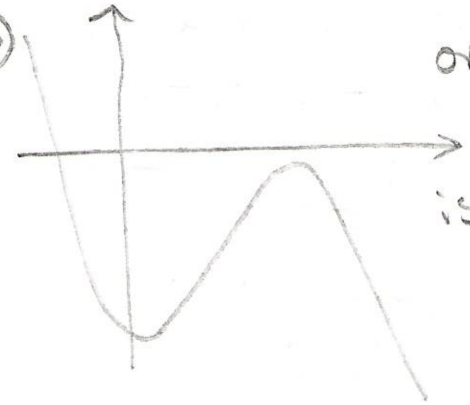
$k = -\frac{9}{108}$

$k = -\frac{1}{12}$

$\therefore y = -\frac{1}{12}(x-4)(x+5)^2$

$a = 4 \quad b = -5 \quad k = -\frac{1}{12}$

b)



one root when max TP is below x-axis

$\therefore d > 9$

PAPER II

1 a) (i) $M = (2, 4)$

(ii) $M_{PM} = \frac{4+4}{2-0} = \frac{8}{2} = 4$

$y - b = m(x-a)$

$y - 4 = 4(x-2)$

$y = 4x - 4$

b) $M_{PR} = \frac{6+4}{10-0} = \frac{10}{10} = 1$

$\therefore M_L = -1$

$(m_1 m_2 = -1)$

$y - b = m(x-a)$

$y - 4 = -(x-2)$

$y = -x + 6$

c) $M_{PR} = (5, 1)$

If $x = 5$, $y = -5+6 = 1$ $\therefore (5,1)$ is on L.

2 For no real roots,

$b^2 - 4ac < 0$

$(-2)^2 - 4(1)(3-p) < 0$

$4 - 4(3-p) < 0$

$4 - 12 + 4p < 0$

$4p - 8 < 0$

$4p < 8$

$p < 2$

3 a)

$$\begin{array}{r|rrrr} -1 & 2 & -9 & 3 & 14 \\ & & -2 & 11 & -14 \\ \hline & 2 & -11 & 14 & \boxed{0} \end{array}$$

$R = 0$ $\therefore (x+1)$ is a factor.

(iii) $(x+1)(2x^2-11x+14)=0$

$(x+1)(2x-7)(x-2)=0$

$x=-1 \quad x=\frac{7}{2} \quad x=2$

b) (i) $A=(-1,0)$ $B=(2,0)$

$C=(\frac{7}{2},0)$

(ii) $\int_{-1}^{2}(2x^3-9x^2+3x+14)\,dx$

$=\left[\frac{x^4}{2}-3x^3+\frac{3x^2}{2}+14x\right]_{-1}^{2}$

$=\left(\frac{2^4}{2}-3(2)^3+\frac{3(2)^2}{2}+14(2)\right)$

$-\left(\frac{(-1)^4}{2}-3(-1)^3+\frac{3(-1)^2}{2}+14(-1)\right)$

$=(8-24+6+28)-(\frac{1}{2}+3+\frac{3}{2}-14)$

$=(18)-(-9)$

$=27$ $\therefore$ Area $=27u^2$

(4) C_1 $(-5,6)$ $r_1=3$

a) C_2 $(3,0)$ $r_2=\sqrt{3^2+0^2+16}$

$=\sqrt{25}$

$=5$

b) Circles do not intersect if

$r_1+r_2 <$ distance between centres.

$r_1+r_2=8$ $\quad d=\sqrt{(-5-3)^2+(6-0)^2}$

$=\sqrt{8^2+6^2}$

$=10$

$r_1+r_2<d$ $\therefore$ circles do not intersect

(5) a) $\vec{AB}=\begin{pmatrix}-8\\16\\2\end{pmatrix}$ $\vec{AC}=\begin{pmatrix}-2\\-8\\16\end{pmatrix}$

b) $\vec{AB}\cdot\vec{AC}=16-128+32$

$=-80$

$|\vec{AB}|=\sqrt{(-8)^2+16^2+2^2}=\sqrt{324}$

$|\vec{AC}|=\sqrt{324}$

$\therefore \cos BAC=\frac{-80}{\sqrt{324}\sqrt{324}}$

$BAC=\cos^{-1}\left(\frac{-80}{324}\right)$

$BAC=104.3^\circ$

(6) a) 200

b) To double, $B(t)=400$

$\therefore 400=200e^{0.107t}$

$2=e^{0.107t}$

$\ln 2=0.107$

$t=\frac{\ln 2}{0.107}$

$t=6.48$ hrs

(7) a) Area $=3x\times 2y$

$=6xy$

$\therefore 6xy=108$

$y=\frac{108}{6x}$

$y=\frac{18}{x}$.

Total length $=9x+8y$

$=9x+8\left(\frac{18}{x}\right)$

$=9x+\frac{144}{x}$

(as required.

b) $L(x)=9x+144x^{-1}$

SP's where $L'(x)=0$

$\therefore 9-144x^{-2}=0$

$$9 = \frac{144}{x^2}$$
$$9x^2 = 144$$
$$x^2 = 16$$
$$x = 4$$

x	$\xrightarrow{3}$ 4 $\xrightarrow{5}$
$L'(x)$	− 0 +
shape	\ 0 /

$\therefore$ Min @ $x = 4$m

(8)

$$5\boxed{\cos x} - 2\sin x = k\cos(x+\alpha)$$
$$= k\cos\alpha\boxed{\cos x} - k\sin\alpha \sin x$$

$\therefore$ ~~$-k\sin\alpha = -2$~~

$$\boxed{\begin{aligned} k\sin\alpha &= 2 \\ k\cos\alpha &= 5 \end{aligned}}$$

$$\tan\alpha = \frac{2}{5}$$
$$\alpha = \tan^{-1}\left(\frac{2}{5}\right)$$
$$\alpha = 21.8^\circ$$

$$k^2 = 5^2 + 2^2$$
$$k = \sqrt{29}$$

S | A
T | C

$k\sin\alpha$ = pos
$k\cos\alpha$ = pos

$\therefore \alpha$ in Q1 $\quad \frac{21.8\pi}{180} = 0.38$

$\therefore \sqrt{29}\cos(x + 0.38)$

b) $10 + 5\cos x - 2\sin x = 12$
$$5\cos x - 2\sin x = 2$$
$$\sqrt{29}\cos(x + 0.38) = 2$$
$$\cos(x+0.38) = \frac{2}{\sqrt{29}}$$
$$x + 0.38 = \cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$$
$$x + 0.38 = 1.19,\ 5.09$$
$$x = 0.81,\ 4.71$$

S | A (x)
T | C ($2\pi - x$)

$2\pi - 1.19$
$= 5.09$

(9) $f'(x) = \frac{2x}{\sqrt{x}} + \frac{1}{\sqrt{x}}$
$$= 2\sqrt{x} + \frac{1}{\sqrt{x}}$$
$$= 2x^{1/2} + x^{-1/2}$$

$$\therefore f(x) = \int (2x^{1/2} + x^{-1/2})\,dx$$
$$= \frac{2x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + C$$
$$= \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} + C$$

If $f(9) = 40$ then
$$\frac{4}{3}\sqrt{9^3} + 2\sqrt{9} + C = 40$$
$$\frac{4}{3}(27) + 2(3) + C = 40$$
$$36 + 6 + C = 40$$
$$C = -2$$

$$\therefore f(x) = \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} - 2$$

(10) a) $\frac{dy}{dx} = \frac{1}{2}(x^2+7)^{-1/2} \times 2x$
$$= \frac{x}{\sqrt{x^2+7}}$$

b) $\int \frac{4x}{\sqrt{x^2+7}}\,dx = 4\int \frac{x}{\sqrt{x^2+7}}\,dx$
$$= 4(x^2+7)^{1/2} + C$$

(11) a) $\sin 2x \tan x$
$$= 2\sin x\cos x \times \frac{\sin x}{\cos x}$$
$$= 2\sin^2 x$$
$$= 1 - (1 - 2\sin^2 x)$$
$$= 1 - \cos 2x \text{ (as required)}$$

b) $f(x) = 1 - \cos 2x$
$$f'(x) = -(-\sin(2x)) \times 2$$
$$= 2\sin 2x$$