① $\underline{u}.\underline{v} = 0$

$8\times(-3) + 2t + (-1)\times(-6) = 0$

$-24 + 2t + 6 = 0$

$2t - 18 = 0$

$\underline{t = 9}$

② $\frac{dy}{dx} = 6x^2$

when $x=-2$, $m = 6(-2)^2$

$= \underline{24}$

when $x=-2$, $y = 2(-2)^3 + 3$

$= -16 + 3$

$= -13$

$\therefore (-2, -13)$

$\therefore y - b = m(x-a)$

$y + 13 = 24(x+2)$

$y + 13 = 24x + 48$

$\underline{y = 24x + 35}$

③

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$R = 0$ $\therefore$ $(x+3)$ is a factor.

$x^3 - 3x^2 - 10x + 24$

$= (x+3)(x^2 - 6x + 8)$

$= \underline{(x+3)(x-2)(x-4)}$

④ $q = 4$

$p = 3$

$r = 1$

⑤ a) $y = 6 - 2x$

$2x = 6 - y$

$x = \frac{6-y}{2}$

$\therefore g^{-1}(x) = \frac{6-x}{2}$

b) $g(g^{-1}(x)) = 6 - 2\left(\frac{6-x}{2}\right)$

$= \underline{x}$

⑥ $\log_6 12 + \frac{1}{3}\log_6 27$

$= \log_6 12 + \log_6 \sqrt[3]{27}$

$= \log_6 12 + \log_6 3$

$= \log_6 36$

$= \underline{2}$

⑦ $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$

$= 3x\sqrt{x} - \frac{2}{x}$

$= \underline{3x^{3/2} - 2x^{-1}}$

$f'(x) = \frac{9}{2}x^{1/2} + 2x^{-2}$

$= \underline{\frac{9}{2}\sqrt{x} + \frac{2}{x^2}}$

$f'(4) = \frac{9}{2}\sqrt{4} + \frac{2}{4^2}$

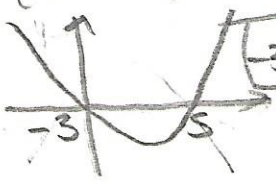
$= \frac{9}{2}(2) + \frac{2}{16}$

$= \underline{9\frac{1}{8}}$

⑧ $x(x-2) < 15$

$x^2 - 2x - 15 < 0$

$(x-5)(x+3) < 0$



$\boxed{-3 < x < 5}$

(9) $y = -\sqrt{3}x \quad \therefore m = -\sqrt{3}$

$m = \tan\theta$

$\tan\theta = -\sqrt{3}$

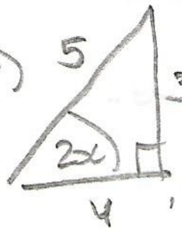
$\theta = 120^\circ$

AB is 120° to x-axis.

BC is 150° to x-axis

$\therefore$ A, B, C <u>not</u> collinear.

(10) a)



$\cos 2x = \frac{4}{5}$

b) $\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

$\cos^2 x = \frac{1}{2}\left[\cos 2x + 1\right]$

$\cos^2 x = \frac{1}{2}\left(\frac{4}{5} + 1\right)$

$\cos^2 x = \frac{1}{2}\left(\frac{9}{5}\right)$

$\cos^2 x = \frac{9}{10}$

$\cos x = \pm\sqrt{\frac{9}{10}}$

$\therefore \cos x = \frac{3}{\sqrt{10}} \quad (0 < x < \frac{\pi}{4})$

(11) a) centre $= (-8, -2)$

$m_R = \frac{-2+5}{-8+2}$

$= \frac{3}{-6}$

$= -\frac{1}{2}$

$\therefore m_T = 2$

$(m_R m_T = -1)$

$\therefore y - b = m(x - a)$

$y + 5 = 2(x + 2)$

$y = 2x - 1$

b) $y = y$

$2x - 1 = -2x^2 + px + 1 - p$

$2x^2 + 2x - px - 2 + p = 0$

$2x^2 + (2-p)x + p - 2 = 0$

$a = 2 \qquad b^2 - 4ac = 0$

$b = 2 - p \qquad (2-p)^2 - 4(2)(p-2) = 0$

$c = p - 2 \qquad 4 - 4p + p^2 - 8p + 16 = 0$

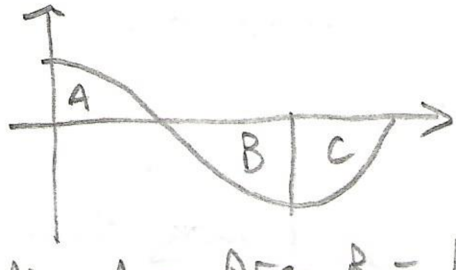
$p^2 - 12p + 20 = 0$

$(p - 10)(p - 2) = 0$

$p = 10, \ p = 2$

$\therefore p = 10 \quad (p > 3)$

(12)



Area A = Area B = Area C.

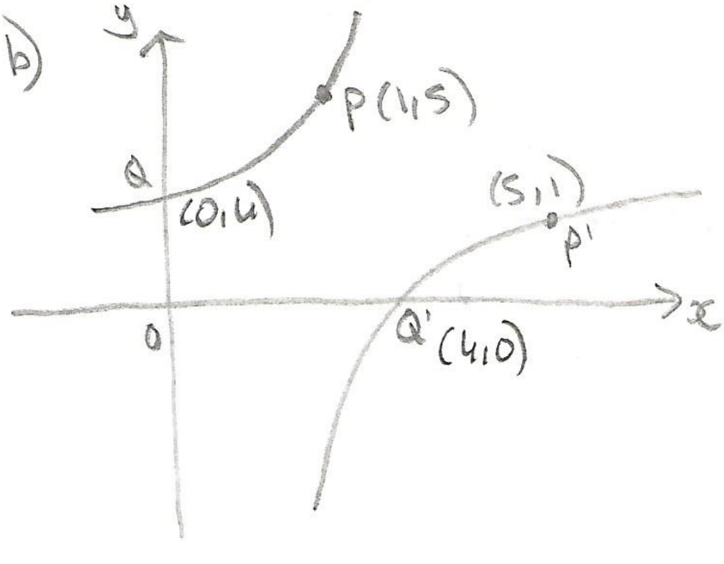
A = +, B, C = −

$\therefore \int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx = -\frac{1}{2}$

(13) a) $f(1) = 2^1 + 3 \quad \therefore b = 5$

$= 5$

b)



c) $y = 4 - f(x+1)$

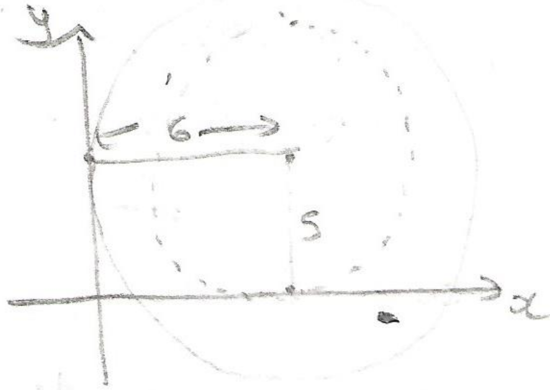
$= -f(x+1) + 4$

Flip in x-axis, slide 1 left, slide up 4

$(3, -11) \to (2, -11) \to (2, -7)$

$\therefore R' = (2, -7)$

(14) centre $= (6, 5)$



Radius $= 6$ (if $r = 5$, circle would only touch once.

$\therefore r = \sqrt{g^2 + f^2 - c}$

$6 = \sqrt{6^2 + 5^2 - k}$

$6 = \sqrt{36 + 25 - k}$

$6 = \sqrt{51 - k}$

$36 = 51 - k$

$k = 25$

(15) $T = \int \left(\frac{1}{25}t - k\right) dt$

$= \frac{1}{50}t^2 - kt + C$.

when $t = 0$, $T = 100$

$\therefore 100 = \frac{1}{50}(0)^2 - k(0) + C$

$C = 100$

$\therefore T = \frac{1}{50}t^2 - kt + 100$.

when $t = 10$, $T = 82$

$\therefore 82 = \frac{1}{50}(10)^2 - 10k + 100$

$82 = \frac{100}{50} - 10k + 100$

$82 = 2 - 10k + 100$

$10k = 102 - 82$

$10k = 20$

$k = 2$

$\therefore T = \frac{1}{50}t^2 - 2t + 100$

HIGHER 2015 PAPER 2

(1) a) $m_{AB} = \frac{7+5}{-5+1}$

$= \frac{12}{-4}$

$= -3$

$\therefore m_{Alt} = \frac{1}{3}$

$(m_{AB} m_{Alt} = -1)$

$y - b = m(x - a)$

$y - 3 = \frac{1}{3}(x - 13)$

$3y - 9 = x - 13$

$x - 3y = 4$ (as required)

b) Midpoint $= \left(\frac{-5+13}{2}, \frac{7+3}{2}\right)$

$= (4, 5)$

$m_{MB} = \frac{5+5}{4+1}$

$= \frac{10}{5}$

$= 2$

$y - b = m(x - a)$

$y - 5 = 2(x - 4)$

$y - 5 = 2x - 8$

$2x - y = 3$

c) $x - 3y = 4$ (1)
$2x - y = 3$ (2)

(1)×2 $2x - 6y = 8$ (3)
$2x - y = 3$ (2)

(3) − (2) $-5y = 5$
$y = -1$
$x + 3 = 4$
$x = 1$

$\therefore (1, -1)$

(2) a) $f(g(x)) = f((1+x)(3-x)+2)$
$= f(3 + 2x - x^2 + 2)$
$= f(5 + 2x - x^2)$
$= 15 + 2x - x^2$

b) $f(g(x)) = -x^2 + 2x + 15$
$= -[x^2 - 2x] + 15$
$= -[(x-1)^2 - 1] + 15$
$= -(x-1)^2 + 1 + 15$
$= 16 - (x-1)^2$

c) $h(x) = \dfrac{1}{15 + 2x - x^2}$
$= \dfrac{1}{(5-x)(3+x)}$
$x \neq 5,\ x \neq -3.$

(3) a) $t_2 = \frac{3}{4} t_1 + 13$
$= \frac{39}{4} + 13$
$= 22\frac{3}{4}$

b) Frog Limit exists as $-1 < \frac{1}{3} < 1$
$L = \frac{1}{3} L + 32$
$\frac{2}{3} L = 32$
$L = 48 m$

Toad Limit exists as $-1 < \frac{3}{4} < 1$
$L = \frac{3}{4} L + 13$
$\frac{1}{4} L = 13$
$L = 52$ feet.

As the well is 50 ft deep, the frog will not escape as it settles at a height of 48 ft. The toad will as it settles at 52 feet.

(4) a) $\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$
$\frac{3}{2}x - \frac{1}{2}x = 5 - 3$
$x = 2$

b) Area $= \int_0^2 f(x) - h(x)\,dx$
$= \int_0^2 \left(\frac{1}{4}x^2 - \frac{1}{2}x + 3\right) - \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right) dx$
$= \int_0^2 \left(-\frac{1}{8}x^2 + \frac{7}{4}x\right) dx$
$= \left[-\frac{x^3}{24} + \frac{7x^2}{8}\right]_0^2$
$= \left(-\frac{8}{24} + \frac{28}{8}\right) - (0)$
$= \left(-\frac{8}{24} + \frac{84}{24}\right)$
$= \frac{76}{24}$
$= 3\frac{1}{6} u^2$
$\therefore$ Area $= 6\frac{1}{3} u^2$

(5) $C_1 = (-3, -5)$

a) $C_2 = (9, 11)$

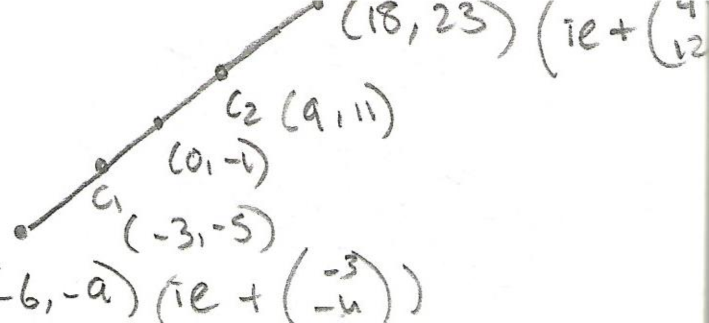
$M = \frac{11+5}{9+3} = \frac{16}{12} = \frac{4}{3}.$

$y - b = m(x-a)$

$y + 5 = \frac{4}{3}(x+3)$

$y + 5 = \frac{4}{3}x + 4$

$y = \frac{4}{3}x - 1$



$\therefore$ Diameter is the line joining $(-6, -9)$ and $(18, 23)$

$\therefore$ centre $= \left(\frac{-6+18}{2}, \frac{-9+23}{2}\right) = (6, 7)$

$\therefore (x-6)^2 + (y-7)^2 = 400$

$x^2 + \left(\frac{4}{3}x - 1\right)^2 + 6x + 10\left(\frac{4}{3}x - 1\right) + 9 = 0$

$x^2 + \frac{16}{9}x^2 - \frac{8}{3}x + 1 + 6x + \frac{40}{3}x - 10 + 9 = 0$

$9x^2 + 16x^2 - 24x + 9 + 54x + 120x - 9 = 0$

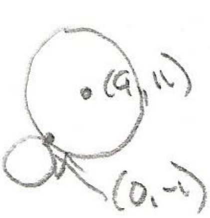
$25x^2 + 150x = 0$

$25x(x+6) = 0$

$x = 0$ or $x = -6$

$\therefore x = 0$ (between -3, 9)

$y = -1 \quad \therefore (0, -1)$



$d = \sqrt{9^2 + 12^2}$

$d = \sqrt{81 + 144}$

$d = \sqrt{225}$

$d = 15$

$\therefore$ radius $= 15$ units.

b)

$C_1 \quad r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{3^2 + 5^2 - 9}$

$= 5$

$\therefore$ Diameter of $C_3 = 2 \times 5 + 2 \times 15$

$= 40$ units

$\therefore$ radius $= 20$ units.

(6) a) $\underline{p} \cdot (\underline{q} + \underline{r})$

$= \underline{p} \cdot \underline{q} + \underline{p} \cdot \underline{r}$

$= (3)(3)\cos 60^\circ + (3)(3)\cos 9$

$= 9\left(\frac{1}{2}\right) + 9(0)$

$= \frac{9}{2}$

b) $\vec{EC} = \vec{ED} + \vec{DC}$

$= \vec{ED} + \vec{EB}$

$= \underline{r} + (-\underline{q} + \underline{p})$

$= \underline{r} + \underline{p} - \underline{q}$

c) $\vec{AE} \cdot \vec{EC}$

$= \underline{q} \cdot (\underline{r} + \underline{p} - \underline{q})$

$= \underline{q} \cdot \underline{r} + \underline{q} \cdot \underline{p} - \underline{q} \cdot \underline{q}$

$= \underline{q} \cdot \underline{r} + \frac{9}{2} - 9$

$= \underline{q} \cdot \underline{r} - \frac{9}{2}$

$$\therefore 9\sqrt{3} - \frac{9}{2} = \underline{q}\cdot\underline{r} - \frac{9}{2}$$

$$\underline{q}\cdot\underline{r} = 9\sqrt{3}$$

$$\underline{q}\cdot\underline{r} = |q||r|\cos\theta$$

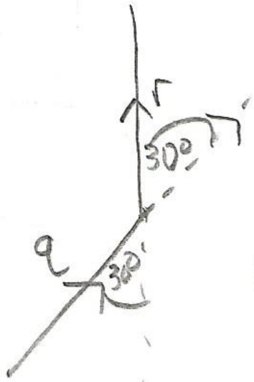
$$9\sqrt{3} = 3|r|\cos 30^\circ$$

$$9\sqrt{3} = 3|r|\left(\frac{\sqrt{3}}{2}\right)$$

$$9\sqrt{3} = \frac{3\sqrt{3}}{2}|r|$$

$$|r| = \frac{18\sqrt{3}}{3\sqrt{3}}$$

$$|r| = 6$$



⑦ a) $\int (3\cos 2x + 1)\,dx$

$$= \frac{3}{2}\sin 2x + x + C$$

b) $3\cos 2x + 1$

$$= 3(\cos^2 x - \sin^2 x) + 1$$

$$= 3\cos^2 x - 3\sin^2 x + 1$$

$$= 3\cos^2 x - 3\sin^2 x + (\cos^2 x + \sin^2 x)$$

$$= 4\cos^2 x - 2\sin^2 x \text{ (as required)}$$

c) $\int (\sin^2 x - 2\cos^2 x)\,dx$

$$= \int -(2\cos^2 x - \sin^2 x)\,dx$$

$$= \int -\frac{1}{2}(4\cos^2 x - 2\sin^2 x)\,dx$$

$$= -\frac{1}{2}\int (3\cos 2x + 1)\,dx$$

$$= -\frac{3}{4}\sin 2x - \frac{1}{2}x + C$$

⑧ a) (i) $x = 20$

$$\therefore T(20) = 5\sqrt{36 + 20^2} + 4(20 - 20)$$

$$= 5\sqrt{436}$$

$$= 104.4 \text{ } \tfrac{1}{10}\text{ths sec}$$

$$= 10.44 \text{ seconds}$$

(ii) $x = 0$

$$T(0) = 5\sqrt{36 + 0} + 4(20 - 0)$$

$$= 5(6) + 80$$

$$= 110 \text{ } \tfrac{1}{10}\text{ths sec}$$

$$= 11 \text{ seconds}$$

b) $T(x) = 5(36 + x^2)^{1/2} + 80 - 4x$

$$T'(x) = \frac{5}{2}(36 + x^2)^{-1/2} \times (2x) - 4$$

$$= 5x(36 + x^2)^{-1/2} - 4$$

sp's @ $T'(x) = 0$

$$\frac{5x}{\sqrt{36 + x^2}} - 4 = 0$$

$$\frac{5x}{\sqrt{36 + x^2}} = 4$$

$$5x = 4\sqrt{36 + x^2}$$

$$\sqrt{36 + x^2} = \frac{5x}{4}$$

$$36 + x^2 = \frac{25x^2}{16}$$

$$36 = \frac{9x^2}{16}$$

$$x^2 = 64$$

$$x = \pm 8$$

x	$\xrightarrow{7}$ 8 $\xrightarrow{9}$
$T'(x)$	−0.20 0.16
shape	\ — /

$\therefore$ min @ $x = 8$

$\therefore T(8) = 5\sqrt{36+64} + 4(20-8)$

$= 50 + 48$

$= 98 \ \tfrac{1}{10}$ second

$= 9.8$ seconds

(9) $36\sin(1.5t) - 15\boxed{\cos(1.5t)}$

$= k\sin(1.5t - \alpha)$

$= k(\sin 1.5t\cos\alpha - \cos 1.5t\sin\alpha)$

$= k\cos\alpha\sin(1.5t) - k\sin\alpha\boxed{\cos(1.5t)}$

$\boxed{\begin{aligned} k\cos\alpha &= 36 \\ k\sin\alpha &= 15 \end{aligned}}$

$\tan\alpha = \dfrac{k\sin\alpha}{k\cos\alpha}$

$k^2 = 36^2 + 15^2$

$\alpha = \tan^{-1}\left(\frac{15}{36}\right)$

$k = 39$

$\alpha = 22.6°$

$\dfrac{22.6\pi}{180} = 0.395$ rads

$\therefore 39\sin(1.5t - 0.395)$

$\therefore 39\sin(1.5t - 0.395) + 65 = 100$

$39\sin(1.5t - 0.395) = 35$

$\sin(1.5t - 0.395) = \dfrac{35}{39}$

$1.5t - 0.395 = \sin^{-1}\left(\frac{35}{39}\right)$

$1.5t - 0.395 = 1.114, 2.028$

$1.5t = 1.509, 2.423$

$t = 1.006, 1.615$