HIGHER 2014 PAPER 1

(1) $u_3 = \frac{1}{3}(15)+1 = 6$ (C)

$u_4 = \frac{1}{3}(6)+1 = \underline{\underline{3}}$

(2) $m_R = \frac{2+1}{1-3}$ $\therefore \underline{\underline{m_T = \frac{2}{3}}}$ (B)

$= -\frac{3}{2}$ $(m_1 m_2 = -1)$

(3) $\log_4 12 - \log_4 x = \log_4 6$

$\log_4\left(\frac{12}{x}\right) = \log_4 6$ (A)

$\frac{12}{x} = 6$

$\underline{\underline{x = 2}}$

(4) $3\sin x - 4\boxed{\cos x}$

$= k\cos(x-\alpha)$

$= k\cos\alpha\boxed{\cos x} + k\sin\alpha \sin x$

$k\sin\alpha = 3$ (B)

$k\cos\alpha = -4$

(5) $\int (2x+9)^5\,dx$

$= \frac{(2x+9)^6}{6\times 2} + C$ (D)

$= \underline{\frac{1}{12}(2x+9)^6 + C}$

(6) $2\underline{u} - 3\underline{v} = \begin{pmatrix}-6\\2\\0\end{pmatrix} - \begin{pmatrix}3\\-3\\6\end{pmatrix}$

$= \begin{pmatrix}-9\\5\\-6\end{pmatrix}$ (A)

(7) $\sin 2a = 2\sin a\cos a$

$= 2\left(\frac{3}{\sqrt{34}}\right)\left(\frac{5}{\sqrt{34}}\right)$

$= \frac{30}{34}$ (C)

$= \frac{15}{17}$

(8) $f(x) = (4-9x^4)^{1/2}$

$f'(x) = \frac{1}{2}(4-9x^4)^{-1/2} \times -36x^3$

$= \underline{-18x^3(4-9x^4)^{-1/2}}$ (D)

(9) Max of $\sin x + \sqrt{3}\cos x = 2$

Max of $\sin 2x + \sqrt{3}\cos 2x = 2$

(2x does NOT change max/min values)

$\therefore 5\sin 2x + 5\sqrt{3}\cos 2x$

$= 5(\sin 2x + \sqrt{3}\cos 2x)$

$\therefore$ Max $= 5\times 2$ (B)

$= \underline{\underline{10}}$

(10) Limit occurs where

$-1 < k-2 < 1$

Add 2 to all 3 terms

$\therefore 1 < k < 3$ (C)

(11) $y = 2f(x) + 1$

stretches by x2 in y-axis

slides 1 up y-axis

$\therefore$ (C)

(12) Denominator $\neq 0$

$\therefore x^2+6x-16 = 0$

$(x+8)(x-2) = 0$

$x = -8 \quad x = 2$ (A)

$\therefore x \neq -8,\ x \neq 2$

(13) $\sin\frac{\pi}{3} - \cos\frac{5\pi}{4} = \sin 60° - \cos 225°$

$= \sin 60 - (-\cos 45°)$

$= \frac{\sqrt{3}}{2} - \left(-\frac{1}{\sqrt{2}}\right)$

(B) $= \underline{\underline{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}}$

(14) $\underline{u}.\underline{v} = 0$

$\therefore -6 + 2k + 5k = 0$

$-6 + 7k = 0$

$7k = 6$

$k = \frac{6}{7}$

(D)

(15) Repeated root @ $x = 2$

$\therefore y = k(x+1)(x-2)^2 \quad (0,-8)$

$\therefore -8 = k(0+1)(0-2)^2$

$-8 = k(1)(4)$

$-8 = 4k$

$k = -2$

$\therefore y = -2(x+1)(x-2)^2$

(B)

(16) $\underline{a}.(\underline{a}+2\underline{b}) = \underline{a}.\underline{a} + \underline{a}.2\underline{b}$

$= \underline{a}.\underline{a} + 2\underline{a}.\underline{b}$

$\underline{a}.\underline{a} = |\underline{a}||\underline{a}|\cos\theta$

$= 1 \times 1 \times \cos 0$

$= 1$

$\therefore \underline{a}.(\underline{a}+2\underline{b}) = 1 + 2\left(\frac{2}{3}\right)$

$= 1 + \frac{4}{3}$

$= \frac{7}{3}$

(C)

(17) $3x^2 + 12x + 17$

$= 3[x^2 + 4x] + 17$

$= 3[(x+2)^2 - 4] + 17$

$= 3(x+2)^2 - 12 + 17$

$= 3(x+2)^2 + 5$

$\therefore q = 5$

(B)

(18) $1 - 2\sin^2 x = \cos 2x$

$\therefore 1 - 2\sin^2(15^\circ) = \cos 2(15)^\circ$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

(C)

(19) $\overrightarrow{SW} = \overrightarrow{SR} + \overrightarrow{RW}$

$= \overrightarrow{WP} + 2\overrightarrow{QP}$

$= -\underline{u} - 2\underline{v}$

(A)

(20) $2 - \log_5 \frac{1}{25}$

$= 2 - \log_5(5^{-2})$

$= 2 - (-2)$

$= 4$

(D)

(21) a) $\frac{dy}{dx} = 6x - 3x^2$

SP's where $\frac{dy}{dx} = 0$

$\therefore 6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0 \quad x = 2$

when $x = 0 \quad y = 0$

$x = 2 \quad y = 3(2)^2 - (2)^3$

$= 12 - 8$

$= 4$

$\boxed{\therefore (0,0) \quad (2,4)}$

x	$\xrightarrow{-1}$	0	$\xrightarrow{1}$	2	$\xrightarrow{3}$
$\frac{dy}{dx}$	-9	0	3	0	-9
shape	\	—	/	—	\

$$\boxed{\begin{array}{l} \therefore \text{Max TP @ } (2,4) \\ \text{Min TP @ } (0,0) \end{array}}$$

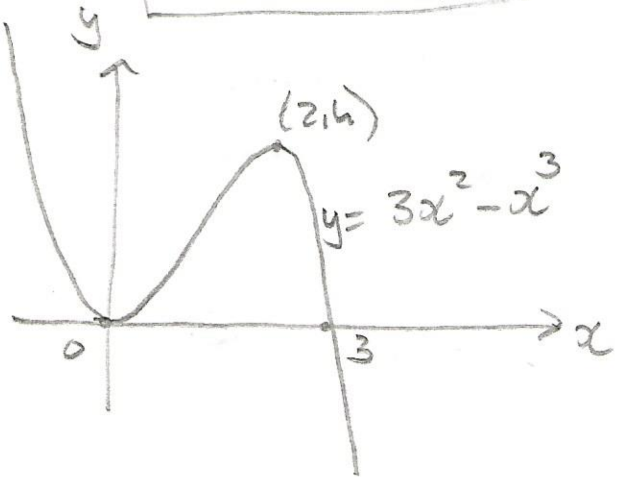
b) when $x=0$ $y=0$ $(0,0)$

when $y=0$ $3x^2-x^3=0$

$x^2(3-x)=0$

$x=0$ $x=3$

$$\boxed{\therefore (0,0)\ (3,0)}$$



(22) a)

$$\begin{array}{r|rrrr} -1 & 6 & 7 & a & b \\ & & -6 & -1 & -a+1 \\ \hline & 6 & 1 & a-1 & \boxed{b-a+1} \end{array}$$

$(x+1)$ is a factor

$\therefore b-a+1=0$

$\underline{\underline{a-b=1}}$

$$\begin{array}{r|rrrr} 2 & 6 & 7 & a & b \\ & & 12 & 38 & 2a+76 \\ \hline & 6 & 19 & a+38 & \boxed{2a+b+76} \end{array}$$

Remainder $=72$

$\therefore 2a+b+76=72$

$\underline{\underline{2a+b=-4}}$

$a-b=1$ — (1)

$2a+b=-4$ — (2)

(1) + 2 $\quad 3a=-3$

$\underline{\underline{a=-1}}$

sub -1 for a in (1)

$-1-b=1$

$-b=2$

$\underline{\underline{b=-2}}$

check in (2): $2a+b$

$=2(-1)+(-2)$

$=\underline{\underline{-4}}$ ✓

$\underline{\underline{a=-1,\ b=-2}}$

b)

$$\begin{array}{r|rrrr} -1 & 6 & 7 & -1 & -2 \\ & & -6 & -1 & 2 \\ \hline & 6 & 1 & -2 & \boxed{0} \end{array}$$

$\therefore 6x^3+7x^2-x-2$

$=(x+1)(6x^2+x-2)$

$=(x+1)(6x^2+4x-3x-2)$

$=(x+1)(2x(3x+2)-1(3x+2))$

$=\underline{\underline{(x+1)(3x+2)(2x-1)}}$

$$\begin{array}{c} 12 \\ \hline 1 \quad 12 \\ 2 \quad 6 \\ 3 \quad 4 \end{array}$$

(23) a)

$x^2+(3x-5)^2+2x-4(3x-5)-15=0$

$x^2+9x^2-30x+25+2x-12x+20-15=0$

$10x^2-40x+30=0$

$10(x^2-4x+3)=0$

$10(x-3)(x-1)=0$

$\underline{x=3 \quad x=1}$

when $x=3$ $y=3(3)-5$

$=4$

$x=1$ $y=3(1)-5$

$=-2$

$$\boxed{\therefore (3,4)\ (1,-2)}$$

b) $T = (-1, 2)$ $P = (3, 4)$ $Q = (1, -2)$

$M_{PT} = \frac{4-2}{3+1}$ $M_{QT} = \frac{-2-2}{1+1}$

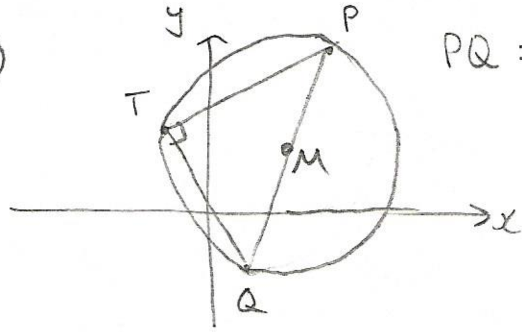
$= \frac{2}{4}$ $= \frac{-4}{2}$

$= \frac{1}{2}$ $= -2$

$M_{PT} \times M_{QT} = \frac{1}{2}(-2)$

$= -1$

$\therefore$ perpendicular (as required)

c) 

PQ = diameter.

$M = \left(\frac{3+1}{2}, \frac{4-2}{2}\right) = (2, 1)$

$r = \sqrt{(2-3)^2 + (1-4)^2}$

$= \sqrt{(-1)^2 + (-3)^2}$

$= \sqrt{10}$

$\therefore (x-2)^2 + (y-1)^2 = 10$

(24) $M = \frac{5-2}{6-0}$ $c = (0, 2)$

$= \frac{3}{6}$

$= \frac{1}{2}$

$\log_9 y = mx + c$

$\log_9 y = \frac{1}{2}x + 2$

$\log_9 y = \log_9 (9^{\frac{1}{2}x}) + \log_9 (9^2)$

$\log_9 y = \log_9 (9^{\frac{1}{2}})^x + \log_9 81$

$\log_9 y = \log_9 3^x + \log_9 81$

$\log_9 y = \log_9 (81(3^x))$

$\therefore y = 81(3^x)$

$k = 81$ $a = 3$

PAPER 2

(1) a) Midpoint $= \left(\frac{3+5}{2}, \frac{0+2}{2}\right)$

$= (4, 1)$

$M_{AB} = \frac{2-0}{5-3}$ $\therefore M_{PB} = -1$

$(M_1 M_2 = -1)$

$= \frac{2}{2}$

$= 1$

$\therefore y - b = m(x - a)$

$y - 1 = -(x - 4)$

$y - 1 = -x + 4$

$y = -x + 5$

b) $y + 2x = 6$

$y = -2x + 6$

Make $y = y$

$\therefore -x + 5 = -2x + 6$

$x = 1$

when $x = 1$, $y = -1 + 5$

$= 4$

$\therefore T = (1, 4)$

c) $M_{AT} = \frac{4-0}{1-3}$

$= \frac{4}{-2}$

$= -2$

$m = \tan\theta$

$\therefore \theta = \tan^{-1}(-2)$

$\theta = 116.6°$

② when $x = 2$, $y = 2^4 - 2(2)^3 + 5$

$= 16 - 2(8) + 5$

$= 5$

$\therefore (2, 5)$.

$\frac{dy}{dx} = 4x^3 - 6x^2$

when $x = 2$, $m = 4(2)^3 - 6(2)^2$

$= 4(8) - 6(4)$

$= 32 - 24$

$= 8$

$\therefore y - b = m(x - a)$

$y - 5 = 8(x - 2)$

③ a) $f(g(x))$

$= f(x+3)$

$= (x+3)(x+3-1) + q$

$= (x+3)(x+2) + q$

$= x^2 + 5x + 6 + q$

b) For equal roots,

$b^2 - 4ac = 0$

$a = 1$ $\quad 5^2 - 4(1)(6+q) = 0$

$b = 5$

$c = 6+q$ $\quad 25 - 24 - 4q = 0$

$1 - 4q = 0$

$q = \frac{1}{4}$

④ a) $C = (11, 12, 6)$

$D = (8, 8, 4)$

b) $\vec{CB} = \begin{pmatrix} 11 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$

$\vec{CD} = \begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$

c) $\vec{CB} \cdot \vec{CD} = 0 \times (-3) + (-8 \times (-4)) + (-4 \times (-2))$

$= 0 + 32 + 8$

$= 40$

$|\vec{CB}| = \sqrt{0^2 + (-8)^2 + (-4)^2}$

$= \sqrt{0 + 64 + 16}$

$= \sqrt{80}$

$|\vec{CD}| = \sqrt{(-3)^2 + (-4)^2 + (-2)^2}$

$= \sqrt{9 + 16 + 4}$

$= \sqrt{29}$

$\cos BCD = \frac{\vec{CB} \cdot \vec{CD}}{|\vec{CB}||\vec{CD}|}$

$BCD = \cos^{-1}\left(\frac{40}{\sqrt{80}\sqrt{29}}\right)$

$BCD = 33.9^\circ$

⑤ $\int_4^t (3x+4)^{-1/2}\, dx = 2$

$\left[\frac{(3x+4)^{1/2}}{\frac{1}{2} \times 3}\right]_4^t = 2$

$\left[\frac{2}{3}\sqrt{3x+4}\right]_4^t = 2$

$\left(\frac{2}{3}\sqrt{3t+4}\right) - \left(\frac{2}{3}\sqrt{16}\right) = 2$

$\frac{2}{3}\sqrt{3t+4} - \frac{8}{3} = 2$

$\frac{2}{3}\sqrt{3t+4} = \frac{6}{3} + \frac{8}{3}$

$\frac{2}{3}\sqrt{3t+4} = \frac{14}{3}$

$\sqrt{3t+4} = 7$

$3t + 4 = 49$

$3t = 45$

$t = 15$

(6) $\sin x - 2\cos 2x = 1$

$\sin x - 2(1 - 2\sin^2 x) = 1$

$\sin x - 2 + 4\sin^2 x = 1$

$4\sin^2 x + \sin x - 3 = 0$

$4s^2 + s - 3 = 0$

$4s^2 + 4s - 3s - 3 = 0$

$4s(s+1) - 3(s+1) = 0$

$(s+1)(4s-3) = 0$

12: 1 12, 2 6, 3 4

$\therefore (\sin x + 1)(4\sin x - 3) = 0$

$\sin x = -1 \qquad \sin x = \frac{3}{4}$

$x = 270^\circ \qquad x = 48.6^\circ, 131.4^\circ$

$\therefore x = \frac{48.6\pi}{180}, \frac{131.4\pi}{180}, \frac{3\pi}{2}$

$x = 0.85,\ 2.29,\ \frac{3\pi}{2}$

(7) a) $2x = 6x - x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$

$\text{Area} = \int_0^4 (\text{TOP} - \text{BOTTOM})\,dx$

$= \int_0^4 (6x - x^2) - (2x)\,dx$

$= \int_0^4 (4x - x^2)\,dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left(2(4)^2 - \frac{4^3}{3}\right) - (0)$

$= \left(32 - \frac{64}{3}\right)$

$= \frac{32}{3} u^2$

$1u^2 = 300 m^2$

$\therefore \text{Area} = 3200 m^2$

b) Parallel to $y = 2x$

$\therefore m = 2$

$\frac{dy}{dx} = 6 - 2x$

$\therefore 6 - 2x = 2$

$-2x = -4$

$x = 2$

when $x = 2$, $y = 6(2) - 2^2$

$= 8$

$\therefore m = 2, \ (2, 8)$

$y - b = m(x - a)$

$y - 8 = 2(x - 2)$

$y - 8 = 2x - 4$

$y = 2x + 4$

$\therefore \text{Area} = \int_0^2 (\text{TOP} - \text{BOTTOM})\,dx$

$= \int_0^2 (2x+4) - (6x - x^2)\,dx$

$= \int_0^2 (x^2 - 4x + 4)\,dx$

$= \left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2$

$= \left(\frac{2^3}{3} - 2(2)^2 + 4(2)\right) - (0)$

$= \frac{8}{3} - 8 + 8$

$= \frac{8}{3} u^2$

$\therefore \text{Area} = 800 m^2$

⑧ $x^2+y^2-2px-4py+3p+2=0$

$g=-p \quad f=-2p \quad c=3p+2$

To be a circle, $r>0$

$\therefore \sqrt{g^2+f^2-c}>0$

$\sqrt{(-p)^2+(-2p)^2-(3p+2)}>0$

$\sqrt{p^2+4p^2-3p-2}>0$

$\sqrt{5p^2-3p-2}>0$

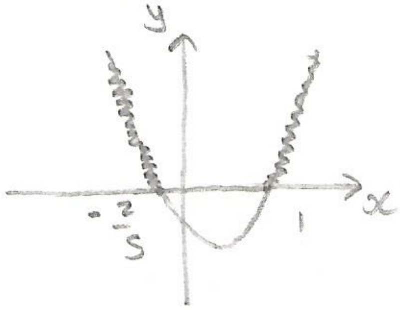
$5p^2-3p-2>0$

$5p^2-5p+2p-2>0$

$5p(p-1)+2(p-1)>0$

$(p-1)(5p+2)>0$

10
1 10
2 5



$\therefore p<-\frac{2}{5},\ p>1$

⑨ a) $a(t)=v'(t)$

$v(t)=8\cos\left(2t-\frac{\pi}{2}\right)$

$\therefore v'(t)=-8\sin\left(2t-\frac{\pi}{2}\right)\times 2$

$=-16\sin\left(2t-\frac{\pi}{2}\right)$

b) $v'(10)=-16\sin\left(20-\frac{\pi}{2}\right)$

$=-16\sin(18.4292\ldots)$

$=-16(-0.40808\ldots)$

$=6.529\ldots$

$v'(10)>0$

$\therefore$ Velocity is INCREASING when $t=10$.

c) $v(t)=s'(t)$

$\therefore s(t)=\int v(t)\,dt$

$=\int 8\cos\left(2t-\frac{\pi}{2}\right)dt$

$=\frac{8\sin\left(2t-\frac{\pi}{2}\right)}{2}+C$

$=4\sin\left(2t-\frac{\pi}{2}\right)+C$

If $s(t)=4$ when $t=0$,

$4=4\sin\left(0-\frac{\pi}{2}\right)+C$

$4=4\sin\left(-\frac{\pi}{2}\right)+C$

$4=4\sin\left(\frac{3\pi}{2}\right)+C$

$4=4(-1)+C$

$4=-4+C$

$C=8$

$\therefore s(t)=4\sin\left(2t-\frac{\pi}{2}\right)+8$