2012 Paper I

① $u_1 = 3(1) + 4 = 7$ Ⓒ
$u_2 = 3(7) + 4 = \underline{25}$

② $\frac{dy}{dx} = 3x^2 - 6$ Ⓓ
$m = 3(-2)^2 - 6$
$= 12 - 6$
$= \underline{6}$

③ $x^2 - 6x + 14 = (x-3)^2 - 9 + 14$
$= \underline{(x-3)^2 + 5}$ Ⓑ

④ $m = \tan 150°$ Ⓑ
$= -\tan 30°$
$= -\frac{1}{\sqrt{3}}$

⑤ $\cos 2a = \cos^2 a - \sin^2 a$
$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$
$= \frac{16}{25} - \frac{9}{25}$ Ⓐ
$= \frac{7}{25}$

⑥ $\frac{dy}{dx} = -6x^{-3} + 3x^{1/2}$ Ⓒ

⑦ $(-3)(1) + (1)(t) + (2t)(-1) = 0$
$-3 + t - 2t = 0$
$-3 - t = 0$ Ⓐ
$\underline{t = -3}$

⑧ $\frac{dv}{dr} = 4\pi r^2$ Ⓒ
when $r = 2$, $\frac{dv}{dr} = \underline{16\pi}$

⑨ $y = a\cos(x - b) + c$ $\therefore$ Ⓐ
$a = 1$ (amplitude)
$b = \frac{\pi}{6}$ $c = -1$

⑩ $\vec{RP} = \vec{RS} + \vec{ST} + \vec{TP}$ Ⓑ
$= -\underline{g} - \underline{f} + h$

⑪ $\int \frac{1}{6x^2}\,dx = \int \frac{1}{6}x^{-2}\,dx$
$= \frac{1}{6}\frac{x^{-1}}{-1} + c$ Ⓓ
$= \underline{-\frac{1}{6}x^{-1} + c}$

⑫ $y = -3\sin\left(x + \frac{\pi}{3}\right) + 2$
$-3 \Rightarrow$ graph from -3 to 3
$+2 \Rightarrow$ up 2 $\quad -1$ to 5
$\therefore$ Max $= 5$
$-3\sin\left(x + \frac{\pi}{3}\right) + 2 = 5$
$-3\sin\left(x + \frac{\pi}{3}\right) = 3$
Ⓑ $\sin\left(x + \frac{\pi}{3}\right) = -1$
$x + \frac{\pi}{3} = \frac{3\pi}{2}$
$x = \frac{3\pi}{2} + \frac{\pi}{3}$
$x = \frac{11\pi}{6}$

⑬ $y = k(x+2)(x+1)$ $(0, 6)$. Ⓓ
$\therefore 6 = k(2)(1)$
$6 = 2k$ $\therefore y = 3(x+2)(x+1)$
$k = 3$

⑭ $\int (2x-1)^{1/2}\,dx = \frac{(2x-1)^{3/2}}{\frac{3}{2}\times 2} + c$
Ⓐ $= \frac{(2x-1)^{3/2}}{3} + c$
$= \underline{\frac{1}{3}(2x-1)^{3/2} + c}$

⑮ $|u| = 1$
$\therefore k\sqrt{3^2 + (-1)^2 + 0^2} = 1$ Ⓓ
$k\sqrt{10} = 1$
$k = \frac{1}{\sqrt{10}}$

(16) $~~\frac{dy}{}~~\, y = 3(\cos x)^4$

$$\frac{dy}{dx} = 12(\cos x)^3 \times (-\sin x)$$

$$= -12\cos^3 x \sin x$$

(C)

(17) $a\cdot(a+b) = a\cdot a + a\cdot b$

$$|a| = \sqrt{3^2+4^2+0}$$

$$= \sqrt{25}$$

$$= 5$$

$$\therefore \underline{a}\cdot\underline{a} = |a||a|\cos 0$$

$$= 25$$

(D)

$$\therefore a.a + a.b = 7$$

$$25 + a.b = 7$$

$$a.b = -18$$

(18) (1) Graph below x axis between s and t ∴ TRUE

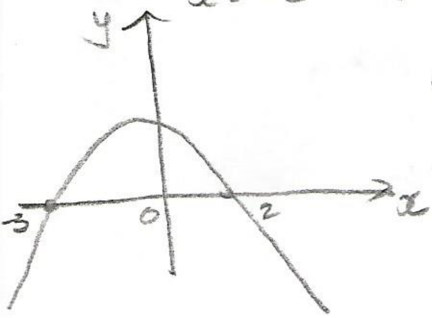
(2) function stationary at P (when $x < q$) so FALSE

(B)

(19) $6 - x - x^2 = 0$

$$(3+x)(2-x) = 0$$

$$x = -3 \quad x = 2$$



$< 0 \Rightarrow$ below x-axis

$\therefore x < -3,\ x > 2$

(B)

(20)

$$\frac{\log_b 9a^2}{\log_b 3a} = \frac{\log_b (3a)^2}{\log_b (3a)}$$

$$= \frac{2\log_b(3a)}{\log_b(3a)}$$

$$= 2$$

(A)

(21)

a)(i)

$$\begin{array}{c|cccc} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -2 \\ \hline & 1 & -1 & -2 & \boxed{0} \\ & x^2 & x & c & \end{array}$$

$R = 0$ ∴ $(x-4)$ is a factor.

(ii) $x^3 - 5x^2 + 2x + 8$

$$= (x-4)(x^2 - x - 2)$$

$$= (x-4)(x-2)(x+1)$$

(iii) $(x-4)(x-2)(x+1) = 0$

$$x = 4,\ x = 2,\ x = -1$$

b) $P = (-1,0)$ $Q = (2,0)$ $R = (4,0)$

$$\therefore A = \int_0^2 (x^3 - 5x^2 + 2x + 8)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x\right]_0^2$$

$$= \left(\frac{16}{4} - \frac{40}{3} + 4 + 16\right) - (0)$$

$$= 24 - \frac{40}{3}$$

$$= \frac{32}{3} = 10\tfrac{2}{3}\,u^2$$

(22) a)

$$\boxed{\cos x} - \sqrt{3}\sin x = k\cos(x+\alpha)$$

$$= k\cos\alpha\boxed{\cos x} - k\sin\alpha\sin x$$

$k\sin\alpha = \sqrt{3}$

$k\cos\alpha = 1$

$k^2 = 3 + 1$

$k = 2$

$$\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$$

$$\tan\alpha = \sqrt{3}$$

$$\alpha = 60°$$

$$\alpha = \frac{\pi}{3}$$

S ✓	A ✓
T	C ✓

$k\sin\alpha = +$

$k\cos\alpha = +$

$$\therefore \cos x - \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{3}\right)$$

b) $y = 2\cos\left(x + \frac{\pi}{3}\right)$

on y-axis, $x = 0$

$\therefore y = 2\cos\left(0 + \frac{\pi}{3}\right)$

$= 2\cos\frac{\pi}{3}$ $\quad \therefore (0, 1)$

$= \underline{\underline{1}}$

on x-axis, $y = 0$

$\therefore 2\cos\left(x + \frac{\pi}{3}\right) = 0$

$\cos\left(x + \frac{\pi}{3}\right) = 0$

$x + \frac{\pi}{3} = \pi, \frac{3\pi}{2}$

$x = \pi - \frac{\pi}{3}, \frac{3\pi}{2} - \frac{\pi}{3}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

$\therefore \left(\frac{\pi}{6}, 0\right), \left(\frac{7\pi}{6}, 0\right)$

23 a) $M = \left(\frac{3-1}{2}, \frac{3+9}{2}\right)$

$= (1, 3)$

$m_{PQ} = \frac{9+3}{-1-3} = \frac{12}{-4} = -3$

$\therefore m_{PB} = \frac{1}{3}$ $(m_1 m_2 = -1)$

$\therefore y - 3 = \frac{1}{3}(x - 1)$

$3y - 9 = x - 1$

$\underline{x - 3y = -8}$

b) $m = -3$ $\quad (1, -2)$

$y + 2 = -3(x - 1)$

$y + 2 = -3x + 3$

$\underline{3x + y = 1}$

c) $x - 3y = -8$ ①

$3x + y = 1$ ②

②×3 $9x + 3y = 3$

$x - 3y = -8$

$10x = -5$

$\underline{x = -\frac{1}{2}}$

$-\frac{1}{2} - 3y = -8$

$-3y = -8 + \frac{1}{2}$

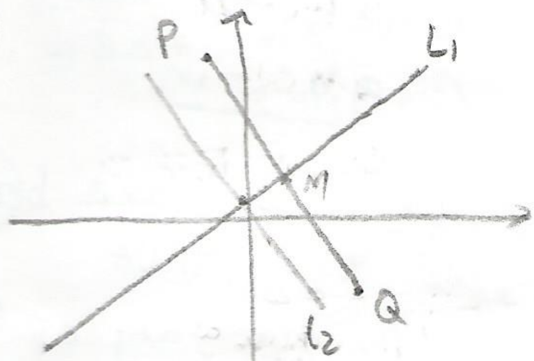
$-3y = -\frac{15}{2}$

$y = \frac{-15}{-6}$

$\underline{y = \frac{5}{2}}$

$\therefore \left(-\frac{1}{2}, \frac{5}{2}\right)$

d)



shortest distance = distance between M and $\left(-\frac{1}{2}, \frac{5}{2}\right)$

$d = \sqrt{\left(1 + \frac{1}{2}\right)^2 + \left(3 - \frac{5}{2}\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{1}{4}}$

$= \sqrt{\frac{10}{4}}$

$= \underline{\frac{\sqrt{10}}{2} \text{ units}}$

Paper 2

① a) (i) $f(g(x)) = f(x+4)$
$= (x+4)^2 + 3$
$= x^2 + 8x + 19$

(ii) $g(f(x)) = g(x^2+3)$
$= x^2 + 7$

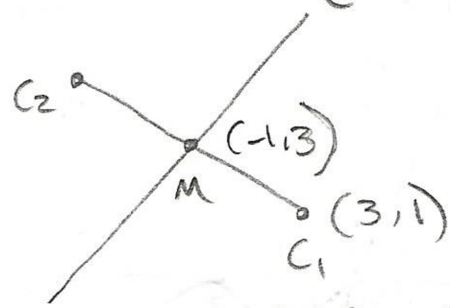
b) $f(g(x)) + g(f(x)) = 0$
$x^2 + 8x + 19 + x^2 + 7 = 0$
$2x^2 + 8x + 26 = 0$
$b^2 - 4ac = 8^2 - 4(2)(26)$
$= 64 - 208$
$= -144$
As $b^2 - 4ac < 0$, no real roots.

② a) $2x - y + 5 = 0$
$y = 2x + 5$
$x^2 + y^2 - 6x - 2y - 30 = 0$
$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$
$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$
$5x^2 + 10x - 15 = 0$
$5(x^2 + 2x - 3) = 0$
$5(x+3)(x-1) = 0$
$x = -3 \qquad x = 1$
$y = 2(-3)+5 \qquad y = 2(1)+5$
$= -1 \qquad = 7$
$\therefore P = (-3, -1) \quad Q = (1, 7)$

b) $x^2 + y^2 - 6x - 2y - 30 = 0$
$2g = -6 \qquad 2f = -2$
$g = -3 \qquad f = -1$
$\therefore$ centre $= (3, 1)$.

$r = \sqrt{g^2 + f^2 - c}$
$= \sqrt{(-3)^2 + (-1)^2 - (-30)}$
$= \sqrt{9 + 1 + 30}$
$= \sqrt{40}$

Congruent $\Rightarrow$ both have radius $= \sqrt{40}$.
Perp bisector of PQ passes through both centres and midpoint is equidistant to both.
$\therefore$ Midpoint $= \left(\frac{-3+1}{2}, \frac{-1+7}{2}\right)$
$= (-1, 3)$.



Stepping out $\Rightarrow C_2 = (-5, 5)$.
$\therefore c = (-5, 5) \quad r = \sqrt{40}$
$(x+5)^2 + (y-5)^2 = 40$

③ $f(0) = 6$
$f(3) = 3^3 - 2(3)^2 - 4(3) + 6$
$= 27 - 18 - 12 + 6$
$= 3$

$f'(x) = 3x^2 - 4x - 4$.
SP's where $f'(x) = 0$
$\therefore 3x^2 - 4x - 4 = 0$
$3x^2 - 6x + 2x - 4 = 0$
$3x(x-2) + 2(x-2) = 0$
$(x-2)(3x+2) = 0$
$x = 2 \qquad \boxed{x = -\frac{2}{3}}$ OUT of RANGE.

12	
1	12
2	6
3	4

$f(2) = 2^3 - 2(2)^2 - 4(2) + 6$
$= 8 - 8 - 8 + 6 \quad = -2$

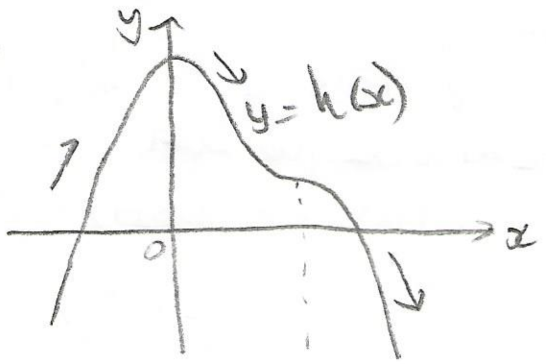
x	$\to$ 2 $\to$
$f'(x)$	-5 0 11
shape	\ — /

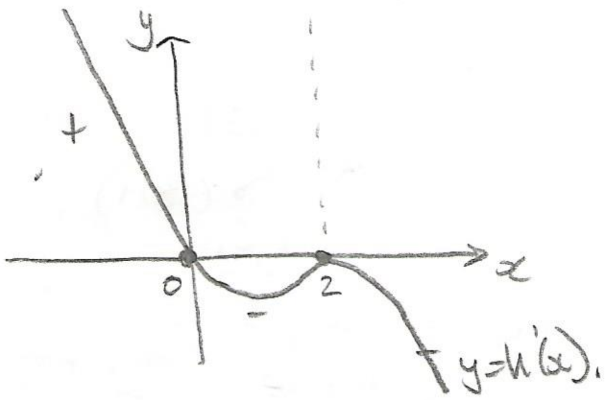
$\therefore$ Min TP @ $(2, -2)$.

$\therefore$ Max $= 6$ (when $x = 0$)

Min $= -2$ (when $x = 2$).

4 a)



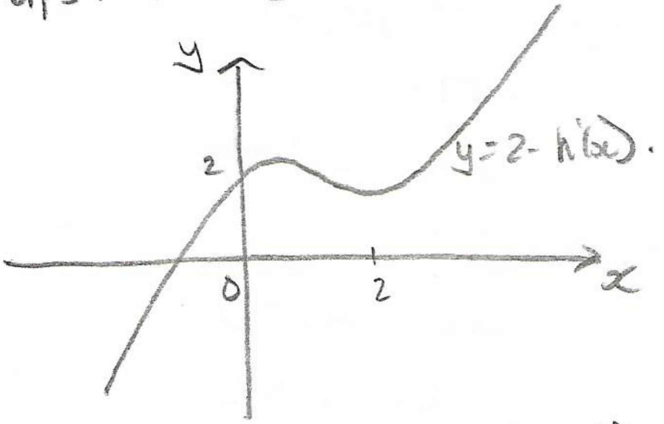


b) $y = 2 - h'(x)$

$= -h'(x) + 2$

flips in x-axis

slides up 2.



5 a) (i) $\overrightarrow{BA} = \underline{a} - \underline{b} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 4 \\ k \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$

(ii) $\cos ABC = \dfrac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$.

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (1)(2) + (0)(k+3) + (-1)(-1)$

$= 2 + 0 + 1$

$= \underline{3}$

$|\overrightarrow{BA}| = \sqrt{1^2 + 0^2 + (-1)^2}$

$= \sqrt{2}$

$|\overrightarrow{BC}| = \sqrt{2^2 + (k+3)^2 + (-1)^2}$

$= \sqrt{4 + k^2 + 6k + 9 + 1}$

$= \sqrt{k^2 + 6k + 14}$

$\therefore \cos ABC = \dfrac{3}{\sqrt{2}\sqrt{k^2+6k+14}}$

$= \dfrac{3}{\sqrt{2(k^2+6k+14)}}$

as required.

b) $\cos 30° = \dfrac{\sqrt{3}}{2}$

$\therefore \dfrac{3}{\sqrt{2(k^2+6k+14)}} = \dfrac{\sqrt{3}}{2}$

$\dfrac{9}{2(k^2+6k+14)} = \dfrac{3}{4}$

$36 = 6(k^2+6k+14)$

$6 = k^2 + 6k + 14$

$k^2 + 6k + 8 = 0$

$(k+2)(k+4) = 0$

$\underline{k = -2,\ k = -4}$

(6) a) Because $-1 < \sin x < 1$ (and $0 < x < \frac{\pi}{2}$)

b) At limit, $u_{n+1} = u_n = L$

$\therefore L = L(\sin x) + \cos 2x$

$L = \frac{1}{2}\sin x$

$\therefore \frac{1}{2}\sin x = \frac{1}{2}\sin x(\sin x) + \cos 2x$

$\frac{1}{2}\sin x = \frac{1}{2}\sin^2 x + \cos 2x$

$\frac{1}{2}\sin x = \frac{1}{2}\sin^2 x + 1 - 2\sin^2 x$

$\frac{1}{2}\sin x = 1 - \frac{3}{2}\sin^2 x$

$\frac{3}{2}\sin^2 x + \frac{1}{2}\sin x - 1 = 0$

$3\sin^2 x + \sin x - 2 = 0$

$(3\sin x - 2)(\sin x + 1) = 0$

$\sin x = \frac{2}{3}$ $\qquad \sin x = -1$

$x = 41.8^\circ, 138.2^\circ$ $\qquad x = 270^\circ$

$= 0.73, 2.41$ $\qquad = \frac{3\pi}{2}$

$\therefore x = 0.73, \cancel{2.41}, \cancel{\frac{3\pi}{2}}$ OUT of interval

(7) a) $4^x = 3^{2-x}$

$\log_a 4^x = \log_a (3^{2-x})$

$x \log_a 4 = (2-x)\log_a 3$

$x \log_a 4 = 2\log_a 3 - x\log_a 3$

$x\log_a 4 + x\log_a 3 = 2\log_a 3$

$x(\log_a 4 + \log_a 3) = 2\log_a 3$

$x(\log_a 12) = \log_a 9$

$x = \frac{\log_a 9}{\log_a 12}$

(as required).

b) let $a = 10$

$x = \frac{\log_{10} 9}{\log_{10} 12} = 0.884.$

$\therefore y = 4^{0.884} = \underline{3.41}$