2011 – Paper 1

① $2p - q - \frac{1}{2}r$

$= \begin{pmatrix} 4 \\ 10 \\ -14 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 10 \\ -13 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$ C

② $3y + 2x = 6$

$3y = -2x + 6$ B

$y = -\frac{2}{3}x + 2$

$m = -\frac{2}{3}$

③ $f(x+2) - 1$ $(1,2) \Rightarrow (-1,1)$

2 left, down 1 $(-2,-3) \Rightarrow (-4,-4)$

D

④ $\frac{dy}{dx} = 3x^2 - 2$ $x = 2$

$\therefore m = 12 - 2$

$= 10$

D

⑤ $x^2 - 8x + 7$ A

$= (x-4)^2 - 16 + 7$

$= (x-4)^2 - 9$

⑥ $m_{cp} = -2 \therefore m_{tan} = \frac{1}{2}$

$y + 3 = \frac{1}{2}(x - 2)$ C

⑦ D

1	1	−1	1	3
		1	0	1
	1	0	1	$\boxed{4}$

⑧ $m = \tan\theta$ A

$= \tan 30°$

$= \frac{1}{\sqrt{3}}$

⑨ $b^2 - 4ac = 23$ B

$23 > 0 \therefore$ (1) is true

23 not square $\therefore$ (2) is false

⑩ $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$ D

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = 30°, 330°$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

⑪ $\int (4x^{\frac{1}{2}} + x^{-3})\,dx$ D

$= \frac{4x^{3/2}}{3/2} + \frac{x^{-2}}{-2} + C$

$= \frac{8}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$

⑫ $\sin(p+q)$

$= \sin p \cos q + \cos p \sin q$

$= \left(\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3}\right) + \left(\frac{1}{\sqrt{5}} \times \frac{2}{3}\right)$

$= \frac{2\sqrt{5}}{3\sqrt{5}} + \frac{2}{3\sqrt{5}}$ C

$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$

⑬ $f(x) = 4\sin 3x$

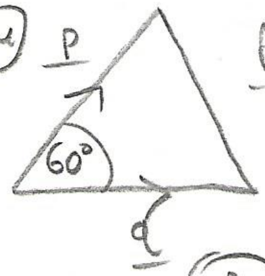
$f'(x) = 12\cos 3x$

$f'(0) = 12\cos 3(0)$ C

$= 12\cos(0)$

$= 12$

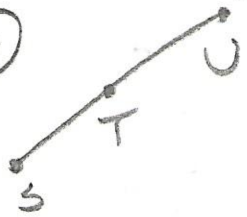
(14)



$$\underset{\sim}{p}\cdot\underset{\sim}{q} = |p||q|\cos\theta$$
$$= 3\times3\times\cos 60^\circ$$
$$= 9\times\frac{1}{2}$$
$$= \frac{9}{2}$$

B

(15)



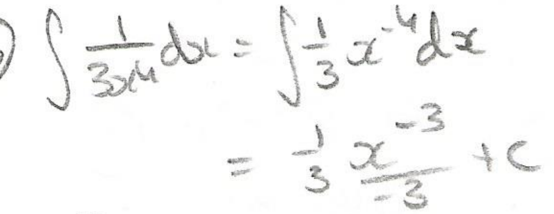
$$\overrightarrow{ST} = \underset{\sim}{t} - \underset{\sim}{s}$$
$$= \begin{pmatrix}-16\\-4\\16\end{pmatrix} - \begin{pmatrix}-4\\5\\1\end{pmatrix}$$
$$= \begin{pmatrix}-12\\-9\\15\end{pmatrix}$$

$$\overrightarrow{TU} = \underset{\sim}{u} - \underset{\sim}{t}$$
$$= \begin{pmatrix}-24\\-10\\26\end{pmatrix} - \begin{pmatrix}-16\\-4\\16\end{pmatrix}$$
$$= \begin{pmatrix}-8\\-6\\10\end{pmatrix}$$

$\frac{-12}{-8} = \frac{3}{2}$

$\frac{-9}{-6} = \frac{3}{2}$ $\quad \therefore \overrightarrow{ST} = \frac{3}{2}\overrightarrow{TU}$

$\frac{15}{10} = \frac{3}{2}$



$\therefore 2:3$

B

(16)

$$\int \frac{1}{3x^4}dx = \int \frac{1}{3}x^{-4}dx$$
$$= \frac{1}{3}\frac{x^{-3}}{-3} + C$$
$$= -\frac{1}{9x^3} + C$$

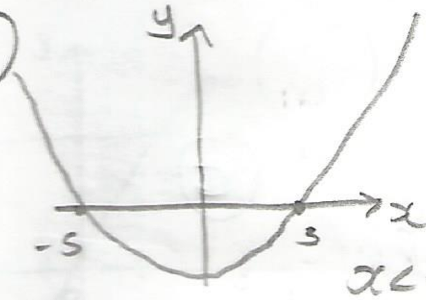
A

(17)

$$y = kx(x+1)(x-2)$$
$$2 = k(2)(-1)$$
$$2 = -2k$$
$$k = -1$$
$$\therefore y = -x(x+1)(x-2)$$

A

(18)



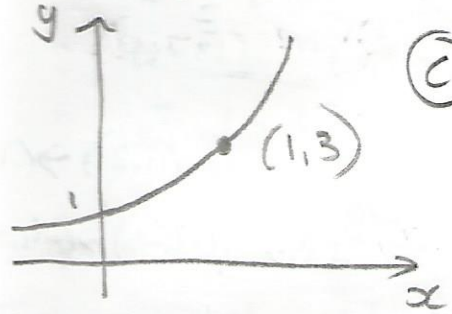
$x < -5$

$x > 3$

C

(19)

$$\log_3 y = x$$
$$y = 3^x$$



C

(20)

$$g(x) = \sin^2\sqrt{x-2}$$

$\sin^2 \Rightarrow$ positive values

$\therefore 0 \le g(x) \le 1$

$\sqrt{x-2} \Rightarrow x-2 \geq 0$

$x \geq 2$

D

(21) a) $B = (7, 12)$

$D = (2, -3)$

$$M_{BD} = \frac{-3-12}{2-7} = \frac{-15}{-5} = 3$$

$$y - b = m(x-a)$$
$$y - 12 = 3(x-7)$$
$$y - 12 = 3x - 21$$
$$3x - y = 9$$

b) $x + 3y = 23$ — (1)
$3x - y = 9$ — (2)

$9x - 3y = 27$ — (3)
$x + 3y = 23$ — (1)

ADD $10x = 50$
$x = 5$

sub 5 for x in (1)
$5 + 3y = 23$
$3y = 18$
$y = 6$

check in (2): $3(5) - 6$
$= 15 - 6$
$= 9$ ✓

$\therefore E = (5,6)$

c) (i) $A = (-1,8)$ $B = (7,12)$

Mid $= \left(\frac{-1+7}{2}, \frac{8+12}{2}\right)$
$= (3,10)$

$M_{AB} = \frac{12-8}{7+1} = \frac{4}{8} = \frac{1}{2}$

$\therefore M_{PB} = -2$ $(m_1 m_2 = -1)$

$y - b = m(x - a)$
$y - 10 = -2(x - 3)$
$y - 10 = -2x + 6$
$2x + y = 16$

d) $E = (5,6)$
$2x + y = 2(5) + 6$
$= 10 + 6$
$= 16$ ✓

$\therefore$ Perpendicular bisector passes through E.

(22) a) (i) $(x-2)(x^2+1) = 0$
$x - 2 = 0$ $x^2 + 1 = 0$
$x = 2$ no sol^{ns}.

$\therefore \boxed{(2,0)}$

(ii) $y = (0-2)(0^2+1)$
$= (-2)(1)$
$= -2$
$\boxed{\therefore (0,-2)}$

b) $f(x) = (x-2)(x^2+1)$
$= x^3 - 2x^2 + x - 2$

$f'(x) = 3x^2 - 4x + 1$

SP's @ $f'(x) = 0$
$3x^2 - 4x + 1 = 0$
$(3x - 1)(x - 1) = 0$
$3x - 1 = 0$ $x - 1 = 0$
$x = \frac{1}{3}$ $x = 1$

when $x = \frac{1}{3}$ $y = \left(\frac{1}{3} - 2\right)\left(\frac{1}{3}^2 + 1\right)$
$= \left(-\frac{5}{3}\right)\left(\frac{1}{9} + 1\right)$
$= \left(-\frac{5}{3}\right)\left(\frac{10}{9}\right)$
$= -\frac{50}{27}$

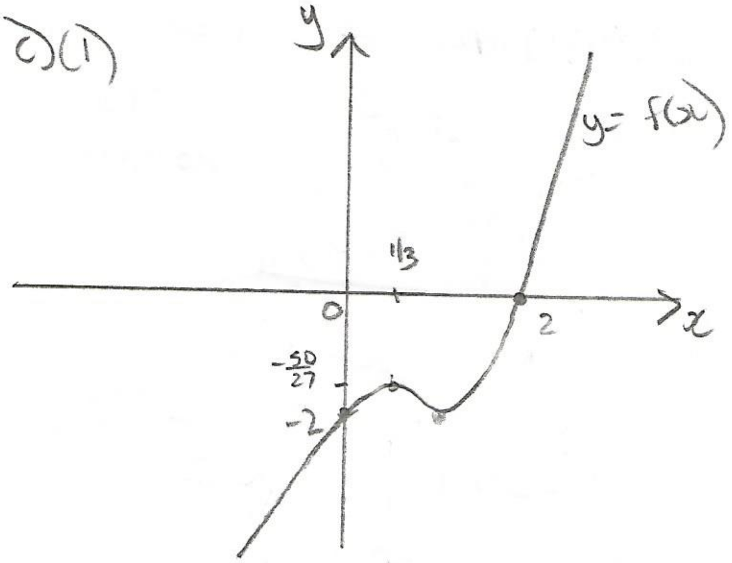
when $x = 1$, $y = (1-2)(1+1)$
$= (-1)(2)$
$= -2$

$\therefore$ TP's @ $\left(\frac{1}{3}, -\frac{50}{27}\right)$
$(1, -2)$

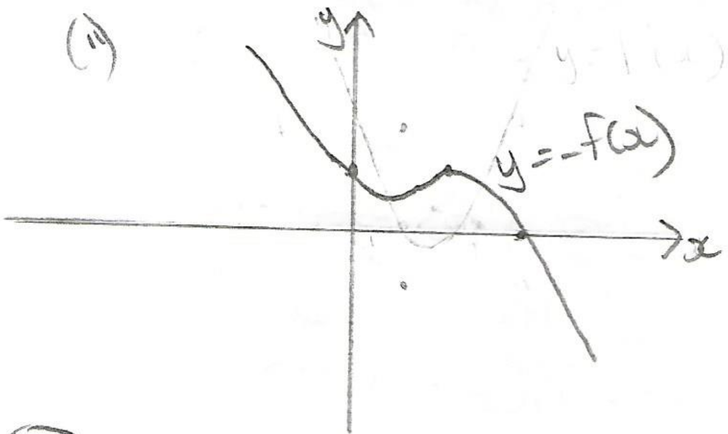
x	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2
$f'(x)$	1	0	$-\frac{1}{4}$	0	5
shape	/	—	\	—	/

$\therefore$ Max TP @ $\left(\frac{1}{3}, -\frac{50}{27}\right)$
Min TP @ $(1, -2)$

c)(i)



(ii)



(23) a) $\cos 2x - 3\cos x + 2 = 0$

$2\cos^2 x - 1 - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ $\quad\quad$ $\cos x = 1$

$x = 60^\circ, 300^\circ$ $\quad\quad$ $x = 0^\circ, \cancel{360^\circ}$ $(<360^\circ)$

$\therefore\ x = 0^\circ, 60^\circ, 300^\circ$

b)

$\cos 4x - 3\cos 2x + 2 = 0 \quad \{0 \le x \le 360^\circ\}$

$\cos 2(2x) - 3\cos(2x) + 2 = 0$

$(2\cos 2x - 1)(\cos 2x - 1) = 0 \quad \{0 \le 2x \le 720^\circ\}$

$\cos 2x = \frac{1}{2}$ $\quad\quad$ $\cos 2x = 1$

$2x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$ $\quad\quad$ $2x = 0^\circ, 360^\circ, 720^\circ$

$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ $\quad\quad$ $x = 0^\circ, 180^\circ, \cancel{360^\circ}$

$\therefore\ x = 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ$

PAPER II

(1) a) $B = (4, 4, 0)$

b) $\overrightarrow{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$

$\overrightarrow{DM} = \underline{m} - \underline{d}$

$= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

c) $|\overrightarrow{DB}| = \sqrt{2^2 + 2^2 + (-6)^2}$

$= \sqrt{4 + 4 + 36}$

$= \sqrt{44}$

$|\overrightarrow{DM}| = \sqrt{0^2 + (-2)^2 + (-6)^2}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$\overrightarrow{DB} \cdot \overrightarrow{DM} = (2 \times 0) + (2 \times (-2)) + ((-6) \times$

$= 0 - 4 + 36$

$= 32$

$\cos BDM = \dfrac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{|\overrightarrow{DB}|\,|\overrightarrow{DM}|}$

$\cos BDM = \dfrac{32}{\sqrt{44}\sqrt{40}}$

$BDM = \cos^{-1}\left(\dfrac{32}{\sqrt{1760}}\right)$

$BDM = 40.3^\circ$

② a) $g(f(x)$
$= g(x^3 - 1)$
$= 3(x^3 - 1) + 1$
$= \underline{\underline{3x^3 - 2}}$

b) $g(f(x)) + xh(x)$
$= 3x^3 - 2 + x(4x - 5)$
$= 3x^3 - 2 + 4x^2 - 5x$
$= 3x^3 + 4x^2 - 5x - 2$

c)(i)

1	3	4	-5	-2
		3	7	2
	3	7	2	0

Since remainder $= 0$, $(x-1)$ is a factor.

(ii) $3x^3 + 4x^2 - 5x - 2$
$= (x-1)(3x^2 + 7x + 2)$
$= (x-1)(3x+1)(x+2)$

d) $g(f(x)) + xh(x) = 0$
$3x^3 + 4x^2 - 5x - 2 = 0$
$(x-1)(3x+1)(x+2) = 0$
$\underline{x = 1, \ x = -\frac{1}{3}, \ x = -2}$

③

a) $u_{n+1} = -\frac{1}{2}u_n \qquad u_0 = -16$
$u_1 = -\frac{1}{2}(-16) = 8$
$u_2 = -\frac{1}{2}(8) = -4$

b) $u_1 = 4, \ u_2 = 5, \ u_3 = 7$

$v_2 = pv_1 + q \qquad v_3 = pv_2 + q$
$5 = 4p + q \qquad 7 = 5p + q$

$5p + q = 7 \quad -①$
$4p + q = 5 \quad -②$

①-② $\underline{p = 2}$
Sub 2 for p in ①
$5p + q = 7$
$10 + q = 7$
$\underline{\underline{q = -3}}$

$p = 2$
$q = -3$

c)(i) $L = -\frac{1}{2}L$
$\frac{3}{2}L = 0$
$\underline{\underline{L = 0}}$

(ii) For limits, $-1 < p < 1$, but $p = 1$ ∴ no limit.

④

(A1) $x^3 - x^2 - 4x + 4 - (2x + 4)$
$= x^3 - x^2 - 4x + 4 - 2x - 4$
$= \underline{x^3 - x^2 - 6x}$

$A = \int_{-2}^{0} (x^3 - x^2 - 6x)\,dx$
$= \left[\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^{0}$
$= (0) - \left(\frac{16}{4} + \frac{8}{3} - 12\right)$
$= -\left(\frac{48}{12} + \frac{32}{12} - \frac{144}{12}\right)$
$= -\left(-\frac{64}{12}\right)$
$= \underline{\underline{\frac{64}{12}u^2}}$

(A2) $2x + 4 - (x^3 - x^2 - 4x + 4)$
$= 2x + 4 - x^3 + x^2 + 4x - 4$
$= \underline{6x + x^2 - x^3}$

$$A = \int_0^3 (6x + x^2 - x^3)\,dx$$

$$= \left[3x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^3$$

$$= \left(27 + \frac{27}{3} - \frac{81}{4}\right) - (0)$$

$$= \frac{324}{12} + \frac{108}{12} - \frac{243}{12}$$

$$= \frac{189}{12}\,u^2$$

$$\therefore \text{Total} = \frac{64}{12} + \frac{189}{12}$$

$$= \frac{253}{12}$$

$$= 21\tfrac{1}{12}\,u^2$$

5 $M = \frac{7-5}{4-0} = \frac{2}{4} = \frac{1}{2}$

$$\log_2 y = \tfrac{1}{2}\log_2 x + 5$$

$$\log_2 y = \log_2 x^{1/2} + \log_2 32$$

$$\log_2 y = \log_2 32x^{1/2}$$

$$y = 32x^{1/2}$$

$$\therefore k = 32$$

$$n = \tfrac{1}{2}$$

6

a) $3\sin x - 5\boxed{\cos x}$

$$= R\sin(x + \alpha)$$

$$= R\sin x\cos\alpha + R\cos x\sin\alpha$$

$$= R\cos\alpha\,\sin x + R\sin\alpha\,\boxed{\cos x}$$

$$\boxed{\begin{aligned} R\cos\alpha &= 3 \\ R\sin\alpha &= -5 \end{aligned}}$$

$$R^2 = 3^2 + (-5)^2$$

$$R^2 = 34$$

$$R = \sqrt{34}$$

$$\tan\alpha = \frac{R\sin\alpha}{R\cos\alpha}$$

$$\tan\alpha = \frac{-5}{3}$$

$$\alpha = \tan^{-1}\left(-\tfrac{5}{3}\right)$$

$$\alpha = \text{"}1.03\text{"}$$

S	A ✓
✓T	C ✓✓

sin = −
cos = +
∴ Q4

$$\therefore \alpha = 2\pi - 1.03$$

$$= 5.25$$

$$\therefore 3\sin x - 5\cos x = \sqrt{34}\sin(x + 5.25)$$

b) $\int_0^t (3\cos x + 5\sin x)\,dx = 3$

$$\left[3\sin x - 5\cos x\right]_0^t = 3$$

$$\left[\sqrt{34}\sin(x + 5.25)\right]_0^t = 3$$

$$\left(\sqrt{34}\sin(t + 5.25)\right) - \left(\sqrt{34}\sin(0 + 5.25)\right) = 3$$

$$\sqrt{34}\sin(t + 5.25) - \sqrt{34}\sin 5.25 = 3$$

$$\sqrt{34}\sin(t + 5.25) = 3 + \sqrt{34}\sin 5.25$$

$$\sin(t + 5.25) = \frac{3 + \sqrt{34}\sin 5.25}{\sqrt{34}}$$

$$t + 5.25 = \sin^{-1}\left(\frac{3 + \sqrt{34}\sin 5.25}{\sqrt{34}}\right)$$

$$t + 5.25 = \sin^{-1}(-0.344439)$$

$$t + 5.25 = \text{"}0.352\text{"}$$

S	A
T	C

$$\pi + 0.352 = 3.49$$

$$2\pi - 0.352 = 5.93$$

$$t + 5.25 = 3.49,\ 5.93$$

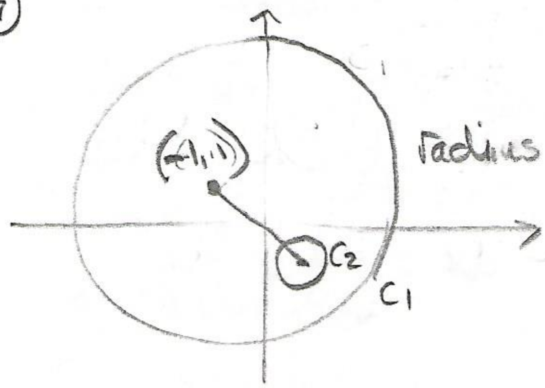
$$t = -1.76,\ 0.68$$

$$t = 0.68,\ 4.52$$

$$\therefore t = 0.68$$

$$(t < 2)$$

(7)



radius $= \sqrt{121}$
$= \underline{\underline{11}}$

C_2 : centre $= (2,-3)$

radius $= \sqrt{g^2+f^2-c}$

$= \sqrt{2^2+3^2-p}$

$= \underline{\sqrt{13-p}}$

From C_1 to C_2:

$d = \sqrt{(-1-2)^2+(1+3)^2}$

$= \sqrt{(-3)^2+4^2}$

$= \underline{\underline{5}}$

For no points of contact,

Distance $C_1 \rightarrow C_2$ + radius $C_2 < 11$

ie $\quad 5+\sqrt{13-p} < 11$

$\sqrt{13-p} < 6$

$13-p < 36$

$-p < 23$

$\underline{\underline{p > -23}}$

Also, $13-p > 0$ (ie $\sqrt{13-p}$)

$-p > -13$

$p < 13$

$\therefore \underline{\underline{-23 < p < 13}}$