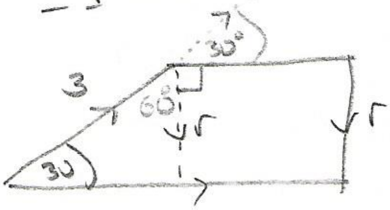
$\underline{p} \cdot \underline{r} = |\underline{p}||\underline{r}| \cos 90^\circ$

$= 0.$

$\therefore \underline{p} \cdot (\underline{q} + \underline{r}) = \underline{\underline{6\sqrt{3}}}$

$\underline{r} \cdot (\underline{p} - \underline{q}) = \underline{r} \cdot \underline{p} - \underline{r} \cdot \underline{q}$

$\underline{r} \cdot \underline{p} = 0.$



$\sin 30^\circ = \frac{r}{3}$

$r = 3 \sin 30^\circ$

$\underline{\underline{r = \frac{3}{2}}}$

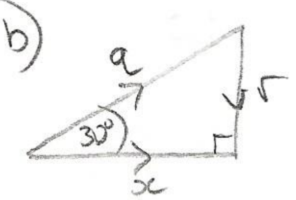
$\underline{r} \cdot \underline{q} = |\underline{r}||\underline{q}| \cos \theta$

$= \frac{3}{2} \times 3 \times \cos 120^\circ$

$= \frac{9}{2} \times \left(-\frac{1}{2}\right)$

$= \underline{\underline{-\frac{9}{4}}}$

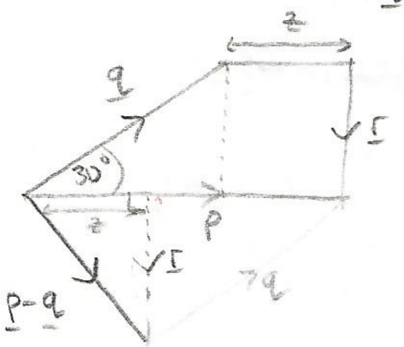
b)



$\underline{x} = \underline{q} + \underline{r}$

$\cos 30^\circ = \frac{|\underline{q}+\underline{r}|}{3}$

$|\underline{q}+\underline{r}| = \frac{3\sqrt{3}}{2}$



$\underline{z} = \underline{p} - (\underline{q} + \underline{r})$

$= 4 - \frac{3\sqrt{3}}{2}$

$(|\underline{p} - \underline{q}|)^2 = |\underline{r}|^2 + |\underline{q} + \underline{r}|^2$

$|\underline{p} - \underline{q}|^2 = \left(\frac{3}{2}\right)^2 + \left(4 - \frac{3\sqrt{3}}{2}\right)^2$

$|\underline{p} - \underline{q}|^2 = \frac{9}{4} + 16 - \frac{24\sqrt{3}}{2} + \frac{27}{4}$

$|\underline{p} - \underline{q}|^2 = \frac{36}{4} + 16 - 12\sqrt{3}$

$|\underline{p} - \underline{q}|^2 = 25 - 12\sqrt{3}$

$\underline{\underline{|\underline{p} - \underline{q}| = \sqrt{25 - 12\sqrt{3}}}}$

2010 Paper I

1. $2x - 3y - 6 = 0$

   $3y = 2x - 6$

   $y = \frac{2}{3}x - 2$

   $\therefore M_{perp} = -\frac{3}{2}$ A

2. $u_0 = 1$

   $u_1 = 2(1) + 3 = 5$

   $u_2 = 2(5) + 3 = \underline{\underline{13}}$ C

3. $3\underline{u} - 2\underline{v}$

   $= \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix}$ D

   $= \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$

4. 3 waves $\Rightarrow b = 3$

   Max 2, min 2 $\Rightarrow a = 2$, $c = 0$ A

   $\therefore 2\cos 3x$

5. $x^2 + 8x + 3$

   $= (x+4)^2 - 16 + 3$ B

   $= (x+4)^2 - 13$

6. $b^2 - 4ac = 0$ $a = k$, $b = -3$, $c = 2$ D

   $9 - 8k = 0$

   $k = \frac{9}{8}$

7. $L = \frac{1}{4}L + 7$

   $\frac{3}{4}L = 7$ C

   $L = \frac{28}{3}$

8. $C = (3, 5) \therefore r = 5$ B

   $\therefore L \Rightarrow y = 10$

9. $\int (2x^{-4} + \cos 5x)\,dx$ C

   $= \frac{2x^{-3}}{-3} + \frac{1}{5}\sin 5x + C$

(10) $\begin{pmatrix} x \\ 5 \\ 7 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$\underline{a} \cdot \underline{b} = -3x + 10 - 7$

$\underline{a} \cdot \underline{b} = 0$ (B)

$\therefore -3x + 3 = 0$

$\underline{\underline{x = 1}}$

(11) $g\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$ (D)

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \underline{\underline{\frac{1}{2}}}$

(12) $f(x) = x^{-1/5}$ (A)

$f'(x) = -\frac{1}{5}x^{-6/5}$

(13) $a > 0 \Rightarrow$ happy face (B)

$b^2 - 4ac > 0 \Rightarrow$ distinct roots

(14) $\int_{-2}^{2} (14 - x^2 - (2x^2 + 2))\,dx$

$= \int_{-2}^{2} (14 - x^2 - 2x^2 - 2)\,dx$ (C)

$= \int_{-2}^{2} (12 - 3x^2)\,dx$

(15)

$f'(x) = x^2 - 9$

(i) $f'(1) = 1 - 9$ $\therefore$ decreasing ✗

$= -8$

(ii) $f'(3) = 3^2 - 9$ $\therefore$ stationary ✓

$= 9 - 9$

$= 0$

only 2 correct (C)

(16) $t = -5$

$y = k(x-1)(x-1)(x-5)$

$10 = k(-1)(-1)(-5)$

$10 = -5k$ (A)

$\underline{\underline{k = -2}}$

(17) $\frac{ds}{dt} = 2t - 5$ (B)

when $t = 3$ $\frac{ds}{dt} = 6 - 5$

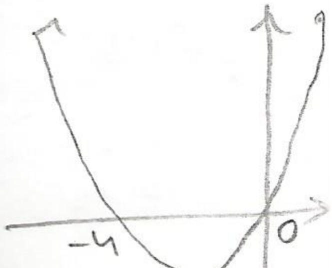
$\underline{\underline{= 1}}$

(18) $x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$ $x = -4$

happy face



(B) $x^2 + 4x > 0$

where $x < -4$, $x > 0$

(19) 3 right $\therefore$ $y = \log_a(x - 3)$

$(6,1)$ should be $(3,1)$

(C) $\therefore f(x) = \log_3(x - 3)$

(20) $y = f(2x) - 3$ $\Rightarrow (6,4)$

$y = f(2x)$ $\Rightarrow (6,7)$ (A)

$y = f(x)$ $\Rightarrow (12,7)$

(21) a) $B = (-4,16)$

$Q =$ mid of $(4,0)$ $(18,20)$

$= (11,10)$

$\therefore M_{BQ} = \frac{16-10}{-4-11} = \frac{6}{-15} = -\frac{2}{5}$

$\therefore y - b = m(x - a)$

$y - 10 = -\frac{2}{5}(x - 11)$

$5y - 50 = -2x + 22$

$2x + 5y = 72$

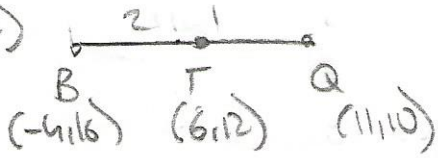
b) $T = (6, 12)$

$2(6) + 5(12)$

$= 12 + 60$

$= 72$ $\therefore$ T is on BQ.

c)



$\overrightarrow{BT} = \begin{pmatrix} -4 \\ 10 \end{pmatrix}$ $\overrightarrow{TQ} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\therefore \overrightarrow{BT} = 2\,TQ$

$\therefore$ T divides BQ 2 : 1

22 a)

(i)

1	2	1	-8	5
		2	3	-5
	2	3	-5	0

$R = 0$ $\therefore$ $(x-1)$ is a factor

(ii) $f(x) = (x-1)(2x^2 + 3x - 5)$

$= (x-1)(2x+5)(x-1)$

b) $2x^3 + x^2 - 8x + 5 = 0$

$(x-1)(2x+5)(x-1) = 0$

$x = 1, \quad x = -5/2$

c) $2x^3 + x^2 - 6x + 2 = 2x - 3$

$2x^3 + x^2 - 8x + 5 = 0$

$(x-1)(2x+5)(x-1) = 0$

$x = 1, \quad x = -5/2$

Tangent @ $x = 1$

$y = 2 - 3$

$= -1$

$\therefore G = (1, -1)$

d) other point of contact

$\Rightarrow x = -5/2$

$y = 2(-5/2) - 3$

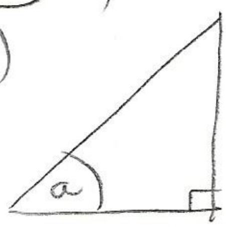
$= -5 - 3$

$= -8$

$\therefore H = (-5/2, -8)$

23 a)

(i)



$3x - 2y = 0$

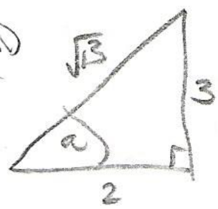
$2y = 3y$

$y = \frac{3}{2}x$

$m = \tan a$

$\therefore \tan a = \frac{3}{2}$

(ii)

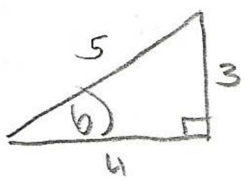


$\sin a = \frac{3}{\sqrt{13}}$

b) $3x - 4y = 0$

$4y = 3x$

$y = \frac{3}{4}x$ $\therefore \tan b = \frac{3}{4}$



$\sin b = \frac{3}{5}$

$\cos b = \frac{4}{5}$

c) (i) $\sin(a - b)$

$= \sin a \cos b - \cos a \sin b$

$= \left(\frac{3}{\sqrt{13}} \times \frac{4}{5}\right) - \left(\frac{2}{\sqrt{13}} \times \frac{3}{5}\right)$

$= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}}$

$= \frac{6}{5\sqrt{13}}$

(ii) $-\frac{6}{5\sqrt{13}}$

2010 Paper 2

① a) $M = (0,1,0)$
$N = (4,2,2)$

b) $\vec{VM} = \underline{m} - \underline{v}$ $V = (0,2,3)$

$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$

$\vec{VN} = \underline{n} - \underline{v}$

$= \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

c) $|\vec{VM}| = \sqrt{1^2+3^2}$
$= \sqrt{10}$

$|\vec{VN}| = \sqrt{16+1}$
$= \sqrt{17}$

$VM \cdot VN = (0 \times 4) + (-1 \times 0) + (-3 \times -1)$
$= \underline{\underline{3}}$

$\therefore \cos\theta = \frac{3}{\sqrt{10}\sqrt{17}}$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{170}}\right)$

$\underline{\underline{\theta = 76.7^\circ}}$

②

a) $12\cos x + 5\sin x = k\cos(x + \alpha)$
$= k\cos\alpha\cos x - k\sin\alpha\sin x$

$k\sin\alpha = 5$

$k\cos\alpha = 12$

$k^2 = 5^2 + 12^2$

$k = 13$

$\tan\alpha = \frac{5}{12}$

$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\alpha = 22.6^\circ$

S A
T C

Q_1

$\therefore 12\cos x - 5\sin x = 13\cos(x + 22.6)^\circ$

b) (i) Max = 13, min = −13

(ii) Max when $\cos(x+22.6)^\circ = 1$

$(x + 22.6) = 0, 360$

$x = -22.6, 337.4$

$\underline{\underline{x = 337.4^\circ}}$

Min when $\cos(x+22.6)^\circ = -1$

$x + 22.6^\circ = 180$

$\underline{\underline{x = 202.6^\circ}}$

③ a) $x^2+y^2+14x+4y-19=0$

(i) $x^2+(3-x)^2+14x+4(3-x)-19=0$

$x^2+9-6x+x^2+14x+12-4x-19=0$

$2x^2+4x+2=0$

$2(x^2+2x+1)=0$

$b^2-4ac = 4-4(1)(1)$
$= 4-4$
$= 0$

$\therefore$ Line is a tangent.

(ii) $2(x^2+2x+1)=0$

$2(x+1)^2=0$

$x=-1$

$y=3-(-1)$
$=4$

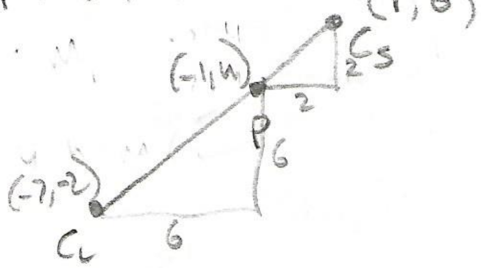
$\underline{\underline{(-1,4)}}$

b) $r_{large} = \sqrt{7^2+2^2+19}$
$= \sqrt{49+4+19}$
$= \sqrt{72}$
$= 6\sqrt{2}$

$\therefore r_{small} = 2\sqrt{2}$

$C_L = (-7,-2)$

$P = (-1,4)$.



$\therefore C_{small} = (1,6)$

$\therefore \underline{(x-1)^2 + (y-6)^2 = 8}$

(4) $2\cos 2x - 5\cos x - 4 = 0$

$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$

$4\cos^2 x - 2 - 5\cos x - 4 = 0$

$4\cos^2 x - 5\cos x - 6 = 0$

$4\cos^2 x - 8\cos x + 3\cos x - 6 = 0$

$4\cos x(\cos x - 2) + 3(\cos x - 2) = 0$

$(\cos x - 2)(4\cos x + 3) = 0$

24	
1	24
2	12
3	8
4	6

$\cos x = 2$ NO SOLN

$\cos x = -3/4$

$x = \cos^{-1}\left(\frac{-3}{4}\right)$

$x = \text{"}41.41°\text{"}$

S	A
T	C

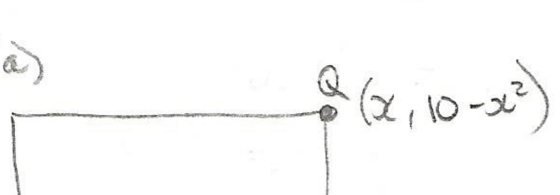
$x = 138.59°, 221.41°$

$\therefore x = \frac{138.59\pi}{180}, \frac{221.41\pi}{180}$

$= \underline{2.42, 3.86}$

(5) a)

(1)



T: $y = \frac{2}{5}(10 - x^2)$ $x = 0$

$= \frac{2}{5}(10)$

$= \underline{4}$ $\therefore P = (x, 4)$

$\therefore QP = 10 - x^2 - 4$

$= 6 - x^2$.

$A = lb = 2x(6 - x^2)$

$= \underline{12x - 2x^3}$

b) $A'(x) = 12 - 6x^2$

SP's @ $A'(x) = 0$

$12 - 6x^2 = 0$

$6x^2 = 12$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\therefore x = \sqrt{2}$

x	1	$\sqrt{2}$	2
$A'(x)$	6	0	-12
shape	/	—	\

$\therefore$ Max when $x = \sqrt{2}$

$\therefore$ Area $= 12\sqrt{2} - 2(\sqrt{2})^3$

$= 12\sqrt{2} - 4\sqrt{2}$

$= \underline{8\sqrt{2}\text{ unit}^2}$

(6) a) $y = (2x - 9)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-1/2} \times 2$

$= (2x-9)^{-1/2}$

$= \frac{1}{(2x-9)^{1/2}}$

$= \frac{1}{\sqrt{2x-9}}$.

when $x = 9$,

$m = \frac{1}{\sqrt{18-9}}$

$= \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

when $x=9$, $y=(18-9)^{1/2}$

$=9^{1/2}$

$=3$

$\therefore (9,3)$

$y-b=m(x-a)$

$y-3=\frac{1}{3}(x-9)$

$3y-9=x-9$

$3y=x$

$y=\frac{1}{3}x$

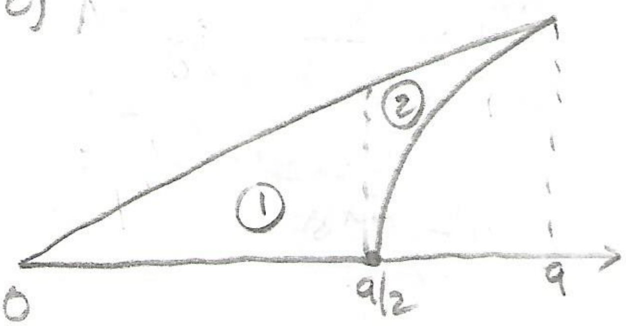
b) $(2x-9)^{1/2}=0$

$2x-9=0$

$2x=9$

$x=\frac{9}{2}$ $\therefore \left(\frac{9}{2},0\right)$

c)



$A_1=\int_0^{9/2}\frac{1}{3}x\,dx$

$=\left[\frac{1}{6}x^2\right]_0^{9/2}$

$=\left(\frac{1}{6}\left(\frac{9}{2}\right)^2\right)-(0)$

$=\left(\frac{1}{6}\left(\frac{81}{4}\right)\right)$

$=\frac{81}{24}u^2$

$A_2=\int_{9/2}^{9}\left(\frac{1}{3}x-(2x-9)^{1/2}\right)dx$

$=\left[\frac{1}{6}x^2-\frac{(2x-9)^{3/2}}{\frac{3}{2}\times 2}\right]_{9/2}^{9}$

$=\left[\frac{1}{6}x^2-\frac{\sqrt{(2x-9)^3}}{3}\right]_{9/2}^{9}$

$=\left(\frac{81}{6}-\frac{\sqrt{(18-9)^3}}{3}\right)-\left(\frac{81}{24}-\frac{\sqrt{(9-9)^3}}{3}\right)$

$=\left(\frac{81}{6}-\frac{\sqrt{9^3}}{3}\right)-\left(\frac{81}{24}\right)$

$=\left(\frac{81}{6}-\frac{27}{3}\right)-\frac{81}{24}$

$=\left(\frac{81}{6}-\frac{54}{6}\right)-\frac{81}{24}$

$=\frac{27}{6}-\frac{81}{24}$

$=\frac{108}{24}-\frac{81}{24}$

$=\frac{27}{24}u^2$

$\therefore$ Total $=\frac{81}{24}+\frac{27}{24}$

$=\frac{108}{24}$

$=4\frac{1}{2}u^2$

7 a) $\log_4 x=p$

$x=4^p$

$\log_{16}x=\log_{16}(4^p)$

$=\log_{16}(16^{1/2p})$

$=\frac{1}{2}p$

b) $\log_3 x+\log_9 x=12$

$\log_3 x+\frac{1}{2}\log_3 x=12$

$\frac{3}{2}\log_3 x=12$

$\log_3 x^{3/2} = 12$

$x^{3/2} = 3^{12}$

$x^{3/2} = 531441$

$x = 531441^{2/3}$

$x = \sqrt[3]{531441^2}$

$x = 81^2$

$\underline{x = 6561}$

2008 SQP1 Paper I

① $y = \frac{x^3 - x}{x^2} = \frac{x^3}{x^2} - \frac{x}{x^2} = x - x^{-1}$

$\frac{dy}{dx} = 1 + x^{-2}$ B

$= 1 + \frac{1}{x^2}$

② $g(f(x)) = g(2x-3)$ A

$= (2x-3)^2$

$= 4x^2 - 12x + 9$

③ $\int \frac{1}{\sqrt[3]{x}}\,dx = \int x^{-1/3}\,dx$

$= \frac{x^{2/3}}{2/3} + c$ C

$= \frac{3}{2}x^{2/3} + C$

④ $d = \sqrt{(2+1)^2 + (3+4)^2 + (-2+0)^2}$

$= \sqrt{3^2 + 7^2 + (-2)^2}$

$= \sqrt{9 + 49 + 4}$ C

$= \sqrt{62}$

⑤ $u_0 = -1$

$u_1 = 3(-1) - 4 = -7$ A

$u_2 = 3(-7) - 4 = -25$

⑥ $y = -3 - f(x)$

$= -f(x) - 3$

Flips in x-axis

slides 3 down y-axis

$\therefore$ A

⑦ $y = 3(x-4)^2 - 5$

TP @ $(4, -5)$ $3x^2 \Rightarrow$ MIN TP

$\therefore$ C

⑧ $\sin 2x = 2\sin x \cos x$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right)$ A

$= \underline{\frac{4\sqrt{2}}{9}}$

⑨ $\sin(a-b) = \sin a \cos b - \cos a \sin b$

$= \left(\frac{1}{\sqrt{5}} \times \frac{4}{\sqrt{17}}\right) - \left(\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{17}}\right)$

$= \frac{4}{\sqrt{85}} - \frac{2}{\sqrt{85}}$ B

$= \underline{\frac{2}{\sqrt{85}}}$

⑩ $r = \sqrt{g^2 + f^2 - c}$ $g = 4$

$= \sqrt{4^2 + (-3)^2 + 12}$ $f = -3$

$c = -12$

$= \sqrt{\cancel{18}16 + 9 + 12}$ C

$= \sqrt{37}$

⑪ $\overrightarrow{PQ} = \underline{q} - \underline{p}$ $\overrightarrow{QR} = \underline{r} - \underline{q}$

$= \begin{pmatrix} 5 \\ 13 \\ 13 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}$ $= \begin{pmatrix} s \\ 33 \\ 25 \end{pmatrix} - \begin{pmatrix} 5 \\ 13 \\ 13 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 10 \\ 6 \end{pmatrix}$ $= \begin{pmatrix} s-5 \\ 20 \\ 12 \end{pmatrix}$

$\overrightarrow{QR} = 2\overrightarrow{PQ}$ (as $10 \to 20$, $6 \to 12$)

$\therefore s - 5 = 8$ C

$s = 13$