$= \left[8x - \frac{2x^3}{3}\right]_{-2}^{2}$

$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$

$= 32 - \frac{32}{3}$

$= \frac{64}{3}$ unit^2

$A_2 = \int_{-3}^{3} (32 - 2x^2 - 14)\,dx$

$= \int_{-3}^{3} (18 - 2x^2)\,dx$

$= \left[18x - \frac{2x^3}{3}\right]_{-3}^{3}$

$= \left(54 - \frac{54}{3}\right) - \left(-54 + \frac{54}{3}\right)$

$= 108 - \frac{108}{3}$

$= \frac{216}{3}$ unit^2

$\therefore$ Area $= \frac{216}{3} - \frac{64}{3}$

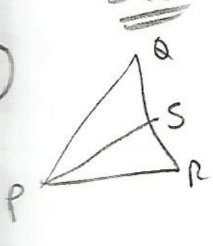
$= \frac{152}{3}$

$= 50\frac{2}{3}$ unit^2

2009 Paper I

1. $u_1 = 2$ (A)
   $u_2 = 3 \times 2 + 4 = 10$
   $u_3 = 3 \times 10 + 4 = 34$

2. $r = \sqrt{g^2 + f^2 - c}$ (B)
   $= \sqrt{16 + 9 + 75}$
   $= \sqrt{100}$
   $= 10$

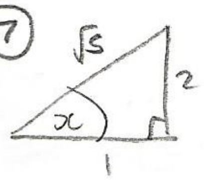
3. 

   $S = (1, 5)$
   $m = \frac{5+2}{1+3} = \frac{7}{4}$ (D)

4. $\frac{dy}{dx} = 15x^2 - 12$ (C)
   $m = 15 - 12$
   $= 3$

5. $m_{ST} = \frac{-1-3}{5-2} = \frac{-4}{3}$
   $d = \sqrt{3^2 + 4^2}$
   $= \sqrt{25}$
   $= 5$
   (1) True (2) false (B)

6. $L = 0.7L + 10$ (A)
   $0.3L = 10$
   $L = \frac{10}{0.3}$
   $L = \frac{100}{3}$

7. 

   $\cos 2x = 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$ (A)
   $= \frac{2}{5} - 1$
   $= -\frac{3}{5}$

8. $\frac{d}{dx}\left(\frac{1}{4}x^{-3}\right) = \frac{-3}{4}x^{-4}$ (D)
   $= \frac{-3}{4x^4}$

9. $x^2 + 4x^2 = 5$ (A)
   $5x^2 - 5 = 0$
   $5(x^2 - 1) = 0$
   $x = \pm 1$
   $x = 1, y = 2$
   $x = -1\ y = -2$

10. (B)

11. $\sin x = \frac{\sqrt{5}}{4}$ or $\sin x = -1$ (B)
    2 answers 1 answer
    $\therefore 3$

12. $b^2 - 4ac$ (C)
    $= 1 - 4 \times 2 \times (-9)$
    $= 46$ $\therefore$ Real, distinct roots

(13) $k^2 = 3+1$ $\tan\alpha = \frac{1}{\sqrt{3}}$ (A)

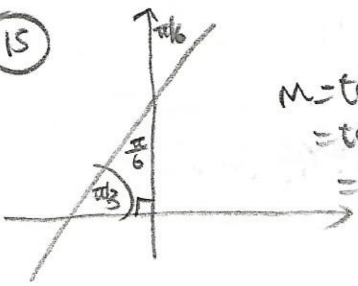
$\underline{k=2}$

$\underline{\alpha = 60^\circ}$

(14) $2\sin(3x-\frac{\pi}{4}) : -2 \to 2$ (C)

$2\sin(3x-\frac{\pi}{2})+5 : 3 \to 7$

(15)



$m = \tan\theta$ (A)

$= \tan\frac{\pi}{3}$

$= \underline{\sqrt{3}}$

(16) $A = \int_0^1 (4x^3 - 9x^2)\,dx$

$= \left[x^4 - 3x^3\right]_0^1$ (B)

Below x-axis,

$\therefore -\left[x^4 - 3x^3\right]_0^1$

(17) $|u| = \sqrt{3^2+4^2}$ (A)

$= \sqrt{25}$

$= \underline{5}$

unit vector $= \frac{1}{5}\begin{pmatrix}-3\\0\\4\end{pmatrix}$

$= \begin{pmatrix}-3/5\\0\\4/5\end{pmatrix}$

$= \underline{-\frac{3}{5}i + \frac{4}{5}k}$

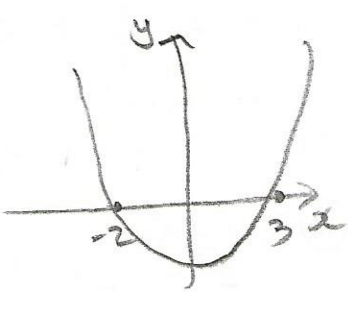
(18) $f'(x) = -\frac{1}{2}(4-3x^2)^{-3/2} \times (-6x)$

$= 3x(4-3x^2)^{-3/2}$ (D)

(19) $6 + x - x^2 < 0$

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$



(C) $x < -2,\ x > 3$

(20) $\frac{dA}{dr} = 4\pi r + 6\pi$ (C)

when $r=2$ $\frac{dA}{dr} = 8\pi + 6\pi$

$= \underline{14\pi}$

(21)

a) $P = (-3, 0)$

b) $m_{QR} = \frac{6+2}{4-8} = \frac{8}{-4} = \underline{-2}$

$\therefore m_{PT} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x+3)$

$\underline{2y = x+3}$

c) $y - 6 = -2(x-4)$

$y - 6 = -2x + 8$

$\underline{2x + y = 14}$

$2x + y = 14$ — ①

$x - 2y = -3$ — ②

① × 2 $4x + 2y = 28$

$x - 2y = -3$

$5x = 25$

$\underline{x = 5}$

$5 - 2y = -3$

$-2y = -8$

$\underline{y = 4}$

check

$10 + 4 = 14$ ✓

$\underline{(5, 4)}$

(22) a)

i) $\overrightarrow{DE} = \underline{e} - \underline{d}$

$= \begin{pmatrix}1\\-2\\-3\end{pmatrix} - \begin{pmatrix}10\\-8\\-15\end{pmatrix}$

$= \begin{pmatrix}-9\\6\\12\end{pmatrix}$

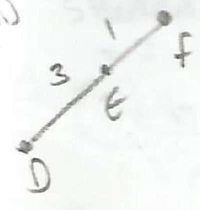
$= 3\overrightarrow{EF}$

$\overrightarrow{EF} = \underline{f} - \underline{e}$

$= \begin{pmatrix}-2\\0\\1\end{pmatrix} - \begin{pmatrix}1\\-2\\-3\end{pmatrix}$

$= \begin{pmatrix}-3\\2\\4\end{pmatrix}$

(ii)



3:1

b) $\overrightarrow{GE} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$

$DE \cdot GE = 0$

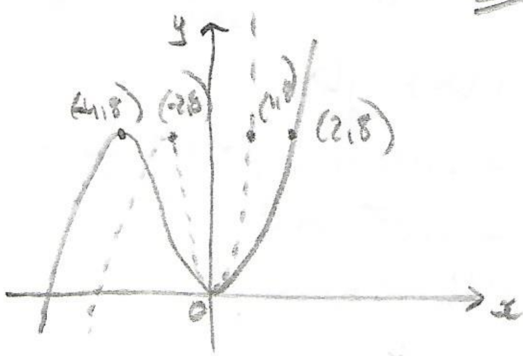
$-9(1-k)+6(-3)+12(-3)=0$

$-9+9k-18-36=0$

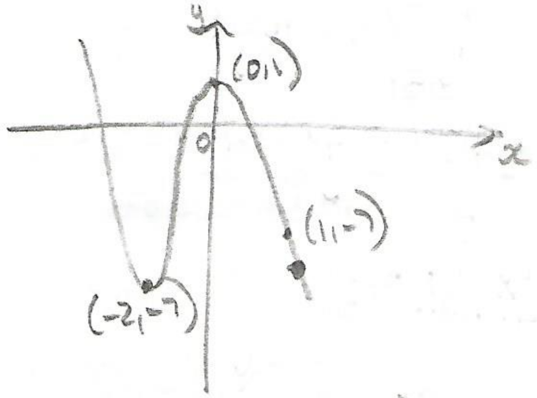
$9k=63$

$\underline{\underline{k=7}}$

(23) a)



b)



(24) a) $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{2}\times\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \underline{\underline{\frac{\sqrt{3}+1}{2\sqrt{2}}}}$

b) $\sin(A+B)+\sin(A-B)$

$= \sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}$

$= \underline{2\sin A\cos B}$

c) (i) $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

(ii) $\sin\frac{7\pi}{12} + \sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= 2\left(\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}\right) \quad = \underline{\underline{\frac{\sqrt{3}}{\sqrt{2}}}}$

2009 Paper II

(1) $\frac{dy}{dx} = 3x^2 - 6x - 9$

SP's where $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 6x - 9 = 0$

$3(x^2-2x-3)=0$

$3(x+1)(x-3)=0$

$x=-1 \quad x=3$

when $x=-1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

when $x=3$, $y = (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

$\boxed{(-1, 17) \quad (3, -15)}$

x	-2	-1	0	3	4
$\frac{dy}{dx}$	15	0	-9	0	15
shape	/	—	\	—	/

$\therefore$ Max TP @ $(-1, 17)$

Min TP @ $(3, -15)$

(2) a)

(i) $f(g(x)) = 3(x^2-2)+1$

$= \underline{3x^2 - 5}$

(ii) $g(f(x)) = (3x+1)^2 - 2$

$= \underline{9x^2 + 6x - 1}$

b) $6x = 18x + 6$

$-12x = 6$

$\underline{\underline{x = -\frac{1}{2}}}$

(3) a)(i)

1	1	8	11	-20
		1	9	20
	1	9	20	0

(ii) $f(x) = (x-1)(x^2+9x+20)$

$= (x-1)(x+4)(x+5)$

b) $\log_2(x+3) + \log_2(x^2+5x-4) = 3$

$\log_2(x+3)(x^2+5x-4) = 3$

$(x+3)(x^2+5x-4) = 2^3$

$x^3+5x^2-4x+3x^2+15x-12 = 8$

$x^3+8x^2+11x-20 = 0$

$(x-1)(x+4)(x+5) = 0$

$x=1,\ x=-4,\ x=-5$

$\therefore \underline{\underline{x=1}}$

4 a) $(x+1)^2+(y-2)^2$

$=(5+1)^2+(10-2)^2$

$=6^2+8^2$

$=\underline{100}$ $\quad\therefore (5,10)$ lies on circle C_1.

b) centre $= (-1,2)$

$\overrightarrow{PC} = \begin{pmatrix}-6\\-8\end{pmatrix} \quad \therefore \underline{q} = \underline{c} + \overrightarrow{PC}$

$= \begin{pmatrix}-1\\2\end{pmatrix} + \begin{pmatrix}-6\\-8\end{pmatrix}$

$= \begin{pmatrix}-7\\-6\end{pmatrix}$

$\therefore Q = (-7,-6)$.

$m_{PQ} = \frac{-6-10}{-7-5} = \frac{-16}{-12} = \frac{4}{3}$

$\therefore m_{tan} = -\frac{3}{4}$

$y+6 = -\frac{3}{4}(x+7)$

$4y+24 = -3x-21$

$\underline{3x+4y+45=0}$

5 a) $m=3$

$n=2$

b) $3\cos 2x = -4\cos 2x + 3$

$7\cos 2x = 3$

$\cos 2x = \frac{3}{7}$

$2x = \cos^{-1}\left(\frac{3}{7}\right)$

$2x = 1.279,\ 5.155\ldots$

$x = 0.6395,\ 2.578$

$y = 3\cos 2x$

$= 1.286$

$\therefore$ Points of intersection

$: (0.6, 1.3)\ (2.6, 1.3)$

c) $f(x) - g(x)$

$= -4\cos 2x + 3 - (3\cos 2x)$

$= -7\cos 2x + 3$

$= 3 - 7\cos 2x$

$A = \int_{0.6}^{2.6} (3 - 7\cos 2x)\,dx$

$= \left[3x - \frac{7}{2}\sin 2x\right]_{0.6}^{2.6}$

$= \left(7.8 - \frac{7}{2}\sin 5.2\right) - \left(1.8 - \frac{7}{2}\sin 1.2\right)$

$= (10.8920913) - (-1.46213680$

$= 12.354228\ldots$

$= \underline{12.4 \text{ unit}^2}$

6 a) $N = N_0 e^{rt}$

$= (61\times10^6)\,e^{0.016\times14}$

$= 76{,}315{,}332$.

b) $2 = e^{0.0043t}$

$\ln 2 = 0.0043t$

$t = \frac{\ln 2}{0.0043}$

$\underline{t = 161.2 \text{ years}}$

7 a) $\underline{p}\cdot(\underline{q}+\underline{r}) = \underline{p}\cdot\underline{q} + \underline{p}\cdot\underline{r}$

$\underline{p}\cdot\underline{q} = |\underline{p}||\underline{q}|\cos\theta$

$= 4\times3\times\cos 30^\circ$

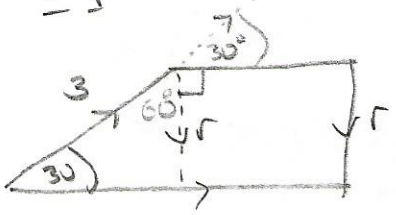
$= \frac{12\sqrt{3}}{2}$

$= 6\sqrt{3}$

$\underline{p}\cdot\underline{r} = |\underline{p}||\underline{r}|\cos 90^\circ$
$= 0.$
$\therefore \underline{p}\cdot(\underline{q}+\underline{r}) = \underline{\underline{6\sqrt{3}}}$

$\underline{r}\cdot(\underline{p}-\underline{q}) = \underline{r}\cdot\underline{p} - \underline{r}\cdot\underline{q}$

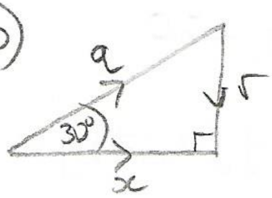
$\underline{r}\cdot\underline{p} = 0.$



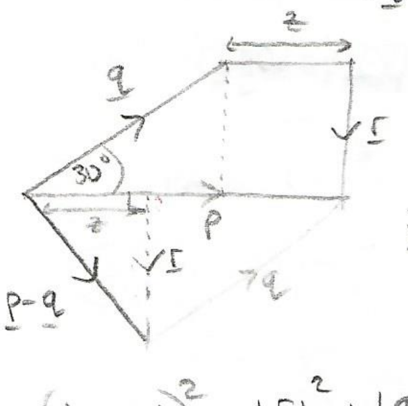
$\sin 30^\circ = \frac{r}{3}$
$r = 3\sin 30^\circ$
$\underline{\underline{r = \frac{3}{2}}}$

$\underline{r}\cdot\underline{q} = |\underline{r}||\underline{q}|\cos\theta$
$= \frac{3}{2} \times 3 \times \cos 120^\circ$
$= \frac{9}{2} \times \left(-\frac{1}{2}\right)$
$= \underline{\underline{-\frac{9}{4}}}$

b)



$\underline{x} = \underline{q} + \underline{r}$
$\cos 30^\circ = \frac{|\underline{q}+\underline{r}|}{3}$
$|\underline{q}+\underline{r}| = \frac{3\sqrt{3}}{2}$



$\underline{z} = \underline{p} - (\underline{q}+\underline{r})$
$= 4 - \frac{3\sqrt{3}}{2}$

$(|\underline{p}-\underline{q}|)^2 = |\underline{r}|^2 + |\underline{q}+\underline{r}|^2$
$|\underline{p}-\underline{q}|^2 = \left(\frac{3}{2}\right)^2 + \left(4 - \frac{3\sqrt{3}}{2}\right)^2$
$|\underline{p}-\underline{q}|^2 = \frac{9}{4} + 16 - \frac{24\sqrt{3}}{2} + \frac{27}{4}$
$|\underline{p}-\underline{q}|^2 = \frac{36}{4} + 16 - 12\sqrt{3}$
$|\underline{p}-\underline{q}|^2 = 25 - 12\sqrt{3}$
$\underline{\underline{|\underline{p}-\underline{q}| = \sqrt{25 - 12\sqrt{3}}}}$

2010 Paper I

① $2x - 3y - 6 = 0$
$3y = 2x - 6$
$y = \frac{2}{3}x - 2$
$\therefore M_{perp} = -\frac{3}{2}$ (A)

② $u_0 = 1$
$u_1 = 2(1) + 3 = 5$ (C)
$u_2 = 2(5) + 3 = \underline{\underline{13}}$

③ $3\underline{u} - 2\underline{v}$
$= \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix}$ (D)
$= \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$

④ 3 waves $\Rightarrow b = 3$ (A)
Max 2, min 2 $\Rightarrow a = 2$
$c = 0$
$\therefore 2\cos 3x$

⑤ $x^2 + 8x + 3$
$= (x+4)^2 - 16 + 3$ (B)
$= (x+4)^2 - 13$

⑥ $b^2 - 4ac = 0$ $a = k$
$9 - 8k = 0$ $b = -3$ (D)
$k = \frac{9}{8}$ $c = 2$

⑦ $L = \frac{1}{4}L + 7$
$\frac{3}{4}L = 7$ (C)
$L = \frac{28}{3}$

⑧ $C = (3, 5) \therefore r = 5$ (B)
$\therefore L \Rightarrow y = 10$

⑨ $\int (2x^{-4} + \cos 5x)\,dx$ (C)
$= \frac{2x^{-3}}{-3} + \frac{1}{5}\sin 5x + C$