⑧ $\log_3(a-1) - 2\cdot2 = 0$

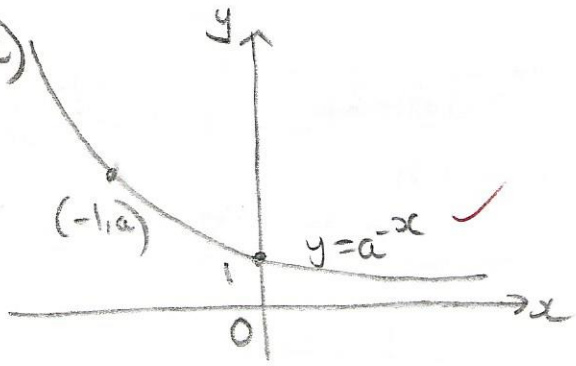
$\log_3(a-1) = 2\cdot2$

$a-1 = 3^{2\cdot2}$

$a = 3^{2\cdot2} + 1$ ✓

$a = 12\cdot2$

⑨ a)

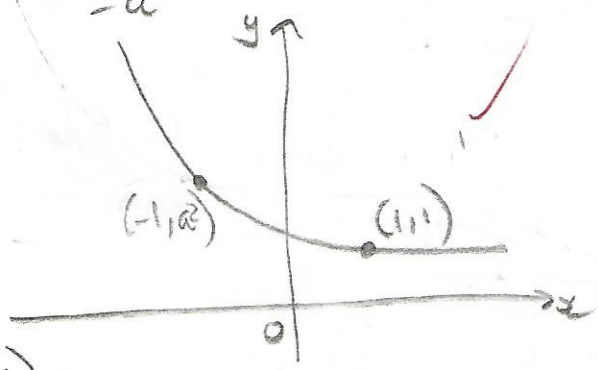


✓

b) $y = a^{1-x}$

$= a^{(-x+1)}$

$= a^{-(x-1)}$



✓ ✓

⑩ a) (i) $a = 2, 4$

(ii) $y = k(x-2)(x-4)$

$6 = k(-2)(-4)$

$8k = 6$

$k = \frac{3}{4}$

b) $\frac{dy}{dx} = \frac{3}{4}(x-2)(x-4)$

$= \frac{3}{4}(x^2 - 6x + 8)$

$= \frac{3x^2}{4} - \frac{9x}{2} + 6$

$y = \int \left(\frac{3x^2}{4} + \frac{9x}{2} + 6\right) dx$

$= \frac{x^3}{4} - \frac{9x^2}{4} + 6x + C$

when $x = 0$, $y = 6$

$\therefore -y = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + 6$ ✓

⑪ a) $y = 3(4^x)$

$6 = 3(4^a)$

$4^a = 2$ ✓

$a = 1/2$

b) $y = 3(4^x)$

$b = 3(4^{-1/2})$

$b = \frac{3}{\sqrt{4}}$

$b = \frac{3}{2}$ ✓

c) $y = 3(4^x)$

$\log_{10} y = \log_{10}[3(4^x)]$

$\log_{10} y = \log_{10} 3 + \log_{10}(4^x)$

$\log_{10} y = \log_{10} 4(x) + \log_{10} 3$

$P = \log_{10} 4$, $Q = \log_{10} 3$

Gradient $= \log_{10} 4$ ✓

$= 0\cdot602$

2008 Paper I

① $u_{10} = 10$

$u_{11} = 0\cdot3 \times 10 + 6 = 9$

$u_{12} = 0\cdot3 \times 9 + 6 = 8\cdot7$ (C)

② $(x+7)^2 + (y-6)^2 = 36$ (D)

③ $(k \times 0) + (-1 \times 4) + (1 \times k) = 0$

$-4 + k = 0$

$k = 4$ (C)

④ $L = 0\cdot4L - 240$

$0\cdot6L = -240$

$L = \frac{-240}{0\cdot6}$

$L = \frac{-2400}{6}$ (B)

$L = -400$

(5) $M_R = \frac{9-5}{7-2}$ $\therefore M_{tan} = -\frac{5}{4}$ (A)

$= \frac{4}{5}$ $y - 9 = -\frac{5}{4}(x-7)$

(6) $2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$ (B)

$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = 60°, 120°$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ $\frac{\pi}{2} \leq x \leq \pi$

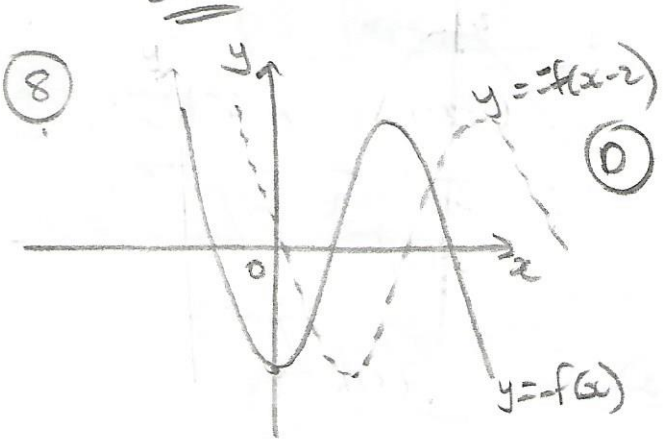
(7) $m = \tan\theta$

$= \tan 135°$ (C)

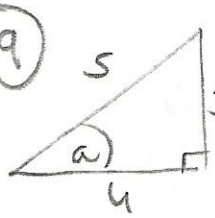
$= -\tan 45°$

$= -1$

S	A
T	C

(8) 

(D)

(9) 

$\sin(x+a)$ (B)

$= \sin x \cos a + \cos x \sin a$

$= \sin x\left(\frac{4}{5}\right) + \cos x\left(\frac{3}{5}\right)$

$= \frac{4}{5}\sin x + \frac{3}{5}\cos x$

(10) $b^2 - 4ac$

$= 1^2 - 4(1)(1)$ (A)

$= 1 - 4$

$= -3$

$\therefore$ roots not real

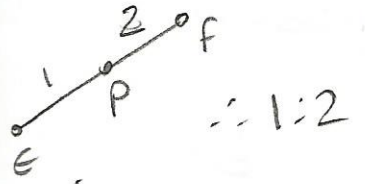
Neither statement one.

(11) $\overrightarrow{EP} = \underline{p} - \underline{e}$ $\overrightarrow{PF} = \underline{f} - \underline{p}$

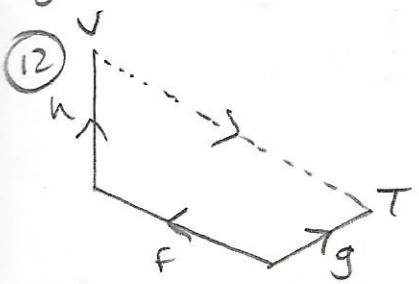
$= \begin{pmatrix}1\\5\\7\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix}$ $= \begin{pmatrix}7\\17\\13\end{pmatrix} - \begin{pmatrix}1\\5\\7\end{pmatrix}$

$= \begin{pmatrix}3\\6\\3\end{pmatrix}$ $= \begin{pmatrix}6\\12\\6\end{pmatrix}$

$= 2\overrightarrow{EP}$



$\therefore 1:2$ (B)

(12) 

(C)

$\overrightarrow{VT} = -h - f + g$

$= -f + g - h$

(13) $y = k(x-1)(x-4)$

$12 = k(-1)(-4)$ (A)

$12 = 4k$

$k = 3$

$\therefore y = 3(x-1)(x-4)$

(14) $\int 4\sin(2x+3)\,dx$ (B)

$= -\frac{4\cos(2x+3)}{2} + C$

$= -2\cos(2x+3) + C$

(15) $y = (x^3+4)^2$ (C)

$\frac{dy}{dx} = 2(x^3+4) \times 3x^2$

$= 6x^2(x^3+4)$

(16) $2x^2 + 4x + 7$

$= 2[x^2 + 2x] + 7$

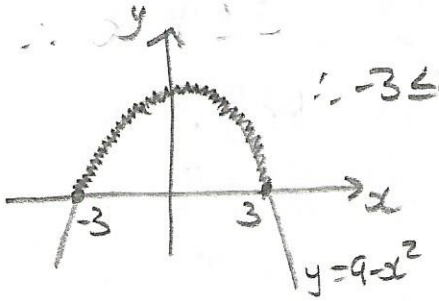
$= 2[(x+1)^2 - 1] + 7$ (A)

$= 2(x+1)^2 - 2 + 7$ $\therefore q = 5$

$= 2(x+1)^2 + 5$

(17) $9-x^2 \geq 0$

$(3+x)(3-x) \geq 0$



$\therefore -3 \leq x \leq 3$ (C)

(18) $\underline{q} \cdot (\underline{p}+\underline{q})$ (C)

$= \underline{q} \cdot \underline{p} + \underline{q} \cdot \underline{q}$

$= 10 + 4^2$

$= 26$

(19) $y = 2m^x$ (B)

$54 = 2m^3$

$m^3 = 27$

$m = 3$

(20) For $y = \log_3 x$ and $(q, 2)$

$2 = \log_3 q$

$3^2 = q$ (D)

$q = 9$

$\therefore$ for $\log_3(x-4)$, $q = 9+4$

$= 13$

(21) a) $f(x) = x^3 - 3x^2 + 2$

SP's when $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, \quad x = 2$

$y = 2, \quad y = 8 - 12 + 2$

$= -2$

$\therefore (0, 2), (2, -2)$

x	-1	0	1	2	3
$f'(x)$	9	0	-3	0	9
shape	/	—	\	—	/

$\therefore$ Max TP @ $(0, 2)$

Min TP @ $(2, -2)$

b)(i)

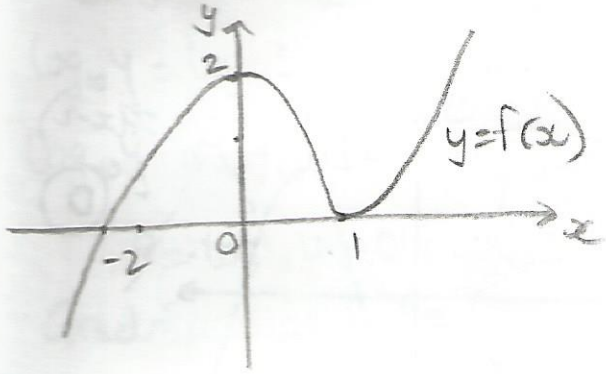
	1	0	-3	2
1		1	1	-2
	1	1	-2	0
	x^2	x	C	R

(ii) $f(x) = (x-1)(x^2+x-2)$

$= (x-1)(x+2)(x-1)$

c) y intercept $= (0, 2)$

x intercepts $= (1, 0)(-2, 0)$.



(22) a) $\frac{dy}{dx} = 3x^2 - 12x + 8$

$\therefore 3x^2 - 12x + 8 = -1$

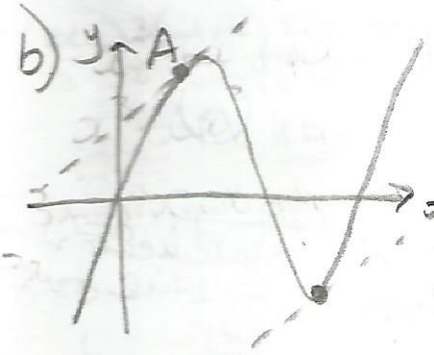
$3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$\therefore x = 3, \quad x = 1$

b)



At A,

$x = 1$

$y = 3$

$\therefore (1, 3)$

(23) a) $h(f(x)) = \log_2(x^2-x+10)$

$h(g(x)) = \log_2(5-x)$

b) $\log_2(x^2-x+10) - \log(5-x) = 3$

$\log_2\left(\frac{x^2-x+10}{5-x}\right) = 3$

$\frac{x^2-x+10}{5-x} = 2^3$

$\frac{x^2-x+10}{5-x} = 8$

$x^2-x+10 = 8(5-x)$

$x^2-x+10 = 40-8x$

$x^2+7x-30 = 0$

$(x+10)(x-3) = 0$

$x = -10, \quad x = 3$

$x > 0, \quad \therefore \underline{x = 3}$

2008 Paper II

① a) midpoint $= (1,-3)$

$m_{BC} = \frac{-1+5}{-3-5} = \frac{4}{-8} = -\frac{1}{2}$

$\therefore m_{PB} = 2$

$y+3 = 2(x-1)$

$y+3 = 2x-2$

$\underline{2x-y=5}$

b) midpoint $= (2,4)$

$m_{CM} = \frac{4+5}{2-5} = \frac{9}{-3} = -3$

$\therefore y-4 = -3(x-2)$

$y-4 = -3x+6$

$\underline{3x+y=10}$

c) $2x-y=5$

$3x+y=10$

$5x = 15$

$x = 3$

$6-y=5$

$y=1$

check $3\times3+1=10$ ✓

$\therefore \underline{(3,1)}$

② a) $P = (8,0,4)$

$Q = (0,4,3)$

b) $\vec{PQ} = q-p$

$= \begin{pmatrix}0\\4\\3\end{pmatrix} - \begin{pmatrix}8\\0\\4\end{pmatrix}$

$= \begin{pmatrix}-8\\4\\-1\end{pmatrix}$

$\vec{PA} = a-p$

$= \begin{pmatrix}8\\0\\0\end{pmatrix} - \begin{pmatrix}8\\0\\4\end{pmatrix}$

$= \begin{pmatrix}0\\0\\-4\end{pmatrix}$

c) $|\vec{PQ}| = \sqrt{64+16+1}$

$= \sqrt{81}$

$= \underline{9}$

$|\vec{PA}| = \sqrt{16}$

$= \underline{4}$

$\vec{PQ}.\vec{PA} = 0+0+4$

$= \underline{4}$

$\therefore \cos QPA = \frac{4}{4\times9}$

$QPA = \cos^{-1}\left(\frac{1}{9}\right)$

$\underline{QPA = 83.6^\circ}$

③

a) (i) $p = \sqrt{7}$

(ii) $q = -3$

b) $\sqrt{7}\cos x - 3\sin x = k\cos(x+\alpha)$

$= k\cos x\cos\alpha - k\sin x\sin\alpha$

$= k\cos\alpha\cos x - k\sin\alpha\sin x$

$k\sin\alpha = 3$

$k\cos\alpha = \sqrt{7}$

$k^2 = 7+9$

$k = \sqrt{16}$

$\underline{k=4}$

$\tan\alpha = \frac{3}{\sqrt{7}}$

$\alpha = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

$\underline{\alpha = 48.6^\circ}$

S ✓	A ✓
T ✓	C ✓

$\sin = +$

$\cos = +$

$\therefore Q_1$

$\therefore f(x)+g(x) = 4\cos(x+48.6)^\circ$

c) If $h(x) = f(x)+g(x)$

then $h'(x) = f'(x)+g'(x)$

$\therefore f'(x)+g'(x) = \underline{-4\sin(x+48.6)^\circ}$

(4) a) $c = (-4, -2)$

$r = \sqrt{16 + 4 + 38}$

$= \sqrt{58}$

b) $c = (4, 6)$

$d = \sqrt{8^2 + 8^2}$

$= \sqrt{128}$

$D = \sqrt{128} = 11.31$

$r_1 + r_2 = \sqrt{58} + \sqrt{26} = 12.71$

$r_1 + r_2 > D$, $\therefore$ intersect

c) $x^2 + y^2 + 8x + 4y - 38 = 0$

$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$

$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x-3)(x+1) = 0$

$x = 3, \quad x = -1$

$y = 1 \quad y = 5$

$\therefore$ Intersect at $(3, 1)$

$(-1, 5)$

(5) $\cos 2x + 2\sin x = \sin^2 x$

$1 - 2\sin^2 x + 2\sin x = \sin^2 x$

$3\sin^2 x - 2\sin x - 1 = 0$

$(3\sin x + 1)(\sin x - 1) = 0$

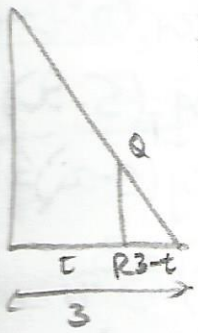
$\sin x = -\frac{1}{3} \quad \sin x = 1$

$x = "19.5°" \quad x = 90°$

$x = 199.5°, 340.5°$

$\therefore x = 90°, 199.5°, 340.5°$

(6) a)



SIMILAR TRIANGLES!

$\frac{QR}{6} = \frac{3-t}{3}$

$QR = \frac{6(3-t)}{3}$

$QR = 2(3-t)$

$QR = 6 - 2t$

b) Area $= \frac{1}{2}bh$

$= t(6-2t)$

$= 6t - 2t^2$

SP's when $\frac{dA}{dt} = 0$

$\therefore 6 - 4t = 0$

$4t = 6$

$t = \frac{3}{2}$

t	1	$3/2$	2
$\frac{dA}{dt}$	2	0	-2
shape	/	—	\

$(\frac{3}{2}, 3)$

$\therefore$ Max area when $t = 3/2$

(7) $24 = 32 - 2x^2$

$2x^2 - 8 = 0$

$2(x^2 - 4) = 0$

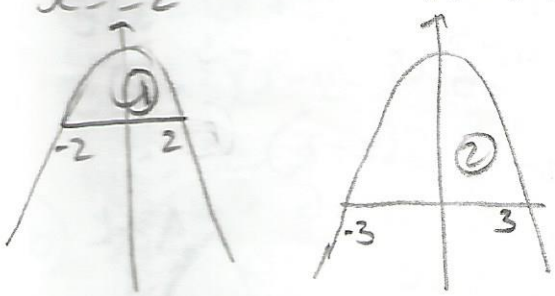
$x = \pm 2$

$14 = 32 - 2x^2$

$2x^2 - 18 = 0$

$2(x^2 - 9) = 0$

$x = \pm 3$



Shaded area $= A_2 - A_1$.

$A_1 = \int_{-2}^{2} (32 - 2x^2 - 24)\,dx$

$= \int_{-2}^{2} (8 - 2x^2)\,dx$

$= \left[8x - \frac{2x^3}{3}\right]_{-2}^{2}$

$= \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$

$= 32 - \frac{32}{3}$

$= \frac{64}{3}$ unit^2

$A_2 = \int_{-3}^{3} (32 - 2x^2 - 14)\,dx$

$= \int_{-3}^{3} (18 - 2x^2)\,dx$

$= \left[18x - \frac{2x^3}{3}\right]_{-3}^{3}$

$= \left(54 - \frac{54}{3}\right) - \left(-54 + \frac{54}{3}\right)$

$= 108 - \frac{108}{3}$

$= \frac{216}{3}$ unit^2

$\therefore$ Area $= \frac{216}{3} - \frac{64}{3}$

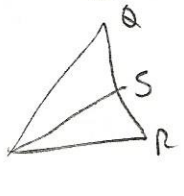
$= \frac{152}{3}$

$= 50\frac{2}{3}$ unit^2

2009 Paper I

(1) $u_1 = 2$
$u_2 = 3\times2+4 = 10$
$u_3 = 3\times10+4 = 34$ (A)

(2) $r = \sqrt{g^2+f^2-c}$
$= \sqrt{16+9+75}$
$= \sqrt{100}$
$= 10$ (B)

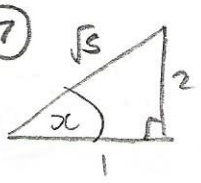
(3) 

$S = (1,5)$
$m = \frac{5+2}{1+3} = \frac{7}{4}$ (D)

(4) $\frac{dy}{dx} = 15x^2 - 12$
$m = 15-12$
$= 3$ (C)

(5) $M_{ST} = \frac{-1-3}{5-2} = \frac{-4}{3}$
$d = \sqrt{3^2+4^2}$
$= \sqrt{25}$
$= 5$
① True ② false (B)

(6) $L = 0.7L + 10$
$0.3L = 10$
$L = \frac{10}{0.3}$
$L = \frac{100}{3}$ (A)

(7) 

$\cos 2x = 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$
$= \frac{2}{5} - 1$
$= -\frac{3}{5}$ (A)

(8) $\frac{d}{dx}\left(\frac{1}{4}x^{-3}\right) = \frac{-3}{4}x^{-4}$
$= \frac{-3}{4x^4}$ (D)

(9) $x^2 + 4x^2 = 5$
$5x^2 - 5 = 0$
$5(x^2-1) = 0$
$x = \pm 1$
$x=1, y=2$
$x=-1\ y=-2$ (A)

(10) (B)

(11) $\sin x = \frac{\sqrt{5}}{4}$ or $\sin x = -1$ (B)
2 answers 1 answer
$\therefore 3$

(12) $b^2 - 4ac$
$= 1 - 4\times2\times(-9)$
$= 46$ (C)
$\therefore$ Real, distinct roots