

(12) a)

(i) $PS = 6 - x$, $RS = 12 - \frac{8}{x}$

(ii) $Area = (6 - x)(12 - \frac{8}{x})$

$= 72 - \frac{48}{x} - 12x + 8$

$= 80 - 12x - \frac{48}{x}$

b) SP's where $\frac{dA}{dx} = 0$

$A = 80 - 12x - 48x^{-1}$

$\frac{dA}{dx} = -12 + \frac{48}{x^2}$

$\therefore -12 + \frac{48}{x^2} = 0$

$48 = 12x^2$

$x^2 = 4$

$x = \pm 2$

x	1	2	3
$\frac{dA}{dx}$	36	0	$-6\frac{2}{3}$
shape	/	-	\

\therefore Max when $x = 2$

ie Max Area = 32 unit²

$1 \leq x \leq 4$

\therefore when $x = 1$, $A = 20$

$x = 4$, $A = 20$

\therefore Minimum = 20 unit² when $x = 1, 4$

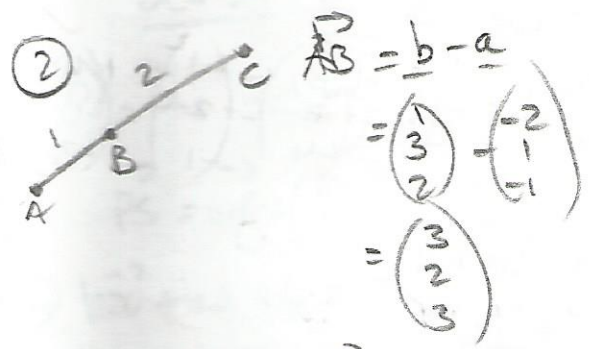
(1) $3x - y + 2 = 0$ $m = 3$

$y = 3x + 2$

$y - 4 = 3(x + 1)$

$y - 4 = 3x + 3$ ✓

$y = 3x + 7$



$\therefore \underline{c} = \underline{b} + 2\underline{AB}$
 $= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix} \therefore c = (7, 7, 8)$ ✓

(3) a) $g(f(x)) = 1 - 2(x^2 + 1)$

$= 1 - 2x^2 - 2$

$= -1 - 2x^2$

b) $g(g(x)) = 1 - 2(1 - 2x)$

$= 1 - 2 + 4x$ ✓

$= 4x - 1$

(4) $kx^2 - x - 1 = 0$

$a = k$ For no real roots,

$b = -1$ $b^2 - 4ac < 0$

$c = -1$ $1 + 4k < 0$

$4k < -1$ ✓

$k < -\frac{1}{4}$

(5) $B = (7, 8)$

$r_{\text{large}} = \sqrt{7^2 + 8^2} - 77$

$= \sqrt{49 + 64} - 77$

$$= \sqrt{113-77}$$

$$= \sqrt{36}$$

$$= \underline{\underline{6}}$$

$$\therefore r_{\text{small}} = 2$$

$$A = (3, 8)$$

$$B = (7, 8)$$

$$C = (11, 8)$$

$$D = (15, 8) \quad \therefore (x-15)^2 + (y-8)^2 = 4$$

$$\textcircled{6} \sin 2x = 6 \cos x$$

$$2 \sin x \cos x = 6 \cos x = 0$$

$$2 \cos x (\sin x - 3) = 0$$

$$2 \cos x = 0 \quad \sin x = 3$$

$$x = 90^\circ, 270^\circ \quad \underline{\text{NO SOLN}}$$

$$\textcircled{7} \text{ a) } u_0 = 0$$

$$u_1 = 16$$

$$u_2 = 20 \quad \checkmark$$

$$u_3 = 21$$

$$\text{b) (i) } -1 \leq \frac{1}{4} < 1 \quad \checkmark$$

$$\text{(ii) } k = \frac{1}{4}k + 16$$

$$\frac{3}{4}k = 16$$

$$k = \underline{\underline{\frac{64}{3}}}$$

$$\textcircled{8} \text{ a) } 3 \left| \begin{array}{ccc|c} 1 & -4 & 1 & 6 \\ & 3 & -3 & -6 \\ \hline 1 & -1 & -2 & 0 \end{array} \right.$$

$$\text{b) } y = (x-3)(x^2-x-2)$$

$$= (x-3)(x-2)(x+1)$$

$$\therefore A = (2, 0)$$

$$\text{c) } A = \int_0^2 (x^3 - 4x^2 + x + 6) dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_0^2$$

$$= \left(\frac{16}{4} - \frac{32}{3} + 2 + 12 \right) - (0)$$

$$= \left(\frac{48}{12} - \frac{128}{12} + 14 \right)$$

$$= \left(\frac{-80}{12} + \frac{168}{12} \right)$$

$$= \frac{88}{12} \quad \checkmark$$

$$= \underline{\underline{\frac{22}{3} \text{ unit}^2}}$$

$$\frac{14}{12}$$

$$\frac{28}{12}$$

$$\frac{140}{168}$$

$$\textcircled{9} \text{ a) } 3x - x^3 = 0$$

$$x(3-x^2) = 0$$

$$x = 0, x = \pm\sqrt{3}$$

$$(0, 0), (\sqrt{3}, 0), (-\sqrt{3}, 0)$$

$$\text{b) } y = 3x - x^3 \quad \text{SPS @ } \frac{dy}{dx} = 0$$

$$\therefore 3 - 3x^2 = 0$$

$$3(1-x^2) = 0$$

$$3(1+x)(1-x) = 0$$

$$x = 1, x = -1$$

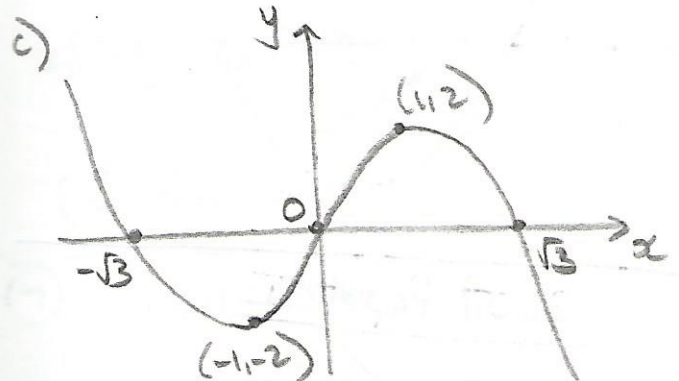
$$y = 2, y = -2$$

x	-2	-1	0	1	2
$\frac{dy}{dx}$	-9	0	3	0	-9
shape	\	-	/	-	\

$$\therefore \begin{pmatrix} 1, 2 \\ -1, -2 \end{pmatrix}$$

$$\therefore \text{Min TP @ } (-1, -2)$$

$$\text{Max TP @ } (1, 2)$$



$$\textcircled{10} y = (3x^2 + 2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} (3x^2 + 2)^{-1/2} \times 6x$$

$$= \frac{3x}{\sqrt{3x^2 + 2}}$$

(1) a)

$$\sqrt{3} \cos x + \sin x = k \cos(x - \alpha)$$

$$= k \cos x \cos \alpha + k \sin x \sin \alpha$$

$$= k \cos x (\cos \alpha) + k \sin x (\sin \alpha)$$

$$k \sin \alpha = 1$$

$$k \cos \alpha = \sqrt{3}$$

$$\tan \alpha = \frac{1}{\sqrt{3}}$$

$$k^2 = 3 + 1$$

$$\alpha = \frac{\pi}{6}$$

$$k = 2$$

$$\therefore \sqrt{3} \cos x + \sin x = 2 \cos(x - \frac{\pi}{6})$$

b) Max = 2 when $\cos(x - \frac{\pi}{6}) = 1$

$$x - \frac{\pi}{6} = 0$$

$$x = \frac{\pi}{6}$$

Min = -2 when $\cos(x - \frac{\pi}{6}) = -1$

$$x - \frac{\pi}{6} = \pi$$

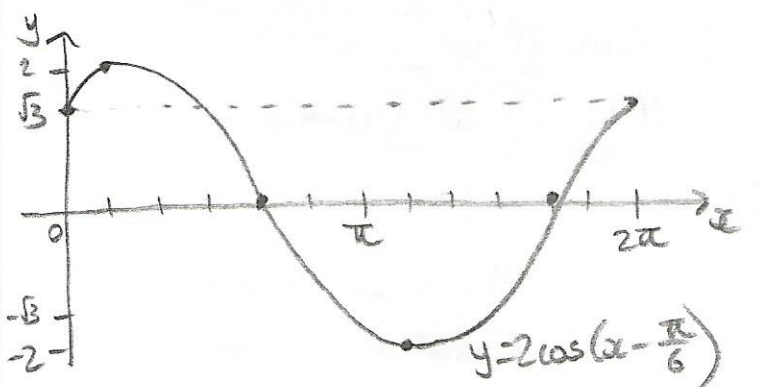
$$x = \frac{7\pi}{6}$$

when $x = 0$, $y = 2 \cos(-\frac{\pi}{6})$
 $= \sqrt{3}$

when $y = 0$, $2 \cos(x - \frac{\pi}{6}) = 0$
 $\cos(x - \frac{\pi}{6}) = 0$

$$x - \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{2\pi}{3}, \frac{5\pi}{3}$$



2007 Paper II

① a) $\underline{a} = (0, 2, 2)$

b) $\underline{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

c) $|\underline{p}| = \sqrt{2}$ $|\underline{q}| = \sqrt{6}$

$$\underline{p} \cdot \underline{q} = 0 + 2 + 1 = 3$$

$$\therefore \cos \theta = \frac{3}{\sqrt{2} \sqrt{6}}$$

$$\theta = \cos^{-1} \left(\frac{3}{\sqrt{12}} \right)$$

$$\theta = \cos^{-1} \left(\frac{3}{2\sqrt{3}} \right)$$

$$\theta = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

$$\theta = 30^\circ$$

② a) $\sin(c+d) = \sin c \cos d + \cos c \sin d$
 $= \left(\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} \right) + \left(\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} \right)$

$$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{1}{\sqrt{2}}$$

b) i) $\sin 2c = 2 \sin c \cos c$

$$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$$

$$= \frac{4}{5}$$

ii) $\cos 2d = 2 \cos^2 d - 1$

$$= 2 \left(\frac{3}{\sqrt{10}} \right)^2 - 1$$

$$= \frac{18}{10} - 1$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

③ $x^2 + y^2 + 6x - 4y - 7 = 0$

$$x^2 + (6 - 2x)^2 + 6x - 4(6 - 2x) - 7 = 0$$

$$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$$

$$5x^2 - 10x + 5 = 0$$

$$5(x^2 - 2x + 1) = 0$$

$$b^2 - 4ac = 4 - 4 \times 1 \times 1$$

$$= 0 \therefore \text{tangent.}$$

$$5(x^2 - 2x + 1) = 0$$

$$5(x-1)^2 = 0$$

$$x = 1$$

$$y = 6 - 2$$

$$= 4$$

$$(1, 4) \checkmark$$

$$\textcircled{4} \text{ a) } a = 2 \quad b = 3 \quad c = -1 \checkmark$$

$$\text{b) } 2\sin 3x - 1 = 0$$

$$\sin 3x = \frac{1}{2}$$

$$3x = 30^\circ, 150^\circ, 390^\circ, 510^\circ, 750^\circ, 870^\circ$$

$$x = 10^\circ, 50^\circ, 130^\circ, 170^\circ, 250^\circ, 290^\circ$$

$$P = (50^\circ, 0) \checkmark$$

$$\textcircled{5} \text{ a) } \frac{dy}{dx} = x - 8$$

$$4 = x - 8$$

$$x = 12$$

$$y = \frac{1}{2} 12^2 - 96 + 34$$

$$= 72 - 96 + 34$$

$$= 10$$

$$(12, 10) \checkmark$$

$$\text{b) } x - 8 = -4 \text{ (symmetry)}$$

$$x = 4$$

$$(4, 10) \checkmark$$

$$\text{c) Centre: } x = \frac{12+4}{2} = 8$$

$$m_{CP} = \frac{1}{4}$$

$$\therefore y - 10 = \frac{1}{4}(x - 4)$$

$$4y - 40 = x - 4$$

$$4y = x + 36$$

$$4y = 8 + 36$$

$$4y = 44$$

$$y = 11$$

$$(8, 11) \checkmark$$

\textcircled{6} a)

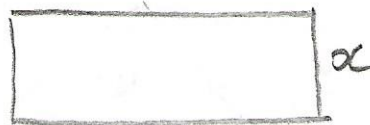
$$\text{i) } ST^2 = 10^2 + 10^2$$

$$ST^2 = 200$$

$$ST = \sqrt{200}$$

$$ST = 10\sqrt{2}$$

\textcircled{ii}



$$10\sqrt{2} - 2x$$

$$\text{Area} = lb$$

$$= x(10\sqrt{2} - 2x)$$

$$= 10\sqrt{2}x - 2x^2$$

$$\text{b) } \frac{dA}{dx} = 10\sqrt{2} - 4x$$

$$\text{SP's @ } \frac{dA}{dx} = 0 \therefore 10\sqrt{2} - 4x = 0$$

$$4x = 10\sqrt{2}$$

$$x = \frac{5\sqrt{2}}{2}$$

x	3	$\frac{5\sqrt{2}}{2}$	4
$\frac{dA}{dx}$	2.14	0	-1.86
shape	/	-	\

$$\therefore \text{Max when } x = \frac{5\sqrt{2}}{2}$$

$$\text{breadth} = \frac{5\sqrt{2}}{2} \text{ m}$$

$$\text{length} = 10\sqrt{2} - 2x$$

$$= 10\sqrt{2} - 5\sqrt{2} \checkmark$$

$$= 5\sqrt{2} \text{ m}$$

$$\textcircled{7} \int_0^2 \sin(\ln x + 1) dx$$

$$= \left[\frac{-\cos(\ln x + 1)}{1} \right]_0^2$$

$$= \left(\frac{-\cos(1)}{1} \right) - \left(\frac{-\cos(1)}{1} \right)$$

$$= 0.363 \text{ unit}^2$$

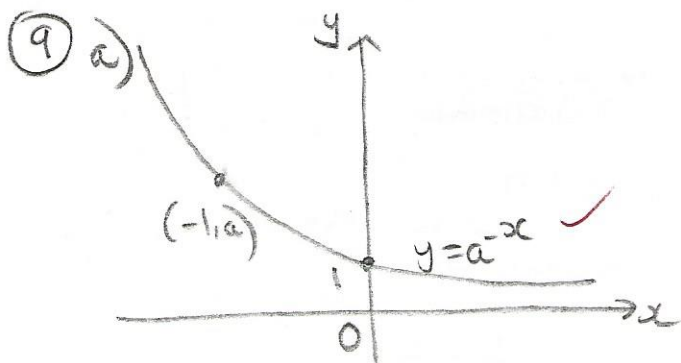
$$\textcircled{8} \log_3(a-1) - 2 \cdot 2 = 0$$

$$\log_3(a-1) = 2 \cdot 2$$

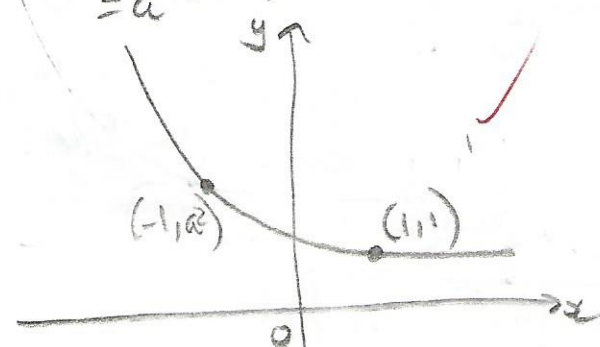
$$a-1 = 3^{2 \cdot 2}$$

$$a = 3^{2 \cdot 2} + 1$$

$$\underline{\underline{a = 12 \cdot 2}}$$



b) $y = a^{1-x}$
 $= a^{-(x-1)}$
 $= a^{-(x-1)}$



$\textcircled{10} \text{ a)}$ (i) $a = 2, 4$

(ii) $y = k(x-2)(x-4)$

$$6 = k(-2)(-4)$$

$$8k = 6$$

$$\underline{\underline{k = \frac{3}{4}}}$$

b) $\frac{dy}{dx} = \frac{3}{4}(x-2)(x-4)$
 $= \frac{3}{4}(x^2 - 6x + 8)$
 $= \frac{3x^2}{4} - \frac{9x}{2} + 6$

$$y = \int \left(\frac{3x^2}{4} - \frac{9x}{2} + 6 \right) dx$$

$$= \frac{x^3}{4} - \frac{9x^2}{4} + 6x + C$$

when $x=0$, $y=6$

$$\therefore y = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + 6$$

$\textcircled{11} \text{ a)}$ $y = 3(4^x)$

$$6 = 3(4^a)$$

$$4^a = 2$$

$$\underline{\underline{a = \frac{1}{2}}}$$

b) $y = 3(4^x)$

$$b = 3(4^{-\frac{1}{4}})$$

$$b = \frac{3}{\sqrt[4]{4}}$$

$$\underline{\underline{b = \frac{3}{2}}}$$

c) $y = 3(4^x)$

$$\log_{10} y = \log_{10} [3(4^x)]$$

$$\log_{10} y = \log_{10} 3 + \log_{10} (4^x)$$

$$\log_{10} y = \log_{10} 4(x) + \log_{10} 3$$

$$P = \log_{10} 4, Q = \log_{10} 3$$

$$\text{Gradient} = \log_{10} 4$$

$$= \underline{\underline{0.602}}$$

2008 Paper I

$\textcircled{1} u_{10} = 10$

$$u_{11} = 0.3 \times 10 + 6 = 9$$

$$u_{12} = 0.3 \times 9 + 6 = 8.7$$

\textcircled{C}

$\textcircled{2} (x+7)^2 + (y-6)^2 = 36$

\textcircled{D}

$\textcircled{3} (k \times 0) + (-1 \times 4) + (1 \times k) = 0$

$$-4 + k = 0$$

$$\underline{\underline{k = 4}}$$

\textcircled{C}

$\textcircled{4} L = 0.4L - 240$

$$0.6L = -240$$

$$L = \frac{-240}{0.6}$$