(8) $k\sin 2x = \sin x$

$2k\sin x\cos x - \sin x = 0$

$\sin x(2k\cos x - 1) = 0$

$\sin x = 0$ or $2k\cos x - 1 = 0$

$x = 0$ $\quad\quad$ $\cos x = \frac{1}{2k}$

$\therefore x = 0, \pi, 2\pi$

(B, D)

$\therefore$ at A, C ; $\cos x = \frac{1}{2k}$

(9) a) £252 million

b) $20 = 252e^{-0.06335t}$

$e^{-0.06335t} = \frac{20}{252}$

$-0.06335t = \ln\left(\frac{20}{252}\right)$

$t = \dfrac{\ln\left(\frac{20}{252}\right)}{-0.06335}$

$t = 40$

40 years

(10) $\underline{a}.(\underline{a}+\underline{b}+\underline{c}) = \underline{a}.\underline{a} + \underline{a}.\underline{b} + \underline{a}.\underline{c}$

$\underline{a}.\underline{a} = 9$ $\quad\quad$ $\underline{a}.\underline{c} = 9\cos 60^\circ = \frac{9}{2}$

$\underline{a}.\underline{b} = 9\cos\theta$
$= 9\cos 90^\circ$
$= 0$

$\therefore \underline{a}.(\underline{a}+\underline{b}+\underline{c}) = 13\frac{1}{2}$

(11) a)

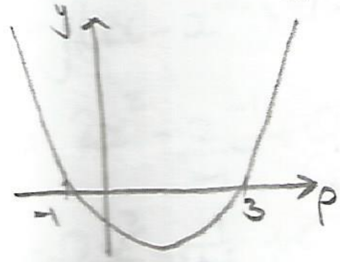
$$\begin{array}{r|rrrr} -1 & 1 & p & p & 1 \\ & & -1 & -p+1 & -1 \\ \hline & 1 & p-1 & 1 & 0 \end{array}$$

b) $x^2 + (p-1)x + 1 = 0$

$a = 1$, $b = p-1$, $c = 1$

Real roots when $b^2 - 4ac \geq 0$

$b^2 - 4ac = (p-1)^2 - 4$
$= p^2 - 2p + 1 - 4$
$= p^2 - 2p - 3$
$= (p-3)(p+1)$



$p \leq -1$, $p \geq 3$

2006 Paper I

(1) a) midpoint $= (3,5)$

$\therefore m_{BD} = \frac{5+5}{3+2} = \frac{10}{5} = 2$

$y - 5 = 2(x-3)$
$y - 5 = 2x - 6$
$y = 2x - 1$

b) $m_{BC} = \frac{-5+2}{-2-7} = \frac{-3}{-9} = \frac{1}{3}$

$\therefore m_{AE} = -3$

$y - 12 = -3(x+1)$
$y - 12 = -3x - 3$
$y = -3x + 9$

c) $3x + y = 9$
$2x - y = 1$

$5x = 10$
$x = 2$ $\quad\quad$ check $2\times 2 - 3 = 1$

$6 + y = 9$
$y = 3$

$(2,3)$

(2) a) $r = \sqrt{3^2 + 3^2}$

$= \sqrt{18}$

$(x+2)^2 + (y-3)^2 = 18$

b) $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \therefore Q = (-5, 0)$

$m_{PQ} = \frac{6-0}{1+5} = \frac{6}{6} = 1$

$\therefore m_{tan} = -1$

$y - 0 = -(x+5)$

$y = -x - 5$

(3) a) (i) $f(g(x)) = 2(2x-3) + 3$

$= 4x - 3$

(ii) $g(f(x)) = 2(2x+3) - 3$

$= 4x + 3$

b) $f(g(x)) \times g(f(x)) = (4x-3)(4x+3)$

$= 16x^2 - 9$

$\therefore$ Minimum value $= -9$

(4) a) $-1 < 0.8 < 1$

b) $L = 0.8L + 12$

$0.2L = 12$

$L = \frac{12}{0.2}$

$L = 60$

(5) $f(x) = (2x-1)^5$

$f'(x) = 5(2x-1)^4 \times 2$

$= 10(2x-1)^4$

SP's where $f'(x) = 0$

$\therefore 10(2x-1)^4 = 0$

$(2x-1)^4 = 0$

$x = 1/2$ $(1/2, 0)$

$y = 0$

x	0	$1/2$	1
$f'(x)$	-1	0	1
shape	/	—	/

$\therefore$ Rising Point of inflection @ $(1/2, 0)$

(6) a) $A_1 = \int_0^1 (x^3 - 6x^2 + 4x + 1)\,dx$

$= \left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_0^1$

$= \left(\frac{1}{4} - 2 + 2 + 1\right) - (0)$

$= \frac{5}{4}$ unit^2

b) $A_2 = \int_1^2 (x^3 - 6x^2 + 4x + 1)\,dx$

$= \left[\frac{x^4}{4} - 2x^3 + 2x^2 + x\right]_1^2$

$= \left(\frac{16}{4} - 16 + 8 + 2\right) - \left(\frac{5}{4}\right)$

$= \left(\frac{16}{4} - \frac{24}{4}\right) - \left(\frac{5}{4}\right)$

$= \frac{-8}{4} - \frac{5}{4}$

$= -\frac{13}{4}$

$\therefore$ Area $= \frac{5}{4} + \frac{13}{4}$

$= \frac{18}{4}$

$= 4\frac{1}{2}$ unit^2

(7) $\sin x^\circ - \sin 2x^\circ = 0$ $\quad 0 \le x \le 360^\circ$

$\sin x^\circ - 2\sin x^\circ \cos x^\circ = 0$

$\sin x^\circ (1 - 2\cos x^\circ) = 0$

$\sin x^\circ = 0$ $\qquad 1 - 2\cos x^\circ = 0$

$x = 0^\circ, 180^\circ, 360^\circ$ $\qquad \cos x^\circ = \frac{1}{2}$

$x = 60^\circ, 300^\circ$

$\therefore x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$

8) a) $2x^2+4x-3$

$= 2(x^2+2x)-3$

$= 2\left[(x+1)^2-1\right]-3$

$= \underline{\underline{2(x+1)^2-5}}$

b) $(-1,-5)$

9) a) $\underline{u}.\underline{v} = k^3+3k^2-(k+2)$

$= k^3+3k^2-k-2$

$\therefore\ \underline{u}.\underline{v} = 1$

$k^3+3k^2-k-2=1$

$\underline{k^3+3k^2-k-3=0}$

b)

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$k^3+3k^2-k-3=(k+3)(k^2-1)$

$\underline{=(k+3)(k+1)(k-1)}$

c) $k=1\ \ (k>0)$

d) $\underline{u}.\underline{v} = |\underline{u}||\underline{v}|\cos\theta$

$\underline{u} = \begin{pmatrix}1\\1\\3\end{pmatrix} \quad \underline{v} = \begin{pmatrix}1\\3\\-1\end{pmatrix}$

$|\underline{u}| = \sqrt{11} \quad \underline{v} = \sqrt{11}$

$\therefore \cos\theta = \frac{1}{\sqrt{11}\sqrt{11}}$

$\cos\theta = \underline{\underline{\frac{1}{11}}}$

10) $\log_4 y = \log_4(a^x)$

$3 = \log_4(a^6)$

$4^3 = a^6$

$64 = a^6$

$a=2$

2006 Paper II

1) a) $m_{PS} = \frac{6-0}{4-2} = \frac{6}{2} = 3$

$\therefore m_{QS} = -\frac{1}{3}$

$y-6 = -\frac{1}{3}(x-4)$

$3y-18 = -x+4$

$\underline{x+3y=22}$

b) $Q:\ x+3(0)=22$

$x=22 \quad (22,0)$

$\vec{PS} = \vec{QR} = \begin{pmatrix}2\\6\end{pmatrix} \quad \therefore R=(24,6)$

2) $kx^2+4x+6=0$

For equal roots, $b^2-4ac=0$

$k^2-24k=0$

$k(k-24)=0$

$k=0,\ k=24$

$\underline{\therefore k=24}$

3) a) $x=8.\ \frac{dy}{dx}=2x-14$

$\therefore m = 16-14$

$\underline{\underline{=2}}$

$y-5=2(x-8)$

$y-5=2x-16$

$\underline{y=2x-11}$

b) $2x-11 = -x^2+10x-27$

$x^2-8x+16=0$

$(x-4)^2=0$

$x=4 \quad \underline{\underline{(4,-3)}}$

$y=-3$

4) centre $=(3,4)$, $r=5$

$x^2+y^2-6x-8y-12=0$

$r=\sqrt{3^2+4^2+12}$

$=\sqrt{37}$

$\therefore$ larger circle $r=\sqrt{37}$

⑤ $y = \int (4x - 6x^2)\,dx$

$= 2x^2 - 2x^3 + c$

$9 = 2(-1)^2 - 2(-1)^3 + c$

$9 = 2 + 2 + c$

$\underline{c = 5}$

$\therefore \underline{y = 2x^2 - 2x^3 + 5}$

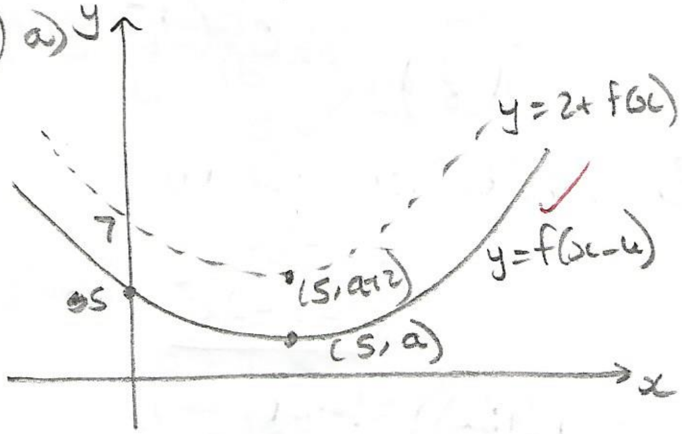
⑥ a) $\overrightarrow{PQ} = \underline{q} - \underline{p} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

b) $|\overrightarrow{PQ}| = \sqrt{16+9}$

$= \underline{5}$

c) $\underline{u} = \begin{pmatrix} 4/5 \\ 0 \\ -3/5 \end{pmatrix}$

⑦ a)



⑧

a)(i) $\sin a = \frac{1}{\sqrt{5}}$

(ii) $\sin 2a = 2\sin a\cos a$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

$= \underline{\frac{4}{5}}$

b) $\sin(3a) = \sin(2a + a)$

$= \sin 2a\cos a + \cos 2a\sin a$

$= \left(\frac{4}{5} \times \frac{2}{\sqrt{5}}\right) + \left(\frac{3}{5} \times \frac{1}{\sqrt{5}}\right)$

$= \frac{8}{5\sqrt{5}} + \frac{3}{5\sqrt{5}} = \underline{\frac{11}{5\sqrt{5}}}$

$\circledast$ $\cos 2a = 1 - 2\sin^2 a$

$= 1 - \frac{2}{5} = \frac{3}{5}$

⑨ $y = \frac{1}{x^3} - \cos 2x$

$= x^{-3} - \cos 2x.$

$\frac{dy}{dx} = -3x^{-4} + 2\sin 2x$

$= \underline{2\sin 2x - \frac{3}{x^4}}$

⑩ a)

$7\sin x - 24\boxed{\cos x} = k\sin(x - \alpha)$

$= k\sin x\cos\alpha - k\cos x\sin\alpha$

$= \underbrace{k\cos\alpha}_{7}\sin x - \underbrace{k\sin\alpha}_{-24}\boxed{\cos x}$

$k\sin\alpha = 24$

$k\cos\alpha = 7$ $\qquad \tan\alpha = \frac{24}{7}$

$k^2 = 7^2 + 24^2$ $\qquad \alpha = 1.29$

$\underline{k = 25}$

$\therefore 7\sin x - 24\cos x = 25\sin(x - 1.29)$

b) $y = 25\sin(x - 1.29)$

$\frac{dy}{dx} = 25\cos(x - 1.29)$

$\therefore 25\cos(x - 1.29) = 1$

$\cos(x - 1.29) = \frac{1}{25}$

$x - 1.29 = \cos^{-1}\left(\frac{1}{25}\right)$

$x - 1.29 = 1.5308\ldots$

$\underline{x = 2.82}$

⑪ $A_t = A_0 e^{-0.000124t}$

$e^{-0.000124t} = \frac{A_t}{A_0}$

$e^{-0.000124t} = 0.88$

$-0.000124t = \ln 0.88$

$t = \frac{\ln 0.88}{-0.000124}$

$t = 1031$

Yes $(1031 > 1000\text{yrs})$

(12) a)

(i) $PS = 6 - x$, $RS = 12 - \frac{8}{x}$

(ii) $\text{Area} = (6-x)(12 - \frac{8}{x})$

$= 72 - \frac{48}{x} - 12x + 8$

$= 80 - 12x - \frac{48}{x}$

b) SP's where $\frac{dA}{dx} = 0$

$A = 80 - 12x - 48x^{-1}$

$\frac{dA}{dx} = -12 + \frac{48}{x^2}$

$\therefore -12 + \frac{48}{x^2} = 0$

$48 = 12x^2$

$x^2 = 4$

$x = \pm 2$

x	1	2	3
$\frac{dA}{dx}$	36	0	$-6\frac{2}{3}$
shape	/	—	\

$\therefore$ Max when $x = 2$

ie Max Area $= 32 \text{ unit}^2$

$1 \le x \le 4$

$\therefore$ when $x = 1$, $A = 20$

$x = 4$, $A = 20$

$\therefore$ Minimum $= 20u^2$ when $x = 1, 4$

2007 Paper 1

(1) $3x - y + 2 = 0$ $\quad m = 3$

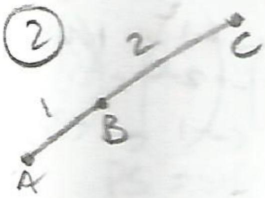
$y = 3x + 2$

$y - 4 = 3(x+1)$

$y - 4 = 3x + 3$

$y = 3x + 7$

(2)



$\vec{AB} = \underline{b} - \underline{a}$

$= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

$\therefore \underline{c} = \underline{b} + 2\vec{AB}$

$= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix}$ $\quad \therefore C = (7, 7, 8)$

(3) a) $g(f(x)) = 1 - 2(x^2 + 1)$

$= 1 - 2x^2 - 2$

$= -1 - 2x^2$

b) $g(g(x)) = 1 - 2(1 - 2x)$

$= 1 - 2 + 4x$

$= 4x - 1$

(4) $kx^2 - x - 1 = 0$

$a = k$
$b = -1$
$c = -1$

For no real roots,

$b^2 - 4ac < 0$

$1 + 4k < 0$

$4k < -1$

$k < -\frac{1}{4}$

(5) $B = (7, 8)$

$r_{\text{large}} = \sqrt{7^2 + 8^2 - 77}$

$= \sqrt{49 + 64 - 77}$