

b) SP's @ $\frac{dW}{dx} = 0$

$$V = 4x - \frac{2}{3}x^3$$

$$\frac{dW}{dx} = 4 - 2x^2$$

$$\therefore 4 - 2x^2 = 0$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

x	1	$\sqrt{2}$	2
$\frac{dW}{dx}$	2	0	-4
shape	/	-	\

\therefore Maximum volume when $x = \sqrt{2}$

⑩ a) $A_t = A_0 e^{-0.002t}$
 $600 = A_0 e^{-0.002 \times 1000}$

$$600 = A_0 e^{-2}$$

$$A_0 = \frac{600}{e^{-2}}$$

$$A_0 = 4433 \mu\text{g}$$

b) $0.5 = e^{-0.002t}$

$$\ln 0.5 = -0.002t$$

$$t = \frac{\ln 0.5}{-0.002}$$

$$t = 346.6 \text{ years}$$

⑪ $2x - \frac{1}{2}x^2 = 1.5$

$$2x - \frac{1}{2}x^2 = \frac{3}{2}$$

$$4x - x^2 = 3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1, x = 3.$$

$$\therefore A = \int_1^3 (2x - \frac{1}{2}x^2 - \frac{3}{2}) dx$$

$$= \left[x^2 - \frac{x^3}{6} - \frac{3x}{2} \right]_1^3$$

$$= \left(9 - \frac{27}{6} - \frac{9}{2} \right) - \left(1 - \frac{1}{6} - \frac{3}{2} \right)$$

$$= \left(\frac{54}{6} - \frac{27}{6} - \frac{27}{6} \right) - \left(\frac{6}{6} - \frac{1}{6} - \frac{9}{6} \right)$$

$$= (0) - \left(-\frac{4}{6} \right)$$

$$= \frac{4}{6}$$

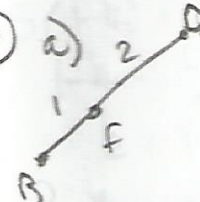
$$= \frac{2}{3} \text{ m}^2$$

200 S Paper I

① $m = \tan \theta$ $y = \sqrt{3}(x+2)$ ✓
 $= \tan 60^\circ$ $y = \sqrt{3}x + 2\sqrt{3}$
 $= \sqrt{3}$

② a) P is midpoint of AB
 $A = (-3, -2)$ $B = (3, 6)$
 $\therefore P = (0, 2)$ ✓

b) $AB = \sqrt{6^2 + 8^2}$
 $= 10$ ✓

③ a)  $f = \frac{1}{3}(2b + d)$
 $= \frac{1}{3} \left(\begin{pmatrix} 24 \\ 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 4 \end{pmatrix} \right)$
 $= \frac{1}{3} \begin{pmatrix} 30 \\ 15 \\ 4 \end{pmatrix}$
 $\therefore f = \begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix}$ ✓

b) $af = f - a = \begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$
 $= \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$ ✓

4) a) $h(x) = g(f(x))$
 $= (3x-1)^2 + 7$

b) (i) $(\frac{1}{3}, 7)$

(ii) $\{y: y \geq 7, y \in \mathbb{R}\}$

5) $f(x) = (1 + 2\sin x)^4$
 $f'(x) = 4(1 + 2\sin x)^3 \times 2\cos x$
 $= 8\cos x (1 + 2\sin x)^3$

6) a) $u_{n+1} = k u_n + 5$

$L = kL + 5$

$L(1-k) = 5$

$4(1-k) = 5$

$1-k = \frac{5}{4}$

$k = 1 - \frac{5}{4}$

$k = -\frac{1}{4}$

b) (i) $u_1 = 3m + 5$

$u_2 = m(3m + 5) + 5$
 $= 3m^2 + 5m + 5$

(ii) $3m^2 + 5m + 5 = 7$

$3m^2 + 5m - 2 = 0$

$(3m-1)(m+2) = 0$

$m = \frac{1}{3}, m = -2$

$\therefore m = -2$

7) a) $a = 4, b = 5$

b) $x > 4$

8) a)
$$\begin{array}{c|ccc} 3 & 2 & -7 & 0 & 9 \\ & 6 & -3 & -9 & \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

$f(x) = (x-3)(2x^2 - x - 3)$
 $= (x-3)(2x-3)(x+1)$

b) $(3,0), (3,2), (0,0), (-1,0)$
 $(0, a)$

c) when $x = -2, y = -16 - 28 + 9 = -35$ $(-2, -35)$
 when $x = 2, y = 16 - 28 + 9 = -3$ $(2, -3)$

SP's when $\frac{dy}{dx} = 0$

$\therefore 6x^2 - 14x = 0$

$2x(3x-7) = 0$

$x = 0, x = \frac{7}{3}$ OUT OF RANGE.

when $x = 0, y = 9$ $(0, 9)$

Max = 9 (when $x = 0$)

Min = -35 (when $x = -2$)

9) $\cos 2x = \frac{7}{25}$

$25 \cos 2x = 7$

$25(2\cos^2 x - 1) = 7$

$50\cos^2 x - 25 = 7$

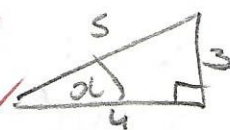
$50\cos^2 x = 32$

$\cos^2 x = \frac{32}{50}$

$\cos^2 x = \frac{16}{25}$

$\cos x = \frac{4}{5}$

$\sin x = \frac{3}{5}$



10) a)

$\sin x - \sqrt{3} \cos x = k \sin(x - \alpha)$
 $= k \sin x \cos \alpha - k \cos x \sin \alpha$
 $= k \cos \alpha \sin x - k \sin \alpha \cos x$

$k \cos \alpha = 1$
 $k \sin \alpha = \sqrt{3}$

$\tan \alpha = \frac{\sqrt{3}}{1}$

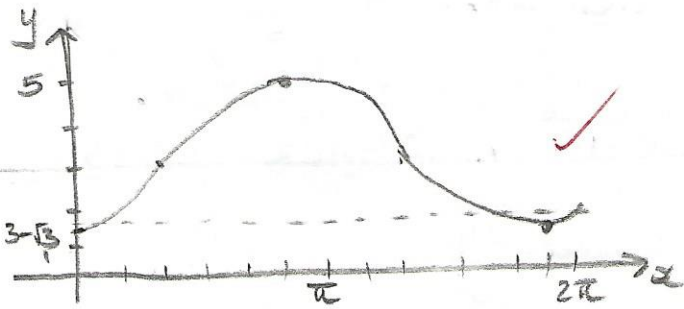
$k^2 = 3 + 1$

$\alpha = \frac{\pi}{3}$

$k = 2$

$\therefore 2 \sin(x - \frac{\pi}{3})$

b) $y = 3 + \sin x - \sqrt{3} \cos x$
 $= 2 \sin(x - \frac{\pi}{3}) + 3$



Max = 5 when $\sin(x - \frac{\pi}{3}) = 1$
 $x - \frac{\pi}{3} = \frac{\pi}{2}$
 $x = \frac{5\pi}{6}$

Min = 1 when $\sin(x - \frac{\pi}{3}) = -1$
 $x - \frac{\pi}{3} = \frac{3\pi}{2}$
 $x = \frac{11\pi}{6}$

y intercept: $x = 0$
 ie $2 \sin(-\frac{\pi}{3}) + 3$
 $= -2 \sin(\frac{\pi}{3}) + 3$
 $= -\sqrt{3} + 3$
 $= 3 - \sqrt{3}$

11) a) $(x-t)^2 + y^2 = 4$
 b) $x^2 - 2tx + t^2 + y^2 - 4 = 0$
 $x^2 - 2tx + t^2 + (2x)^2 - 4 = 0$
 $x^2 + 4x^2 - 2tx + t^2 - 4 = 0$
 $5x^2 - 2tx + t^2 - 4 = 0$

For tangency, $b^2 - 4ac = 0$
 $a = 5$ $4t^2 - 20(t^2 - 4) = 0$
 $b = -2t$ $4t^2 - 20t^2 + 80 = 0$
 $c = t^2 - 4$ $-16t^2 + 80 = 0$
 $-16(t^2 - 5) = 0$
 $t = \pm \sqrt{5}$
 $\therefore t = \sqrt{5}$

① $\int \frac{4x^3 - 1}{x^2} dx$
 $= \int (\frac{4x^3}{x^2} - \frac{1}{x^2}) dx$
 $= \int (4x - x^{-2}) dx$
 $= 2x^2 - \frac{x^{-1}}{-1} + C$
 $= 2x^2 + \frac{1}{x} + C$

② a) $\sin p = \frac{15}{17}$ $\sin q = \frac{3}{5}$
 $\cos p = \frac{8}{17}$ $\cos q = \frac{4}{5}$
 $\therefore \sin(p+q) = \sin p \cos q + \cos p \sin q$
 $= (\frac{15}{17} \times \frac{4}{5}) + (\frac{8}{17} \times \frac{3}{5})$
 $= \frac{60}{85} + \frac{24}{85}$
 $= \frac{84}{85}$

b) i) $\cos(p+q) = \cos p \cos q - \sin p \sin q$
 $= (\frac{8}{17} \times \frac{4}{5}) - (\frac{15}{17} \times \frac{3}{5})$
 $= \frac{32}{85} - \frac{45}{85}$
 $= \frac{-13}{85}$

ii) $\tan(p+q) = \frac{\frac{84}{85}}{\frac{-13}{85}}$
 $= \frac{84}{-13}$

③ a) $m = \frac{4-0}{5-1} = \frac{4}{4} = 1 \therefore m_{PB} = -1$
 (m₁m₂ = -1)
 midpoint = (3, 2)
 $\therefore y - 2 = -1(x - 3)$
 $y - 2 = -x + 3$

$$x+y=5$$

$$b) x+3y=1$$

$$3y=1-x$$

$$y = \frac{1}{3} - \frac{1}{3}x \quad \therefore m = -\frac{1}{3}$$

$$\text{so } m_p = 3$$

$$y-0 = 3(x-1)$$

$$y = 3x-3$$

$$c) (i) x+y=5 \quad \text{--- (1)}$$

$$3x-y=3 \quad \text{--- (2)}$$

$$\text{we } = 8$$

$$x = 2$$

$$y = 3$$

$$C = (2, 3)$$

$$\text{check } 3+2=3$$

$$= 3 \quad \checkmark$$

$$(ii) r = \sqrt{12+32}$$

$$= \sqrt{10}$$

$$\therefore (x-2)^2 + (y-3)^2 = 10 \quad \checkmark$$

$$d) a) \vec{TA} = \underline{a} - \underline{c} \quad \vec{TB} = \underline{b} - \underline{c}$$

$$= \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -12 \\ 0 \\ 9 \end{pmatrix} - \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix} \quad \checkmark$$

$$b) |\vec{TA}| = \sqrt{25+225+1} \quad |\vec{TB}| = \sqrt{1600+225+4}$$

$$= \sqrt{251}$$

$$= \sqrt{1829}$$

$$\vec{TA} \cdot \vec{TB} = 200 + 225 + 2$$

$$= 427$$

$$\therefore \cos ATB = \frac{\vec{TA} \cdot \vec{TB}}{|\vec{TA}| |\vec{TB}|}$$

$$\cos ATB = \frac{427}{\sqrt{251} \sqrt{1829}}$$

$$ATB = \cos^{-1} \left(\frac{427}{\sqrt{459079}} \right)$$

$$= 50.90$$

$$5) 2x^2 - 9 = x^2$$

$$x^2 - 9 = 0$$

$$x = \pm 3$$

$$x^2 - (2x^2 - 9) = -x^2 + 9$$

$$\therefore A = \int_{-3}^3 (9 - x^2) dx$$

$$= \left[9x - \frac{x^3}{3} \right]_{-3}^3$$

$$= (27 - 9) - (-27 + 9)$$

$$= 18 - (-18)$$

$$= 36 \text{ unit}^2 \quad \checkmark$$

$$6) y = 24x^{-1/2}$$

$$\frac{dy}{dx} = -12x^{-3/2}$$

$$= \frac{-12}{\sqrt{x^3}}$$

$$\text{when } x=4$$

$$m = \frac{-12}{\sqrt{4^3}}$$

$$= \frac{-12}{8}$$

$$= -\frac{3}{2}$$

$$\text{when } x=4,$$

$$y = \frac{24}{\sqrt{4}} \quad (4, 12)$$

$$= 12$$

$$\therefore y - 12 = -\frac{3}{2}(x - 4)$$

$$2y - 24 = -3x + 12$$

$$3x + 2y = 36 \quad \checkmark$$

$$7) \log_4(5-x) - \log_4(3-x) = 2$$

$$\log_4 \left(\frac{5-x}{3-x} \right) = 2$$

$$\frac{5-x}{3-x} = 4^2$$

$$5-x = 16(3-x)$$

$$5-x = 48 - 16x$$

$$15x = 43$$

$$x = \frac{43}{15} \quad \checkmark$$

⑧ $k \sin 2x = \sin x$

$2k \sin x \cos x - \sin x = 0$

$\sin x (2k \cos x - 1) = 0$

$\sin x = 0$ or $2k \cos x - 1 = 0$

$x = 0$ $\cos x = \frac{1}{2k}$

$\therefore x = 0, \pi, 2\pi$

(B, D)

\therefore at A, C; $\cos x = \frac{1}{2k}$

⑨ a) £252 million

b) $20 = 252e^{-0.06335t}$

$e^{-0.06335t} = \frac{20}{252}$

$-0.06335t = \ln\left(\frac{20}{252}\right)$

$t = \frac{\ln\left(\frac{20}{252}\right)}{-0.06335}$

$t = 40$ ✓

40 years

⑩ $a(a+bc) = a \cdot a + a \cdot b + a \cdot c$

$a \cdot a = 9$

$a \cdot c = 9 \cos 60^\circ$

$a \cdot b = 9 \cos 90^\circ$

$= \frac{9}{2}$

$= 9 \cos 90^\circ$

$= 0$

\therefore $a \cdot (a+bc) = 13\frac{1}{2}$ ✓

⑪ a)
$$\begin{array}{c|cccc} -1 & 1 & p & p & 1 \\ & -1 & -p+1 & -1 & \\ \hline & 1 & p-1 & 1 & 0 \end{array}$$

b) $x^2 + (p-1)x + 1 = 0$

$a=1, b=p-1, c=1$

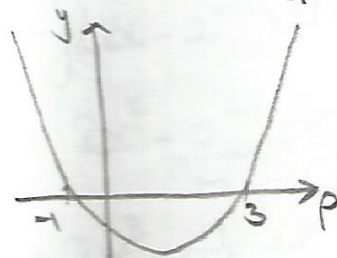
Real roots when $b^2 - 4ac \geq 0$

$b^2 - 4ac = (p-1)^2 - 4$

$= p^2 - 2p + 1 - 4$

$= p^2 - 2p - 3$

$= (p-3)(p+1)$



$p \leq -1, p \geq 3$ ✓

2006 Paper I

① a) midpoint = (3, 5)

$\therefore M_{BD} = \frac{5+5}{3+2} = \frac{10}{5} = 2$

$y - 5 = 2(x - 3)$

$y - 5 = 2x - 6$

$y = 2x - 1$

b) $M_{BC} = \frac{-5+2}{-2-7} = \frac{-3}{-9} = \frac{1}{3}$

$\therefore M_{AC} = -3$

$y - 12 = -3(x + 1)$

$y - 12 = -3x - 3$

$y = -3x + 9$

c) $3x + y = 9$

$2x - y = 1$

$5x = 10$

$x = 2$

check $2+2-3 = 1$ ✓

$6+y=9$

$y=3$ ✓

(2, 3)