$\frac{dA}{dx} = 2x - 432000x^{-2}$

$= 2x - \frac{432000}{x^2}$

$\therefore 2x - \frac{432000}{x^2} = 0$

$2x = \frac{432000}{x^2}$

$2x^3 = 432000$

$x^3 = 216000$

$x = 60$

x	59	60	61
$\frac{dA}{dx}$	-6.1	0	5.9
shape	\	—	/

$\therefore$ Minimum when $x = 60$ cm

(9) $\underline{a}.(\underline{a}+\underline{b}) = \underline{a}.\underline{a} + \underline{a}.\underline{b}$

$36 = 25 + \underline{a}.\underline{b}$

$\underline{a}.\underline{b} = 11$

$\cos\theta = \frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$

$\cos\theta = \frac{11}{5\times4}$

$\theta = \cos^{-1}\left(\frac{11}{20}\right)$

$\theta = 56.6°$

(10) $3\cos 2x + 10\cos x - 1 = 0$

$6\cos^2 x - 3 + 10\cos x - 1 = 0$

$6\cos^2 x + 10\cos x - 4 = 0$

$2(3\cos^2 x + 5\cos x - 2) = 0$

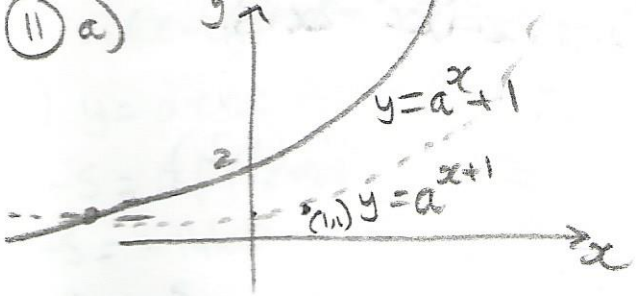
$2(3\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{3}$ $\quad$ $\cos x = -2$ NO SOLUTION!

$x = \cos^{-1}\left(\frac{1}{3}\right)$

$x = 1.23$

(11) a)



b) $a^{x+1} = a^x + 1$

$a^{x+1} - a^x = 1$

$a(a^x) - a^x = 1$

$a^x(a-1) = 1$

$a^x = \frac{1}{a-1}$

$\log_a(a^x) = \log_a\left(\frac{1}{a-1}\right)$

$x = \log_a\left(\frac{1}{a-1}\right)$

2004 PI

(1) a) $2x + 5y = 0$

$5y = -2x$

$y = -\frac{2x}{5}$

$\therefore B = (5, -2)$

$\therefore x + 3\left(\frac{-2x}{5}\right) + 1 = 0$

$x - \frac{6x}{5} + 1 = 0$

$-\frac{1}{5}x = -1$

$x = 5$

$y = -2$

$\therefore M_{AB} = \frac{-2-4}{5-7} = \frac{-6}{-2} = 3$

b) $x + 3y + 1 = 0$

$y = -\frac{x}{3} - \frac{1}{3}$ $\quad \therefore M = -\frac{1}{3}$

$\therefore$ tan to AB

$2x + 5y = 0$

$y = -\frac{2}{5}x$ $\quad \therefore$ Not tan to AB

(2)

a) (1) -1

	1	−1	−5	−3
		−1	2	3
	−1	−2	−3	0

(11) $f(x) = (x+1)(x^2-2x-3)$

$= (x+1)(x-3)(x+1)$

b) $(-1, 0)$

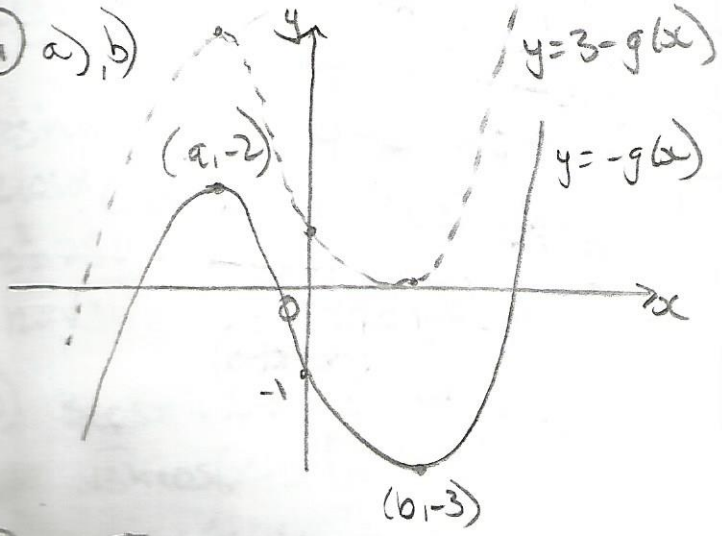
(3) $\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \tan^{-1}(\sqrt{3})$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(4) a), b)



(5) a) $\vec{AB} = \underline{b} - \underline{a}$ $\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2\vec{BC}$ $= \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

∴ as $\vec{AB} = 2\vec{BC}$ and B is common, points are collinear.

b) $\vec{AD} = 4\vec{AB}$ ∴ $\underline{d} = \underline{a} + \vec{AD}$

$= \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix}$

∴ $D = (5, 20, -9)$ $= \begin{pmatrix} 5 \\ 20 \\ -9 \end{pmatrix}$

(6) $y = 3\sin x + \cos 2x$

$\frac{dy}{dx} = 3\cos x - 2\sin 2x$

(7) $\int_0^2 \sqrt{4x+1}\, dx$

$= \int_0^2 (4x+1)^{1/2} dx$

$= \left[\frac{(4x+1)^{3/2}}{3/2 \times 4}\right]_0^2$

$= \left[\frac{\sqrt{(4x+1)^3}}{6}\right]_0^2$

$= \left(\frac{\sqrt{9^3}}{6}\right) - \left(\frac{\sqrt{1^3}}{6}\right)$

$= \frac{27}{6} - \frac{1}{6}$

$= \frac{26}{6}$

$= 4\frac{1}{3}$

(8) a) $x^2 - 10x + 27$

$= (x-5)^2 + 2$

b) $g'(x) = x^2 - 10x + 27$

$= (x-5)^2 + 2$

As $g'(x) > 0$, function is always increasing.

(9) $\log_2(x+1) - 2\log_2 3 = -3$

$\log_2(x+1) - \log_2 9 = -3$

$\log_2 \frac{(x+1)}{9} = -3$

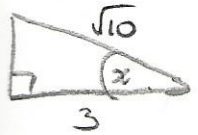
$\frac{x+1}{9} = 2^{-3}$

$\frac{x+1}{9} = 8$

$x+1 = 72$

$x = 71$

(10) $\angle DEA = 2x + 90°$



$\cos \angle DEA = \cos(2x + 90)°$

$= \cos 2x \cos 90 - \sin 2x \sin 90$

$= (\cos 2x \times 0) - (\sin 2x \times 1)$

$= -\sin 2x$

$-\sin 2x = -2\sin x\cos x$

$= -2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right)$

$= \frac{-6}{10}$

$= \frac{-3}{5}$

(11) a) $y = a(x-b)$

$-6 = a(1-2)$

$-6 = -a$

$a = 6$ $\therefore a = 6,\ b = 2$

b) $f(x) = \int 6x(x-2)\,dx$

$= \int (6x^2 - 12x)\,dx$

$= 2x^3 - 6x^2 + C.$

$\therefore 4 = 2 - 6 + C$

$4 = -4 + C$

$C = 8$

$\therefore f(x) = 6x^2 - 12x + 8$

2004 Paper II

(1) a) $x - 2y = 0$

$x = 2y$

$y = \frac{1}{2}x$ $\therefore m = \frac{1}{2}$

$m = \tan\theta$

$\therefore \theta = \tan^{-1}\frac{1}{2}$

$\theta = 30°.$

b) $m_{OB} = \tan 60°$

$= 0.9$

(2) a) $\overrightarrow{QP} = \underline{p} - \underline{q}$ $\overrightarrow{QR} = \underline{r} - \underline{q}$

$= \begin{pmatrix}1\\3\\-1\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix}$ $= \begin{pmatrix}-3\\1\\2\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix}$

$= \begin{pmatrix}-1\\3\\-2\end{pmatrix}$ $= \begin{pmatrix}-5\\1\\1\end{pmatrix}$

b) $|\overrightarrow{QP}| = \sqrt{1+9+4}$ $|\overrightarrow{QR}| = \sqrt{25+1+1}$

$= \sqrt{14}$ $= \sqrt{27}$

$\overrightarrow{QP}.\overrightarrow{QR} = 5 + 3 - 2$

$= 6$

$\therefore \cos\theta = \frac{6}{\sqrt{14}\sqrt{27}}$

$\theta = \cos^{-1}\left(\frac{6}{\sqrt{14}\sqrt{27}}\right)$

$\theta = 72°$

(3) $2x^2 + px - 3 = 0$

For real roots, $b^2 - 4ac \geq 0.$

$a = 2$ $b = p$ $c = -3$

$b^2 - 4ac$

$= p^2 - 4\times 2\times(-3)$

$= p^2 + 24$

$p^2 + 24$ always ≥ 24 $\therefore$ roots always real.

(4) a) $-1 < k < 1$

b) $u_{n+1} = ku_n + 3$

$L = kL + 3$

$5 = 5k + 3$

$5 - 5k = 3$

$5(1-k) = 3$

$1 - k = \frac{3}{5}$

$k = 1 - \frac{3}{5}$

$k = \frac{2}{5}$

(5) a) $y = 6x^2 - x^3$

$\frac{dy}{dx} = 12x - 3x^2$ $\therefore 12x - 3x^2 = 12$

$3x^2 - 12x + 12 = 0$

$3(x^2 - 4x + 4) = 0$

$3(x-2)^2 = 0$

$x = 2$

b) when $x=2$, $y=6(2^2)-(2)^3$

$=24-8$

$=16$

$\therefore y-16=12(x-2)$

$y-16=12x-24$

$y=12x-8$

(6) a)

$3\cos x+5\sin x=k\cos(x-\alpha)$

$=k\cos\alpha\cos x+k\sin\alpha\sin x$

$k\sin\alpha=5$ (5)

$k\cos\alpha=3$ (3)

$\tan\alpha=\frac{5}{3}$

$\alpha=59°$

$k^2=25+9$

$k=\sqrt{34}$ $\therefore \sqrt{34}\cos(x-59)^\circ$

b) $3\cos x^\circ+5\sin x^\circ=4$

$\sqrt{34}\cos(x-59)^\circ=4$

$\cos(x-59)^\circ=\frac{4}{\sqrt{34}}$

$x-59^\circ=\cos^{-1}\left(\frac{4}{\sqrt{34}}\right)$

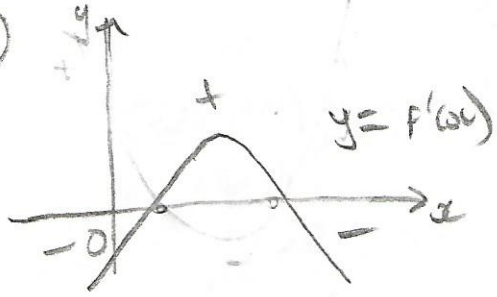
$x-59^\circ=46.7^\circ, 313.3^\circ$

$x=105.7^\circ, 372.3^\circ$

$x=12.3^\circ, 105.7^\circ$

$\therefore x=12.3^\circ$

(7)



(8) a) centre $=(6,1)$

$\therefore M_r=\frac{1+1}{6-5}=\frac{2}{1}=2$

$\therefore M_T=-\frac{1}{2}$

$y+1=-\frac{1}{2}(x-5)$

$2y+2=-x+5$

$x+2y=3$

b) $x=3-2y$

$x^2+y^2+10x+2y+6=0$

$(3-2y)^2+y^2+10(3-2y)+2y+6=0$

$9-12y+4y^2+y^2+30-20y+2y+6=0$

$5y^2-30y+45=0$

$5(y^2-6y+9)=0$

b^2-4ac

$=36-4\times1\times9$

$=0$

$\therefore$ tangent

c) $5(y-3)^2=0$

$y=3$

$x=-3$

$\therefore (-3,3)$

$d=\sqrt{(5+3)^2+(-1-3)^2}$

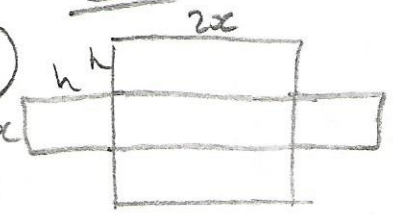
$=\sqrt{8^2+(-4)^2}$

$=\sqrt{64+16}$

$=\sqrt{80}$

$=4\sqrt{5}$

(9)



a) $SA=2x^2+2xh+4xh$

$=2x^2+6xh$

$SA=12$ $\therefore 2x^2+6xh=12$

$6xh=12-2x^2$

$h=\frac{12-2x^2}{6x}$

$h=\frac{6-x^2}{3x}$

$V=lbh$

$=2x\times x\times\left(\frac{6-x^2}{3x}\right)$

$=2x^2\left(\frac{6-x^2}{3x}\right)$

$=\frac{2x^2(6-x^2)}{3x}$

$=\frac{2}{3}x(6-x^2)$

b) SP's @ $\frac{dV}{dx} = 0$

$V = 4x - \frac{2}{3}x^3$

$\frac{dV}{dx} = 4 - 2x^2$

$\therefore 4 - 2x^2 = 0$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

x	1	$\sqrt{2}$	2
$\frac{dV}{dx}$	2	0	−4
shape	/	—	\

$\therefore$ Maximum volume when $x = \sqrt{2}$

(10) a) $A_t = A_0 e^{-0.002t}$

$600 = A_0 e^{-0.002 \times 1000}$

$600 = A_0 e^{-2}$

$A_0 = \frac{600}{e^{-2}}$

$A_0 = 4433$ mg

b) $0.5 = e^{-0.002t}$

$\ln 0.5 = -0.002t$

$t = \frac{\ln 0.5}{-0.002}$

$t = 346.6$ years

(11) $2x - \frac{1}{2}x^2 = 1.5$

$2x - \frac{1}{2}x^2 = \frac{3}{2}$

$4x - x^2 = 3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, x = 3.$

$\therefore A = \int_1^3 (2x - \frac{1}{2}x^2 - \frac{3}{2})\,dx$

$= \left[x^2 - \frac{x^3}{6} - \frac{3x}{2}\right]_1^3$

$= \left(9 - \frac{27}{6} - \frac{9}{2}\right) - \left(1 - \frac{1}{6} - \frac{3}{2}\right)$

$= \left(\frac{54}{6} - \frac{27}{6} - \frac{27}{6}\right) - \left(\frac{6}{6} - \frac{1}{6} - \frac{9}{6}\right)$

$= (0) - \left(-\frac{4}{6}\right)$

$= \frac{4}{6}$

$= \frac{2}{3}$ m²

2005 Paper I

(1) $m = \tan\theta$

$= \tan 60°$

$= \sqrt{3}$

$y = \sqrt{3}(x+2)$ ✓

$y = \sqrt{3}x + 2\sqrt{3}$

(2) a) P is midpoint of AB

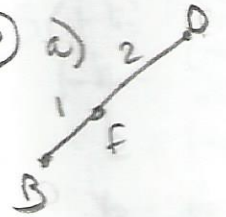
$A = (-3, -2)$ $B = (3, 6)$ ✓

$\therefore P = (0, 2)$

b) $AB = \sqrt{6^2 + 8^2}$ ✓

$= 10$ ✓

(3) a)



$\underline{f} = \frac{1}{3}(2\underline{b} + \underline{d})$

$= \frac{1}{3}\left(\begin{pmatrix}24\\12\\0\end{pmatrix} + \begin{pmatrix}6\\3\\9\end{pmatrix}\right)$

$= \frac{1}{3}\begin{pmatrix}30\\15\\9\end{pmatrix}$

$\therefore F = (10, 5, 3) = \begin{pmatrix}10\\5\\3\end{pmatrix}$ ✓

b) $\underline{Af} = \underline{f} - \underline{a} = \begin{pmatrix}10\\5\\3\end{pmatrix} - \begin{pmatrix}12\\0\\0\end{pmatrix}$

$= \begin{pmatrix}-2\\5\\3\end{pmatrix}$ ✓