$\therefore$ Total $= \frac{845}{24} + \frac{2035}{72}$

$= \frac{2535}{72} + \frac{2035}{72}$

$= \frac{4750}{72}$

$= 63\frac{17}{36} u^2$

2003 PI

① $y = 1 - 4x \quad m_1 = -4 \therefore m_2 = \frac{1}{4}$

$y - b = m(x - a)$

$y - 3 = \frac{1}{4}(x+1)$

$4y - 12 = x + 1$

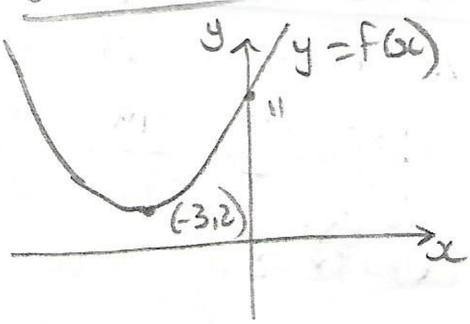
$x - 4y = -13$

② a) $x^2 + 6x + 11$

$= (x+3)^2 - 9 + 11$

$= (x+3)^2 + 2$

b)



③ $\underline{u} \cdot \underline{v} = 6 - 6 + 0$

$= 0$

$\underline{u}$ and $\underline{v}$ are perpendicular.

④ a) $u_0 = 12,\ u_1 = 15,\ u_2 = 16$

$12a + b = 15$ — ①

$15a + b = 16$ — ②

② − ① $3a = 1$

$a = \frac{1}{3}$

$5 + b = 16$

$b = 11$

check

$12(\frac{1}{3}) + 4$

$= 4 + 11$

$= 15$ ✓

$a = \frac{1}{3},\ b = 11$

b) $u_{n+1} = \frac{1}{3}u_n + 11$

$L = \frac{1}{3}L + 11$

$\frac{2}{3}L = 11$

$L = \frac{33}{2}$

⑤ $f(x) = \sqrt{x} + \frac{2}{x^2}$

$= x^{1/2} + 2x^{-2}$

$f'(x) = \frac{1}{2}x^{-1/2} - 4x^{-3}$

$= \frac{1}{2\sqrt{x}} - \frac{4}{x^3}$

$f'(4) = \frac{1}{2\sqrt{4}} - \frac{4}{4^3}$

$= \frac{1}{4} - \frac{4}{64}$

$= \frac{1}{4} - \frac{1}{16}$

$= \frac{3}{16}$

⑥ $\overrightarrow{AB} = \underline{b} - \underline{a} \qquad \overrightarrow{AD} = 3\overrightarrow{AB}$

$= \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\therefore \underline{d} = \underline{a} + \overrightarrow{AD}$

$= \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix}$

$\therefore D = (8, 3, -1)$

⑦ $x^2 + 3x + 4 = 2x + 1$

$x^2 + x + 3 = 0$

$b^2 - 4ac = 1 - 12$

$= -11$

$b^2 - 4ac < 0$, $\therefore$ no intersection.

⑧ $\int_0^1 \frac{dx}{(3x+1)^{1/2}}$

$= \int_0^1 (3x+1)^{-1/2}\,dx$

$= \left[\frac{(3x+1)^{1/2}}{1/2 \times 3} \right]_0^1$

$= \left[\frac{2}{3}\sqrt{3x+1}\right]_0$

$= \left(\frac{2}{3}\sqrt{4}\right) - \left(\frac{2}{3}\sqrt{1}\right)$

$= \frac{4}{3} - \frac{2}{3}$

$= \frac{2}{3}$

9 a) $h(x) = f(g(x))$

$= \frac{1}{(2x+3)-4}$

$= \frac{1}{2x-1}$

b) $\{x : x \neq \frac{1}{2}, x \in \mathbb{R}\}$

10 a) (i) $\sin 2p = 2\sin p \cos p$

$= 2 \times \frac{4}{\sqrt{80}} \times \frac{8}{\sqrt{80}}$

$= \frac{64}{80}$

$= \frac{4}{5}$

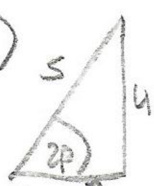
(ii) $\cos 2p = \cos^2 p - \sin^2 p$

$= \frac{64}{80} - \frac{16}{80}$

$= \frac{48}{80}$

$= \frac{3}{5}$

b)



$m_{OB} = \frac{4}{3}$

11 a) centre $= (12, -5)$

(i) $d = \sqrt{12^2 + (-5)^2}$

$= \sqrt{169}$

$= 13$

(ii) $OB = 24$ units $\therefore B = (24, 0)$

$OA = 13.$ $13 - 5 = 8$ $\therefore r_B = 8.$

$\therefore (x-24)^2 + y^2 = 64$

b) $y = px(x-24)$

$-5 = 12p(12-24)$

$-5 = -144p$

$p = \frac{5}{144}$

$\therefore p = \frac{5}{144},\ q = -24$

12 $3\log_e 2e - 2\log_e 3e$

$= \log_e (2e)^3 - \log_e (3e)^2$

$= \log_e 8e^3 - \log_e 9e^2$

$= \log_e e^3 + \log_e 8 - (\log_e 9 + \log_e e^2)$

$= 3 + \log_e 8 - \log_e 9 - 2$

$= 1 + \log_e 8 - \log_e 9$

2003 P II

1 a)

$$\begin{array}{c|cccc} 2 & 6 & -5 & -17 & 6 \\ & & 12 & 14 & -6 \\ \hline & 6 & 7 & -3 & 0 \end{array}$$

b) $f(x) = (x-2)(6x^2 + 7x - 3)$

$= (x-2)(2x+3)(3x-1$

2 $y = 4\sin 2x + 1$

$a = 4,\ b = 2,\ c = 1$

3 $x^2 + 2x - (x^3 - x^2 - 6x)$

$= 2x^2 + 8x - x^3$

$A = \int_0^4 (2x^2 + 8x - x^3)\,dx$

$= \left[\frac{2x^3}{3} + 4x^2 - \frac{x^4}{4}\right]_0^4$

$= \left(\frac{128}{3} + 64 - 64\right) - (0)$

$= 42\frac{2}{3}$ unit^2

(4) a) $y = x^3 + 2x^2 - 3x + 2$

$= 1 + 2 - 3 + 2$

$= 2$ $\therefore (1, 2)$

$\frac{dy}{dx} = 3x^2 + 4x - 3$

$\therefore m = 3 + 4 - 3$

$= 4$

$y - b = m(x - a)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = 4x - 2$

b) $x^2 + y^2 - 12x - 10y + 44 = 0$

$x^2 + (4x-2)^2 - 12x - 10(4x-2) + 44 = 0$

$x^2 + 16x^2 - 16x + 4 - 12x - 40x + 20 + 44 = 0$

$17x^2 - 68x + 68 = 0$

$17(x^2 - 4x + 4) = 0$

$b^2 - 4ac = 16 - 4 \times 1 \times 4$

$= 0$

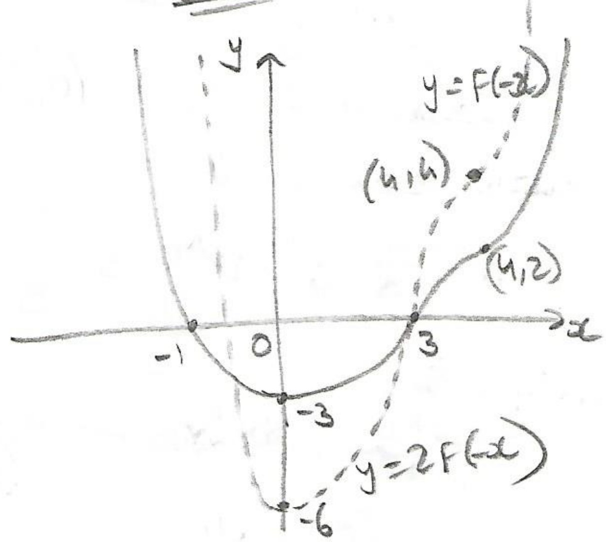
$\therefore$ tangent.

$17(x-2)^2 = 0$

$x = 2$

$y = 6$ $(2, 6)$

(5)



(6) $F(x) = \cos 2x - 3\sin 4x$

$F'(x) = -2\sin 2x - 12\cos 4x$

$F'\left(\frac{\pi}{6}\right) = -2\sin\left(\frac{\pi}{3}\right) - 12\cos\left(\frac{2\pi}{3}\right)$

$= -2\left(\frac{\sqrt{3}}{2}\right) - 12\left(-\frac{1}{2}\right)$

$= 6 - \sqrt{3}$

(7) a)

$2\sin x + 5\boxed{\cos x} = k\sin(x + \alpha)$

$= k\sin x\cos\alpha + k\cos x\sin\alpha$

$= \underbrace{k\cos\alpha}_{2}(\sin x) + \underbrace{k\sin\alpha}_{5}\boxed{\cos x}$

$k\sin\alpha = 5$

$k\cos\alpha = 2$

$k^2 = 2^2 + 5^2$

$k^2 = 4 + 25$

$k = \sqrt{29}$

$\tan\alpha = \frac{5}{2}$

$\alpha = \tan^{-1}\left(\frac{5}{2}\right)$

$\alpha = 68.2^\circ$

S | A ✓

T | C ✓

$s = +$

$c = +$

$\therefore Q_1$

$\therefore 2\sin x + 5\cos x = \sqrt{29}\sin(x + 68.2)^\circ$

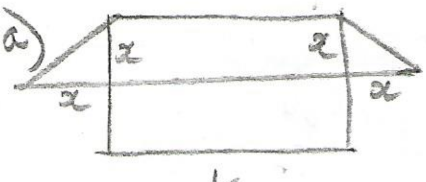
b) Minimum when $\sin(x + 68.2)^\circ = -1$

$x + 68.2^\circ = 270^\circ$

$x = 201.8^\circ$

$(201.8^\circ, -\sqrt{29})$

(8) a)



$A_T = \frac{1}{2}x^2$ $\quad A_R = xL$

$\therefore$ Total $= 2A_T + 2A_R$

$= x^2 + 2xL$

Volume $= AL$

$108000 = \frac{1}{2}x^2 L$

$x^2 L = 216000$

$L = \frac{216000}{x^2}$

$\therefore$ Area $= x^2 + 2x\left(\frac{216000}{x^2}\right)$

$= x^2 + \frac{432000}{x}$

b) sp's when $\frac{dA}{dx} = 0$

$A = x^2 + 432000x^{-1}$

$$\frac{dA}{dx} = 2x - 432000x^{-2}$$
$$= 2x - \frac{432000}{x^2}$$
$$\therefore 2x - \frac{432000}{x^2} = 0$$
$$2x = \frac{432000}{x^2}$$
$$2x^3 = 432000$$
$$x^3 = 216000$$
$$\underline{x = 60}$$

x	59	60	61
$\frac{dA}{dx}$	−6.1	0	5.9
shape	\	—	/

∴ Minimum when $\underline{x = 60\text{ cm}}$

(9) $\underline{a}.(\underline{a}+\underline{b}) = \underline{a}.\underline{a} + \underline{a}.\underline{b}$

$$36 = 25 + \underline{a}.\underline{b}$$
$$\underline{a}.\underline{b} = 11$$
$$\cos\theta = \frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$$
$$\cos\theta = \frac{11}{5\times 4}$$
$$\theta = \cos^{-1}\left(\frac{11}{20}\right)$$
$$\underline{\theta = 56.6^\circ}$$

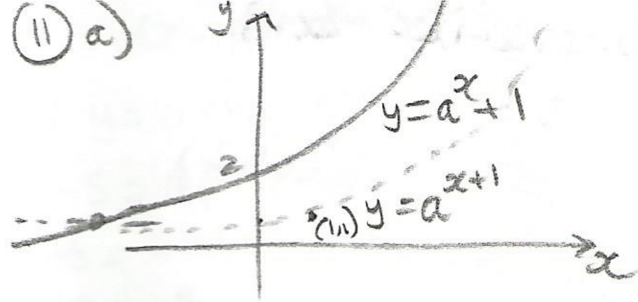
(10) $3\cos 2x + 10\cos x - 1 = 0$

$$6\cos^2 x - 3 + 10\cos x - 1 = 0$$
$$6\cos^2 x + 10\cos x - 4 = 0$$
$$2(3\cos^2 x + 5\cos x - 2) = 0$$
$$2(3\cos x - 1)(\cos x + 2) = 0$$

$\cos x = \frac{1}{3}$ $\quad$ $\cos x = -2$ NO SOLUTION!

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$
$$\underline{x = 1.23}$$

(11) a)



b) $a^{x+1} = a^x + 1$

$$a^{x+1} - a^x = 1$$
$$a(a^x) - a^x = 1$$
$$a^x(a-1) = 1$$
$$a^x = \frac{1}{a-1}$$
$$\log_a(a^x) = \log_a\left(\frac{1}{a-1}\right)$$
$$\underline{x = \log_a\left(\frac{1}{a-1}\right)}$$

2004 PI

(1) a) $2x + 5y = 0$

$$5y = -2x$$
$$y = -\frac{2x}{5}$$

$\therefore x + 3\left(\frac{-2x}{5}\right) + 1 = 0$

$$x - \frac{6x}{5} + 1 = 0$$
$$-\frac{1}{5}x = -1$$
$$\underline{x = 5}$$
$$\underline{y = -2}$$

$\therefore B = (5, -2)$

$\therefore m_{AB} = \frac{-2-4}{5-7} = \frac{-6}{-2} = \underline{3}$

b) $x + 3y + 1 = 0$

$y = -\frac{x}{3} - \frac{1}{3}$ $\quad$ $\therefore m = -\frac{1}{3}$

∴ $\perp$ar TO AB

$2x + 5y = 0$

$y = -\frac{2}{5}x$ $\quad$ ∴ Not $\perp$ar TO AB

(2)

a)(i)

−1	1	−1	−5	−3
		−1	2	3
	1	−2	−3	0