$= -x^2 + px - x + p$

$= p + (p-1)x - x^2$

b) $-x^2 + px - x + p = x + p$

$x^2 + 2x - px = 0$

$x^2 + (2-p)x = 0$

$a = 1$ $b = 2-p$ $c = 0$

for tangency, $b^2 - 4ac = 0$

$(2-p)^2 = 0$

$p = 2$

2002 Paper I

① Centre $= (-1, 4)$ $\therefore M_r = \frac{4+3}{-1+2} = \frac{2}{3}$

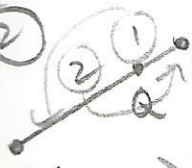
$\therefore M_T = -\frac{3}{2}$ $y - b = m(x - a)$

$y - 3 = -\frac{3}{2}(x - 2)$

$2y - 6 = -3x + 6$

$3x + 2y = 12$

②



$Q = (3, 1, -2)$

$\underline{q} = \frac{1}{3}(2\underline{r} + \underline{p})$

$= \frac{1}{3}\left[\begin{pmatrix}10\\4\\-6\end{pmatrix} + \begin{pmatrix}-1\\-3\\0\end{pmatrix}\right]$

$= \frac{1}{3}\begin{pmatrix}9\\3\\-6\end{pmatrix}$

$= \begin{pmatrix}3\\1\\-2\end{pmatrix}$

③ a) (i) $f(g(x)) = \sin 2x$

$g(f(x)) = 2\sin x$

b) $2f(g(x)) = g(f(x))$

$2\sin 2x = 2\sin x$

$2\sin 2x - 2\sin x = 0$

$4\sin x \cos x - 2\sin x = 0$

$2\sin x(2\cos x - 1) = 0$ $\quad 0 \le x \le 360$

$\sin x = 0$ $\quad \cos x = \frac{1}{2}$

$x = 0°, 180°, 360°$ $\quad x = 60°, 300°$

$\therefore x = 0°, 60°, 180°, 300°, 360°.$

④ $M = \tan\theta$ $\quad \therefore \frac{dy}{dx} = 1$

$= \tan 45°$

$= 1$

$\therefore 4x - 7 = 1$ $\quad y = 2(2)^2 - 7(2) + 10$

$4x = 8$ $\quad = 8 - 14 + 10$

$x = 2$ $\quad = 4$

$(2, 4)$

⑤ $\sin\alpha = \frac{1}{\sqrt{2}}$ $\quad \sin b = \frac{1}{\sqrt{10}}$

$\cos\alpha = \frac{1}{\sqrt{2}}$ $\quad \cos b = \frac{3}{\sqrt{10}}$

$\therefore \sin(\alpha + b) = \sin\alpha\cos b + \cos\alpha\sin b$

$= \left(\frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}\right)$

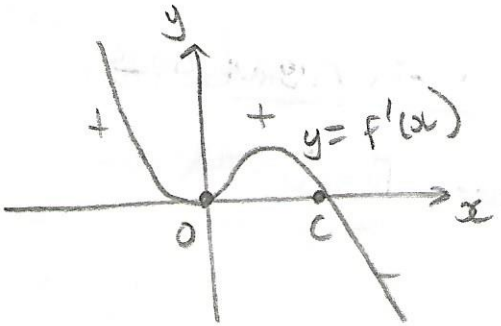
$= \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}}$

$= \frac{4}{\sqrt{20}}$

$= \frac{4}{2\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

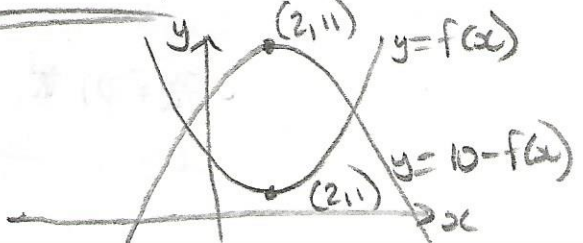
⑥



⑦ a) $x^2 - 4x + 5$

$= (x-2)^2 + 1$

b)



c) $10 - f(x)$

$= 10 - (x^2 - 4x + 5)$

$= 5 + 4x - x^2$

$= (5 - x)(1 + x)$

$\therefore$ positive when $-1 < x < 5$

(8) a) $y = 2\cos 2x$

b) $2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad 0 \le 2x \le 2\pi$

$2x = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{12}, \frac{11\pi}{12}$

$\therefore B = \left(\frac{11\pi}{12}, \sqrt{3}\right)$

(9) a)

$\sin x - \cos x = k\sin(x - \alpha)$

$= k\sin x\cos\alpha - k\cos x\sin\alpha$

$= k\cos\alpha(\sin x) - k\sin\alpha(\cos x)$

$1 \qquad\qquad -1$

$k\sin\alpha = 1$

$k\cos\alpha = 1$

$k = \sqrt{2}$

$\tan\alpha = \frac{1}{1}$

$\alpha = \frac{\pi}{4}$

S | A
T | C

$S = +$

$C = +$

$\therefore$ Q1

$\therefore \sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

b) On y-axis: $\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$

$= \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$

$= -1 \quad \therefore (0, -1)$

On x-axis: $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = 0$

$x - \frac{\pi}{4} = \sin^{-1}(0)$

$x - \frac{\pi}{4} = 0, \pi, 2\pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Max $= \sqrt{2}$ when $\sin\left(x - \frac{\pi}{4}\right) = 1$

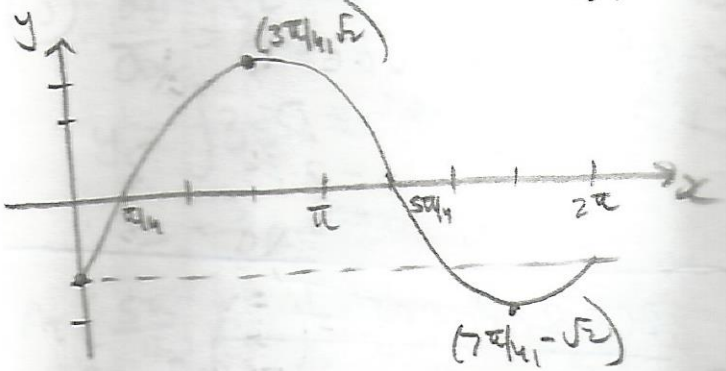
$x - \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{3\pi}{4}$

Min $= -\sqrt{2}$ when $\sin\left(x - \frac{\pi}{4}\right) = -1$

$x - \frac{\pi}{4} = \frac{3\pi}{2}$

$x = \frac{7\pi}{4}$.



(10) a) $f(x) = (8 - x^3)^{1/2}$

$f'(x) = \frac{1}{2}(8 - x^3)^{-1/2} \times (-3x^2)$

$= \frac{-3x^2}{2\sqrt{8 - x^3}}$

b) $-\frac{2}{3}(8 - x^3)^{1/2} + C$

(11) $m = \frac{1 - 0}{0 - 0.5} = -2$

$\therefore \log_5 y = -2\log_5 x + 1$

$\log_5 y = \log_5 x^{-2} + \log_5 5$

$\log_5 y = \log_5 \frac{5}{x^2}$

$y = \frac{5}{x^2}$

$\therefore k = 5, \; n = -2$

2002 Paper II

(1) a) midpoint $= (-2, 2)$

$M_p = \frac{2 - 2}{-2 - 5} = 0 \quad \therefore y = 2$

b) $M_{BC} = \frac{2 + 2}{5 + 3} = \frac{4}{8} = \frac{1}{2} \quad \therefore M_{PB} = -2$

$(m_1 m_2 = -1)$

midpoint $=(1,0)$.

$\therefore y=-(x-1)$

$y=1-x$

c) $2=1-x$ $\quad (-1,2)$

$x=-1$

(2) a) $B=(6,6,0)$

b) $\vec{DA}=\underline{a}-\underline{d}$ $\quad \vec{DB}=\underline{b}-\underline{d}$

$=\begin{pmatrix}6\\0\\0\end{pmatrix}-\begin{pmatrix}3\\3\\8\end{pmatrix}$ $\quad =\begin{pmatrix}6\\6\\0\end{pmatrix}-\begin{pmatrix}3\\3\\8\end{pmatrix}$

$=\begin{pmatrix}3\\-3\\-8\end{pmatrix}$ $\quad =\begin{pmatrix}3\\3\\-8\end{pmatrix}$

c) $\vec{DA}.\vec{DB}=9-9+64$

$=64$

$|\vec{DA}|=\sqrt{9+9+64}$ $\quad |\vec{DB}|=\sqrt{9+9+64}$

$=\sqrt{82}$ $\quad =\sqrt{82}$

$\therefore \cos ADB=\frac{64}{\sqrt{82}\sqrt{82}}$

$ADB=\cos^{-1}\left(\frac{64}{82}\right)$

$ADB=38.7^0$

(3) a) $y=2x^3-7x^2+4x+4$

sp's @ $\frac{dy}{dx}=0$ $\quad \therefore 6x^2-14x+4=0$

$2(3x^2-7x+2)=0$

$2(3x-1)(x-2)=0$

$x=1/3 \quad x=2$

x	0	$1/3$	1	2	3
$\frac{dy}{dx}$	4	0	-4	0	18
shape	/	—	\	—	/

$\therefore$ Max @ $x=1/3$

b) $2 \mid 2 \quad -7 \quad 4 \quad 4$

$\quad\quad\quad 4 \quad -6 \quad -4$

$\quad 2 \quad -3 \quad -2 \quad 0$

$y=(x-2)(2x^2-3x-2)$

$=(x-2)(2x+1)(x-2)$

c) $A=(-\frac{1}{2},0)$

$x<-\frac{1}{2}$.

(4) a) $u_{n+1}=0.8u_n+0.5$

$L=0.8L+0.5$

$0.2L=0.5$ Height settles at 2.5m

$L=\frac{0.5}{0.2}$

$L=2.5$

b) $L=aL+0.5$

$2=2a+0.5$

$2-2a=0.5$

$2(1-a)=0.5$ 25% trimmed of per year.

$1-a=0.25$

$a=0.75$

(5) $1+10x-2x^2=1+5x-x^2$ (TOP, BOTTOM)

$5x-x^2=0$

$x(5-x)=0$

$x=0, x=5$

$\therefore A=\int_0^5 (5x-x^2)dx$

$=\left[\frac{5x^2}{2}-\frac{x^3}{3}\right]_0^5$

$=\left(\frac{125}{2}-\frac{125}{3}\right)-(0)$

$=\frac{125}{6}$ unit^2

(6) $y = 2\sin\left(x - \frac{\pi}{6}\right)$

$= 2\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

$= 2\sin\left(\frac{\pi}{6}\right)$

$= \underline{1}$ $\therefore \left(\frac{\pi}{3}, 1\right)$

$\frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{6}\right)$

$\therefore$ when $x = \frac{\pi}{3}$, $m = 2\cos\left(\frac{\pi}{6}\right)$

$= \underline{\sqrt{3}}$

$\therefore y - 1 = \sqrt{3}\left(x - \frac{\pi}{3}\right)$

(7) $\log_3(x-2) + 1 = 0$

$(x-2) + 3 = 0$

$x + 1 = 0$

$\underline{x = -1}$

(8) $v = \int a\,dt$

$= \int 2(4-t)^{1/2}\,dt$

$= \frac{2(4-t)^{3/2}}{-3/2} + C$

$= \underline{-\frac{4}{3}(4-t)^{3/2} + C}$

(9) $(1-2k)x^2 - 5kx - 2k = 0$

$a = 1-2k$

$b = -5k$

$c = -2k$

For real roots,

$b^2 - 4ac \geq 0$

$(-5k)^2 - 4(1-2k)(-2k) \geq 0$

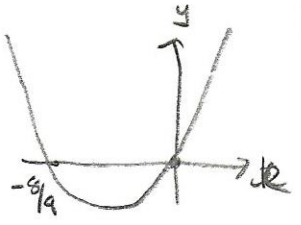
$25k^2 + 8k(1-2k) \geq 0$

$25k^2 + 8k - 16k^2 \geq 0$

$9k^2 + 8k \geq 0$

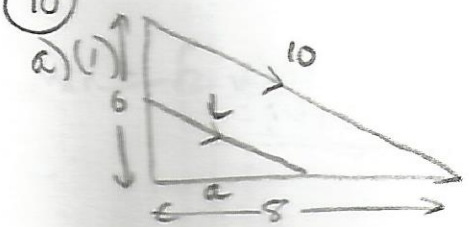
$k(9k+8) \geq 0$

$\therefore$ No real roots when $-8/9 < k < 0$



ie Real for all integer values of k.

(10) a)(i)



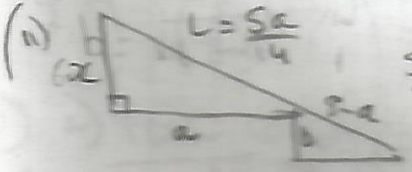
Similar triangles, $\therefore$

$\frac{L}{a} = \frac{10}{8}$

$L = \frac{10a}{8}$

$L = \underline{\frac{5a}{4}}$

(ii)



similar triangles

$x^2 = \frac{25a^2}{16} - a^2$ Sp $\frac{b}{\frac{3a}{4}} = \frac{8-a}{\frac{5a}{4}}$

$x^2 = \frac{9a^2}{16}$ $\frac{4b}{3a} = \frac{32-4a}{5a}$

$x = \frac{3a}{4}$ $b = \underline{\frac{3a(32-4a)}{20a}}$

$\therefore$ Area $= \frac{1}{2}Lb$

$= \frac{1}{2} \times \frac{5a}{4} \times \left(\frac{3a(32-4a)}{20a}\right)$

$= \frac{12a(8-a)}{16}$

$= \underline{\frac{3a}{4}(8-a)}$

(11) $A = 6a - \frac{3}{4}a^2$

$\frac{dA}{da} = 6 - \frac{3}{2}a$

SP's @ $\frac{dA}{da} = 0$ $\therefore 6 - \frac{3}{2}a = 0$

$\frac{3}{2}a = 6$

$\underline{a = 4}$

a	3	4	5
$\frac{dA}{da}$	3	0	-3
shape	/	−	\

$\therefore$ Max @ $a = 4$ m