2001 Paper 1

① $2x+3y=5$

$3y=5-2x$

$y=\frac{5}{3}-\frac{2}{3}x \quad \therefore m=-\frac{2}{3}$

$y+1=-\frac{2}{3}(x-2)$

$3y+3=-2x+4$

$2x+3y=1$

② $x^2-5x+(k+6)=0$

$a=1$, $b=-5$, $c=k+6$

for equal roots, $b^2-4ac=0$

$25-4(k+6)=0$

$25-4k-24=0$

$-4k+1=0$

$k=\frac{1}{4}$

③ a) $\vec{AB}=\underline{b}-\underline{a}$ $\quad \vec{BC}=\underline{c}-\underline{b}$

$=\begin{pmatrix}-2\\-1\\1\end{pmatrix}-\begin{pmatrix}-8\\-10\\-2\end{pmatrix}$ $\quad =\begin{pmatrix}6\\11\\5\end{pmatrix}-\begin{pmatrix}-2\\-1\\1\end{pmatrix}$

$=\begin{pmatrix}6\\9\\3\end{pmatrix}$ $\quad =\begin{pmatrix}8\\12\\4\end{pmatrix}$

$=\frac{4}{3}\vec{AB}$

$\vec{BC}=\frac{4}{3}\vec{AB}$ and B is common,

$\therefore$ collinear

$\therefore$ road is straight.

b) $\vec{DB}=\underline{b}-\underline{d}$

$=\begin{pmatrix}-2\\-1\\1\end{pmatrix}-\begin{pmatrix}1\\-4\\4\end{pmatrix}$

$=\begin{pmatrix}-3\\3\\-3\end{pmatrix}$

$\vec{AB}\cdot\vec{DB}=-18+27-9$

$=0 \quad \therefore$ perpendicular.

④ $x^2+2x-8=(x+1)^2-9$

⑤ a) $\sin 2x-\cos x=0$

$2\sin x\cos x-\cos x=0$

$\cos x(2\sin x-1)=0$

$\cos x=0 \quad \sin x=\frac{1}{2}$

$x=90^\circ \quad x=30^\circ, 150^\circ$

$\therefore x=30^\circ, 90^\circ, 150^\circ$

b) P $x=150^\circ$ $\quad y=\cos 150^\circ$

$=-\cos 30^\circ$

$=-\frac{\sqrt{3}}{2}$

$\therefore x=\left(150, -\frac{\sqrt{3}}{2}\right)$

⑥ $P=12x^3-x^4$

$\frac{dP}{dx}=36x^2-4x^3$

SP's @ $\frac{dP}{dx}=0 \quad \therefore 36x^2-4x^3=0$

$4x^2(9-x)=0$

$x=0, \quad x=9$

x	8	9	10
$\frac{dP}{dx}$	256	0	-400
shape	/	—	\

$\therefore$ Max when $x=9$

⑦ a)

(i) $f(h(x))=f(x+\frac{\pi}{4})$

$=\sin(x+\frac{\pi}{4})$

(ii) $g(h(x))=\cos(x+\frac{\pi}{4})$

b) (i) $\sin(x+\frac{\pi}{4})=\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}$

$=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$

(ii) $\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x$

$f(h(x))-g(h(x))=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x-\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)$

$=\sqrt{2}\sin x$

$\therefore \sqrt{2}\sin x = 1 \quad 0 \le x \le 2\pi$

$\sin x = \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

(8) $4\log_x 6 - 2\log_x 4 = 1$

$\log_x 6^4 - \log_x 4^2 = 1$

$\log_x 1296 - \log_6 16 = 1$

$\log_x\left(\frac{1296}{16}\right) = 1$

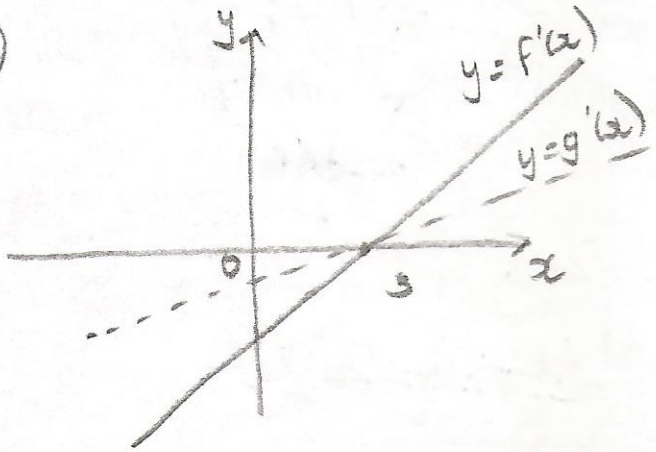
$\frac{1296}{16} = x^1$

$5x = 81$

36
$216\ 6$
1296

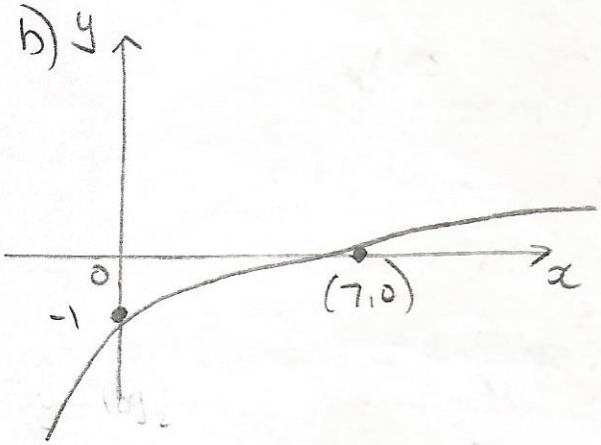
81
$16\overline{)1296}$
1280
16
16

(9)



(10) a) $a = 1, b = 3$

b)



(11) a)

(i) $r = \sqrt{4^2 + 5^2 - 9}$

$= \sqrt{32}$

$= 4\sqrt{2}$

(ii) $PQ = \sqrt{(4+2)^2 + (5+1)^2}$

$= \sqrt{6^2 + 6^2}$

$= \sqrt{72}$

$= 6\sqrt{2}$

$2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$

$\therefore$ circles touch.

b) $m_R = \frac{-1-1}{-2+4} = \frac{-2}{2} = -1$

$\therefore m_T = 1 \quad (m_1 m_2 = -1)$

$y - 1 = x + 4$

$y = x + 5$

b) $x^2 + y^2 - 8x - 10y + 9 = 0$

$x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$

$x^2 + x^2 + 10x + 25 - 8x - 10x - 50 + 9 = 0$

$2x^2 - 8x - 16 = 0$

$2(x^2 - 4x - 8) = 0$

$x^2 - 4x - 8 = 0$

$(x-2)^2 - 12 = 0$

$(x-2)^2 = 12$

$x - 2 = \pm\sqrt{12}$

$x - 2 = \pm 2\sqrt{3}$

$x = 2 \pm 2\sqrt{3}$

2001 Paper II

(1)

a)

-2	2	1	k	2
		-4	6	-2k-12
	2	-3	k+6	-2k-10

$-2k - 10 = 0$

$k = -5$

b)

-2	2	1	-5	2
		-4	6	-2
	2	-3	1	0

$\therefore (x+2)(2x^2-3x+1)=0$

$(x+2)(2x-1)(x-1)=0$

$x=-2,\ x=\frac{1}{2},\ x=1$

---

(2) $y=x-\frac{16}{\sqrt{x}}$

$=x-16x^{-1/2}$

when $x=4$, $y=4-\frac{16}{2}$

$=-4$

$(4,-4)$

$\frac{dy}{dx}=1-8x^{-1/2}$

$=1-\frac{8}{\sqrt{x}}$

when $x=4$, $m=1-\frac{8}{2}$

$=-3$

$\therefore y-b=m(x-a)$

$y+4=-3(x-4)$

$y+4=-3x+12$

$3x+y=8$

---

(3) a) $u_{n+1}=1.015u_n-300$

$(u_0=2500)$

b) 1/3 $u_0=2500$

1/4 $u_1=2237.50$

1/5 $u_2=1971.06$

1/6 $u_3=1700.63$

1/7 $u_4=1426.14$

1/8 $u_5=1147.53$

1/9 $u_6=864.74$

1/10 $u_7=577.71$

1/11 $u_8=286.38$

Final payment on December 1st

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(4) $\vec{BA}=\begin{pmatrix}0\\5\\6\end{pmatrix}-\begin{pmatrix}6\\0\\7\end{pmatrix}$ $\vec{BC}=\begin{pmatrix}4\\5\\0\end{pmatrix}-\begin{pmatrix}6\\0\\7\end{pmatrix}$

$=\begin{pmatrix}-6\\5\\-1\end{pmatrix}$ $=\begin{pmatrix}-2\\5\\-7\end{pmatrix}$

$|\vec{BA}|=\sqrt{36+25+1}$ $|\vec{BC}|=\sqrt{4+25+49}$

$=\sqrt{62}$ $=\sqrt{78}$

$\vec{BA}\cdot\vec{BC}=12+25+7$

$=44$

$\therefore \cos ABC=\frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}||\vec{BC}|}$

$\cos ABC=\frac{44}{\sqrt{62}\sqrt{78}}$

$ABC=\cos^{-1}\left(\frac{44}{\sqrt{62}\sqrt{78}}\right)$

$=41.3^\circ$

---

(5)

$8\cos x^\circ+6\sin x^\circ=k\cos(x+\alpha)$

$=k\cos\alpha\cos x-k\sin\alpha\sin x$

$8 \qquad -6$

$k\sin\alpha=6$

$k\cos\alpha=8$

$k^2=6^2+8^2$

$k=10$

$\tan\alpha=\frac{6}{8}$

$\alpha=\tan^{-1}\left(\frac{6}{8}\right)$

$\alpha=36.9^\circ$

S	A
T	C

$s=+$

$c=+$

$\therefore 8\cos x^\circ+6\sin x^\circ=10\cos(x-36.9)^\circ$

---

(6) $\int\frac{(x^2-2)(x^2+2)}{x^2}dx$

$=\int\frac{x^4-4}{x^2}dx$

$=\int(x^2-4x^{-2})dx$

$=\frac{x^3}{3}-\frac{4x^{-1}}{-1}+C$

$=\frac{x^3}{3}+\frac{4}{x}+C$

---

(7) a) $x=8$

b) mid $=(5,4)$ $m_{AC}=\frac{6-2}{8-2}=\frac{4}{6}=\frac{2}{3}$

$\therefore y-4=-\frac{3}{2}(x-5)$

$2y-8=-3x+15$

$3x+2y=23$

c) $3x+2y=23$, $x=7$

$\therefore 21+2y=23$

$2y=2$ (7,1)

$y=1$

d) $r=\sqrt{(7-8)^2+(1-6)^2}$

$=\sqrt{(-1)^2+(-5)^2}$

$=\sqrt{26}$

$\therefore (x-7)^2+(y-1)^2=26$

(8) $y=(x+1)(x-1)(x-3)$

$=(x^2-1)(x-3)$

$=x^3-3x^2-x+3$

A_1 $x^3-3x^2-x+3-(5x-5)$

$=x^3-3x^2-x+3-5x+5$

$=x^3-3x^2-6x+8$

$A=\int_{-2}^{1}(x^3-3x^2-6x+8)dx$

$=\left[\frac{x^4}{4}-x^3-3x^2+8x\right]_{-2}^{1}$

$=\left(\frac{1}{4}-1-3+8\right)-\left(\frac{16}{4}+8-12-16\right)$

$=\left(\frac{17}{4}\right)-(-16)$

$=16\frac{17}{4}$

$=20\frac{1}{4}u^2$

Symmetrical $\therefore A_2=A_1$

$\therefore$ Total $=40\frac{1}{2}$ units^2

(9) $A=A_0e^{kt}$

$2=e^{1.5k}$

$1.5k=\ln 1.2$

$k=\frac{\ln 1.2}{1.5}$

$\therefore k=0.122$

(10) $\frac{dy}{dx}=3\sin 2x$

$y=\int 3\sin 2x\,dx$

$=-\frac{3}{2}\cos 2x+C$

At $\left(\frac{5\pi}{12},\sqrt{3}\right)$

$\sqrt{3}=-\frac{3}{2}\cos\left(\frac{5\pi}{6}\right)+C$

$\sqrt{3}=-\frac{3}{2}\left(-\cos\frac{\pi}{6}\right)+C$

$\sqrt{3}=-\frac{3}{2}\left(-\frac{\sqrt{3}}{2}\right)+C$

$\sqrt{3}=\frac{3\sqrt{3}}{4}+C$

$C=\sqrt{3}-\frac{3\sqrt{3}}{4}$

$C=\frac{4\sqrt{3}-3\sqrt{3}}{4}$

$C=\frac{\sqrt{3}}{4}$

$\therefore y=\frac{\sqrt{3}}{4}-\frac{3}{2}\cos 2x$

(11) a) $y=k(x+1)(x-p)$

when $x=0$,

$p=k(1)(-p)$

$p=-pk$

$k=-1$

$\therefore y=-(x+1)(x-p)$

$=-(x^2-px+x-p)$

$= -x^2 + px - x + p$

$= p + (p-1)x - x^2$

b) $-x^2 + px - x + p = x + p$

$x^2 + 2x - px = 0$

$x^2 + (2-p)x = 0$

$a = 1$ $b = 2-p$ $c = 0$

for tangency, $b^2 - 4ac = 0$

$(2-p)^2 = 0$

$p = 2$

2002 Paper I

① Centre $= (-1, 4)$ $\therefore M_r = \frac{4+3}{-1+2} = \frac{2}{3}$

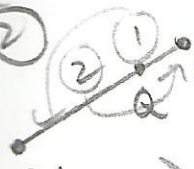
$\therefore M_T = -\frac{3}{2}$ $y - b = m(x - a)$

$y - 3 = -\frac{3}{2}(x - 2)$

$2y - 6 = -3x + 6$

$3x + 2y = 12$

②



$q = \frac{1}{3}(2\underline{r} + \underline{p})$

$= \frac{1}{3}\left[\begin{pmatrix}10\\4\\-6\end{pmatrix} + \begin{pmatrix}-1\\-1\\0\end{pmatrix}\right]$

$= \frac{1}{3}\begin{pmatrix}9\\3\\-6\end{pmatrix}$

$= \begin{pmatrix}3\\1\\-2\end{pmatrix}$

$Q = (3, 1, -2)$

③ a) (i) $f(g(x)) = \sin 2x$

$g(f(x)) = 2\sin x$

b) $2f(g(x)) = g(f(x))$

$2\sin 2x = 2\sin x$

$2\sin 2x - 2\sin x = 0$

$4\sin x \cos x - 2\sin x = 0$

$2\sin x(2\cos x - 1) = 0$ $\quad 0 \le x \le 360$

$\sin x = 0$ $\quad \cos x = \frac{1}{2}$

$x = 0°, 180°, 360°$ $\quad x = 60°, 300°$

$\therefore x = 0°, 60°, 180°, 300°, 360°.$

④ $M = \tan\theta$

$= \tan 45°$

$= 1$

$\therefore \frac{dy}{dx} = 1$

$\therefore 4x - 7 = 1$

$4x = 8$

$x = 2$

$y = 2(2)^2 - 7(2) + 10$

$= 8 - 14 + 10$

$= 4$

$(2, 4)$

⑤ $\sin\alpha = \frac{1}{\sqrt{2}}$ $\quad \sin b = \frac{1}{\sqrt{10}}$

$\cos\alpha = \frac{1}{\sqrt{2}}$ $\quad \cos b = \frac{3}{\sqrt{10}}$

$\therefore \sin(\alpha + b) = \sin\alpha\cos b + \cos\alpha\sin b$

$= \left(\frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}\right)$

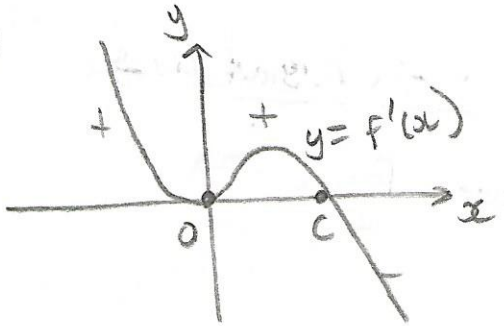
$= \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}}$

$= \frac{4}{\sqrt{20}}$

$= \frac{4}{2\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

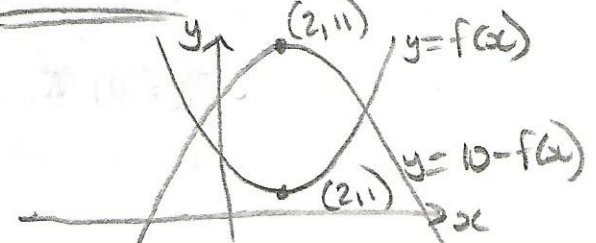
⑥



⑦ a) $x^2 - 4x + 5$

$= (x-2)^2 + 1$

b)



c) $10 - f(x)$

$= 10 - (x^2 - 6x + 5)$

$= 5 + 6x - x^2$

$= (5 - x)(1 + x)$

$\therefore$ positive when $-1 < x < 5$

(8) a) $y = 2\cos 2x$

b) $2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad 0 \le 2x \le 2\pi$

$2x = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{12}, \frac{11\pi}{12}$

$\therefore B = \left(\frac{11\pi}{12}, \sqrt{3}\right)$

(9) a)

$\sin x - \cos x = k\sin(x - \alpha)$

$= k\sin x\cos\alpha - k\cos x\sin\alpha$

$= k\cos\alpha(\sin x) - k\sin\alpha(\cos x)$

$1 \qquad\qquad -1$

$k\sin\alpha = 1$

$k\cos\alpha = 1$

$k = \sqrt{2}$

$\tan\alpha = \frac{1}{1}$

$\alpha = \frac{\pi}{4}$

S | A
T | C

$S = +$

$C = +$

$\therefore$ Q1

$\therefore \sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$

b) On y-axis: $\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$

$= \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$

$= -1 \qquad \therefore (0, -1)$

On x-axis: $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = 0$

$x - \frac{\pi}{4} = \sin^{-1}(0)$

$x - \frac{\pi}{4} = 0, \pi, 2\pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Max $= \sqrt{2}$ when $\sin\left(x - \frac{\pi}{4}\right) = 1$

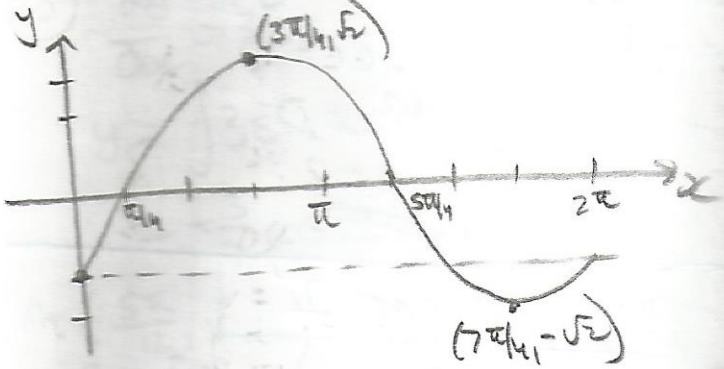
$x - \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{3\pi}{4}$

Min $= -\sqrt{2}$ when $\sin\left(x - \frac{\pi}{4}\right) = -1$

$x - \frac{\pi}{4} = \frac{3\pi}{2}$

$x = \frac{7\pi}{4}$.



(10) a) $f(x) = (8 - x^3)^{1/2}$

$f'(x) = \frac{1}{2}(8 - x^3)^{-1/2} \times (-3x^2)$

$= \frac{-3x^2}{2\sqrt{8 - x^3}}$

b) $-\frac{2}{3}(8 - x^3)^{1/2} + C$

(11) $m = \frac{1 - 0}{0 - 0.5} = -2$

$\therefore \log_5 y = -2\log_5 x + 1$

$\log_5 y = \log_5 x^{-2} + \log_5 5$

$\log_5 y = \log_5 \frac{5}{x^2}$

$y = \frac{5}{x^2}$

$\therefore k = 5, \; n = -2$

2002 Paper II

(1) a) midpoint $= (-2, 2)$

$M_p = \frac{2 - 2}{-2 - 5} = 0 \qquad \therefore y = 2$

b) $M_{BC} = \frac{2 + 2}{5 + 3} = \frac{4}{8} = -\frac{1}{2} \qquad \therefore M_{PB} = -2$

$(m_1 m_2 = -1)$