(11) $A_c = \frac{1}{2}\pi x^2 \quad \therefore \text{light} = 1 \times \frac{1}{2}\pi x^2$

$= \frac{1}{2}\pi x^2$

$A_R = 2x\left(\frac{10 - x(\pi+2)}{2}\right)$

$= 10x - x^2(\pi+2)$

$\therefore \text{light} = 20x - 2x^2(\pi+2)$

$\therefore \text{Total} = 20x - 2\pi x^2 - 4x^2 + \frac{1}{2}\pi x^2$

$= 20x - 4x^2 - \frac{3}{2}\pi x^2$

b) SP's when $\frac{dL}{dx} = 0$.

$\frac{dL}{dx} = 20 - 8x - 3\pi x$

$\therefore 20 - 8x - 3\pi x = 0$

$3\pi x + 8x = 20$

$x(3\pi + 8) = 20$

$x = \frac{20}{3\pi+8}$

$x = 1.15\text{m}$

x	1	1.15	2
$\frac{dL}{dx}$	2.6	0	−14.8
shape	/	—	\

$\therefore$ max when $x = \frac{20}{3\pi+8}$

$\therefore h = \frac{10 - x(\pi+2)}{2}$

$= \frac{10 - \frac{20}{3\pi+8}(\pi+2)}{2}$

$= 2.05\text{m}$

(10) $m = \frac{10.00 - 7.00}{3.10 - 2.10} = \frac{3}{1} = 3$

$Y - b = m(X - a)$

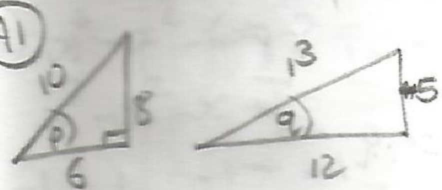
$Y - 10 = 3(X - 3.1)$

$Y - 10 = 3X - 9.3$

$Y = 3X + 0.7$

2000 Paper I

(A1)



$\sin(p+q) = \sin p \cos q + \cos p \sin q$

$= \left(\frac{8}{10} \times \frac{12}{13}\right) + \left(\frac{6}{10} \times \frac{5}{13}\right)$

$= \frac{96}{130} + \frac{30}{130}$

$= \frac{126}{130}$

$= \frac{63}{65}$

(A2) $f'(x) = 3x^2 - 12x + 9$

a) SP's @ $f'(x) = 0 \quad \therefore 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

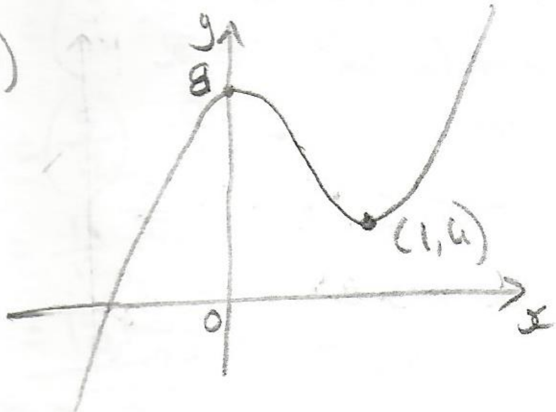
$3(x-3)(x-1) = 0$

$x = 3, x = 1$

when $x = 1$, $y = 1 - 6 + 9$
$= 4$

$\therefore A = (1, 4)$.

b)



c) $4 < k < 8$

(A3) $m = \frac{2+1}{3\sqrt{3} - 0} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

$m = \tan\theta$

$\therefore \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

ie $\theta = 30^\circ$

A4) a)

(i) $3x^2 - 12 = 10x - 15$

$3x^2 - 10x + 3 = 0$

$(3x-1)(x-3) = 0$

$x = \frac{1}{3},\ x = 3$

(ii) If $x = 3$, $y = 27 - 36 + 1$

$= \underline{\underline{-8}}$

$y = 45 - 45 - 8$

$= \underline{\underline{-8}}$

If $x = \frac{1}{3}$, $y = \frac{1}{27} - 4 + 1$

$= \underline{-2\frac{26}{27}}$

$y = \frac{5}{9} - 5 - 8$

$= \underline{-12\frac{4}{9}}$

$\therefore$ At $x=3$, common tangent

At $x = \frac{1}{3}$, parallel lines.

b) $x^3 - 12x + 1 - (5x^2 - 15x - 8)$

$= x^3 - 5x^2 + 3x + 9$

$\therefore A = \int_{-1}^{3} (x^3 - 5x^2 + 3x + 9)\,dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 9x\right]_{-1}^{3}$

$= \left(\frac{81}{4} - \frac{135}{3} + \frac{27}{2} + 27\right) - \left(\frac{1}{4} + \frac{5}{3} + \frac{3}{2} - 9\right)$

$= \left(\frac{243}{12} - \frac{540}{12} + \frac{162}{12} + \frac{324}{12}\right) - \left(\frac{3}{12} + \frac{20}{12} + \frac{18}{12} - \frac{108}{12}\right)$

$= \frac{189}{12} - \left(-\frac{67}{12}\right)$

$= \frac{256}{12}$

$= 21\frac{4}{12}$

$= 21\frac{1}{3}$ units^2

A5) $L = aL + 10$ $\qquad$ $L = a^2L + 16$

$L(1-a) = 10$ $\qquad$ $L(1-a^2) = 16$

$L = \frac{10}{1-a}$ $\qquad$ $L = \frac{16}{1-a^2}$

$\therefore \frac{10}{1-a} = \frac{16}{1-a^2}$

$10 - 10a^2 = 16 - 16a$

$10a^2 - 16a + 6 = 0$

$2(5a^2 - 8a + 3) = 0$ $\qquad$ $\frac{15}{\begin{matrix}1 & 15\\ 3 & 5\end{matrix}}$

$2(5a-3)(a-1) = 0$

$a = \frac{3}{5},\ a = -1$

$\therefore a = \frac{3}{5}$, as $-1 < \frac{3}{5} < 1$

$L = \frac{3}{5}L + 10$

$\frac{2}{5}L = 10$

$\underline{\underline{L = 25}}$

A6) $(2k)^2 + (-k)^2 - (-k-2) > 0$

$4k^2 + k^2 + k + 2 > 0$

$5k^2 + k + 2 > 0$

$5k^2 + k + 2 = 5\left[k^2 + \frac{1}{5}k\right] + 2$

$= 5\left[\left(k + \frac{1}{10}\right)^2 - \frac{1}{100}\right] + 2$

$= 5\left(k + \frac{1}{10}\right)^2 - \frac{1}{20} + 2$

$= \underline{5\left(k + \frac{1}{10}\right)^2 + \frac{39}{20}}$

$\therefore 5k^2 + k + 2$ always > 0

$\therefore$ circle for all values of k.

B7) $\vec{VE} = \vec{VA} + \vec{AB} + \frac{1}{4}\vec{AD}$

$= \begin{pmatrix}-7\\-13\\-11\end{pmatrix} + \begin{pmatrix}6\\6\\-6\end{pmatrix} + \frac{1}{4}\begin{pmatrix}8\\-4\\4\end{pmatrix}$

$= \begin{pmatrix}-1\\-7\\-17\end{pmatrix} + \begin{pmatrix}2\\-1\\1\end{pmatrix} = \underline{\begin{pmatrix}1\\-6\\-16\end{pmatrix}}$

B8 $f'(x) = \sin 3x$

$f(x) = \int \sin 3x \, dx$

$= -\frac{1}{3}\cos 3x + C$

$\therefore -\frac{1}{3}\cos 3\left(\frac{\pi}{9}\right) + C = 1$

$-\frac{1}{3}\cos\frac{\pi}{3} + C = 1$

$-\frac{1}{3}\left(\frac{1}{2}\right) + C = 1$

$C - \frac{1}{6} = 1$

$C = \frac{7}{6}$

$\therefore y = \frac{7}{6} - \cos 3x$

B9 $\log_5 2 + \log_5 50 - \log_5 4$

$= \log_5 100 - \log_5 4$

$= \log_5 25$

$= 2$

B10 $\boxed{\cos x} - \sin x = k\cos(x - \alpha)$

$= k\cos\alpha \boxed{\cos x} + k\sin\alpha \sin x$

$k\sin\alpha = -1$

$k\cos\alpha = 1$

$k^2 = (-1)^2 + 1^2$

$k = \sqrt{2}$

$\tan\alpha = \frac{-1}{1}$

$\alpha = \tan^{-1}(-1)$

$\alpha = \frac{\pi}{4}$

S	A
T	C

$S = -$

$C = +$

$\therefore \cos x - \sin x = \sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)$

Max $= \sqrt{2}$ when $\cos\left(x - \frac{7\pi}{4}\right) = 1$

$x - \frac{7\pi}{4} = 0, 2\pi$

$x = \frac{7\pi}{4}, \frac{15\pi}{4}$

$\therefore$ Max $= \sqrt{2}$ when $x = \frac{7\pi}{4}$

2000 Paper II

A1

a) $\frac{dy}{dx} = 3x^2 - 6x + 2$

$\therefore m = 3 - 6 + 2$

$= -1$

when $x = 1$, $y = 1 - 3 + 2$

$= 0$

$\therefore m = -1, \ (1, 0)$

$y = -(x - 1)$

$y = 1 - x$

b) $x^3 - 3x^2 + 2x = 2x - 4$

$x^3 - 3x^2 + 4 = 0$

2	1	-3	0	4
		2	-2	-4
	1	-1	-2	0

$(x-2)(x^2 - x - 2) = 0$

$(x-2)^2(x+1) = 0$

$x = 2, \ x = -1$

when $x = -1$, $y = -1 - 3 - 2$

$= -6$

$(-1, -6)$

A2

a) $M_{PQ} = \frac{9-1}{1+3} = \frac{8}{4} = 2$

$\therefore M_{AB} = -\frac{1}{2}$ $(M_1 M_2 = -1)$

Midpoint $= (-1, 5)$

$\therefore y - 5 = -(x + 1)$

$y - 5 = -x - 1$

$x + y = 4$

) $QC: x = 1$

$x+y=4$

$1+y=4$

$\underline{y=3}$ $\quad \therefore C=(1,3)$

$Q=(1,9) \therefore r=6$

$(x-1)^2+(y-3)^2=36$

) (i) $y=9$

(ii) $x+9=4$

$\underline{x=-5}$ $\quad \underline{(-5,9)}$

3) a) $p(x)=f(g(x))$

$=f\left(\frac{3}{x}\right)$

$=3-\frac{3}{x}$

$=\underline{\underline{\frac{3x-3}{x}}}$

b) $p(q(x)) = \dfrac{3\left(\frac{3}{3-x}\right)-3}{\frac{3}{3-x}}$

$=\dfrac{\frac{9}{3-x}-\frac{9-3x}{3-3x}}{\frac{3}{3-x}}$

$=\dfrac{\frac{9-9-3x}{3-3x}}{\frac{3}{3-3x}}$

$=\dfrac{\frac{3x}{3-3x}}{\frac{3}{3-3x}}$

$=\frac{3x}{3-3x}\times\frac{3-3x}{3}$

$=\frac{3x}{3}$

$=\underline{\underline{x}}$

) a) $y=kx(x-4)$

$4=2k(-2)$

$4=-4k$

$k=-1$ $\quad \therefore \underline{y=x(4-x)}$

b) $A=\int_2^k (4x-x^2)\,dx$

$=\left[2x^2-\frac{x^3}{3}\right]_2^k$

$=\left(2k^2-\frac{k^3}{3}\right)-\left(8-\frac{8}{3}\right)$

$=2k^2-\frac{k^3}{3}-\left(\frac{24}{3}-\frac{8}{3}\right)$

$=\underline{\underline{2k^2-\frac{k^3}{3}-\frac{16}{3}}}$

A5) $3\cos 2x^\circ + \cos x^\circ = -1$

$3(2\cos^2 x^\circ - 1) + \cos x^\circ + 1 = 0$

$6\cos^2 x^\circ + \cos x^\circ - 2 = 0$

$(3\cos x^\circ + 2)(2\cos x^\circ - 1) = 0$

$\cos x^\circ = -\frac{2}{3}$ $\quad \cos x^\circ = \frac{1}{2}$

$x = 131.8^\circ, 228.1^\circ$

$x = 60^\circ, 300^\circ$.

$\therefore \underline{x = 60^\circ, 131.8^\circ, 228.1^\circ, 300^\circ}$

$\frac{12}{1\ 12}$
$2\ 6$
$3\ 4$

$6c^2+c-2=0$

$6c^2+4c-3c-2=0$

$2c(3c+2)-(3c+2)=0$

$(3c+2)(2c-1)=0$

A6) $A(x)=\frac{3\sqrt{3}}{2}\left(x^2+\frac{16}{x}\right)$

$=\frac{3\sqrt{3}x^2}{2}+\frac{48\sqrt{3}}{2x}$

$=\underline{\frac{3\sqrt{3}x^2}{2}}+24\sqrt{3}x^{-1}$

$A'(x)=3\sqrt{3}x-\frac{24\sqrt{3}}{x^2}$

sp's @ $A'(x)=0$

$\therefore 3\sqrt{3}x-\frac{24\sqrt{3}}{x^2}=0$

$3\sqrt{3}x=\frac{24\sqrt{3}}{x^2}$

$3\sqrt{3}x^3=24\sqrt{3}$

$x^3=\frac{24\sqrt{3}}{3\sqrt{3}}$

$x^3=8$

$\underline{\underline{x=2}}$

x	1	2	3
$A'(x)$	$-21\sqrt{3}$	0	$\sqrt{3}$
shape	$\searrow$	$-$	$\nearrow$

$\therefore$ minimum when $\underline{x=2}$

(B7) Perpendicular when $u.v=0$

$\therefore 2t-20+3t=0$

$5t=20$

$\underline{t=4}$

(B8) $f(x)=(5x-4)^{1/2}$

$f'(x)=\frac{1}{2}(5x-4)^{-1/2}\times 5$

$=\dfrac{5}{2\sqrt{5x-4}}$

$f'(4)=\dfrac{5}{2\sqrt{16}}$

$=\underline{\dfrac{5}{8}}$

(B9) a) $B=(3,2,15)$

b) $\overrightarrow{BA}=\begin{pmatrix}0\\9\\8\end{pmatrix}-\begin{pmatrix}3\\2\\15\end{pmatrix}$ $\quad\overrightarrow{BC}=\begin{pmatrix}17\\0\\8\end{pmatrix}-\begin{pmatrix}3\\2\\15\end{pmatrix}$

$=\begin{pmatrix}-3\\7\\-7\end{pmatrix}$ $\quad=\begin{pmatrix}14\\-2\\-7\end{pmatrix}$

$|\overrightarrow{BA}|=\sqrt{9+49+49}$ $\quad|\overrightarrow{BC}|=\sqrt{196+4+49}$

$=\sqrt{107}$ $\quad=\sqrt{249}$

$\overrightarrow{BA}.\overrightarrow{BC}=-42-14+49$

$=\underline{-7}$

$\therefore \cos ABC=\dfrac{-7}{\sqrt{107}\sqrt{249}}$

$ABC=\cos^{-1}\left(\dfrac{-7}{\sqrt{107}\sqrt{249}}\right)$

$=\underline{92.5^\circ}$

(B10) $\displaystyle\int\frac{1}{(7-3x)^2}\,dx$

$=\displaystyle\int(7-3x)^{-2}\,dx$

$=\dfrac{(7-3x)^{-1}}{-1\times-3}+C$

$=\dfrac{1}{3(7-3x)}+C$

(B11) a) $m=\frac{1.8}{3}=0.6$

$P=0.6Q+1.8$

b) $\log_e P=0.6\log_e q+1.8$

$\log_e P=\log_e q^{0.6}+\log_e(e^{1.8})$

$\log_e P=\log_e q^{0.6}+\log_e 6.05$

$\log_e P=\log_e(6.05q^{0.6})$

$P=6.05q^{0.6}$

$\therefore \underline{a=6.05,\ q=0.6}$