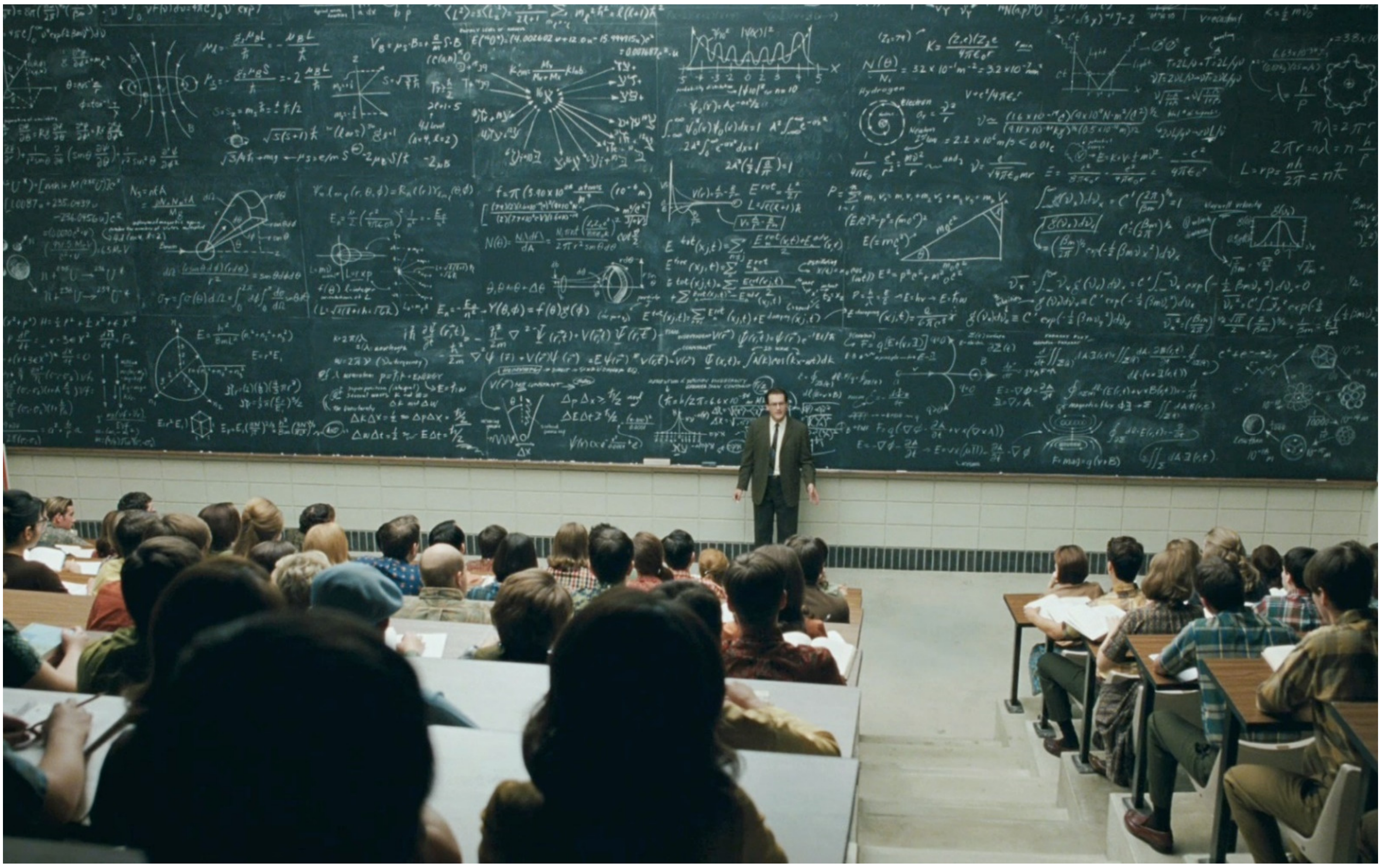
Coatbridge High School
Maths Department



Higher Maths
Formal Homework Exercises

FORMULAE LIST (As provided in the exam booklet)

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre ($-g$, $-f$) and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a , b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between the $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$, where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

OTHER USEFUL FORMULAE (NOT provided in the exam booklet: these MUST BE MEMORISED!)

Trig Identities: $\tan x = \frac{\sin x}{\cos x}$ $\quad \sin^2 x + \cos^2 x = 1 \rightarrow$ $\sin^2 x = 1 - \cos^2 x$, $\cos^2 x = 1 - \sin^2 x$

Distance Formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Section Formula: $\underline{p} = \frac{1}{n+m}(n\underline{a} + m\underline{b})$ where P divides AB in the ratio m:n

Laws of Logs: $\log_a x + \log_a y = \log_a xy$ $\quad \log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $\quad n\log_a x = \log_a x^n$

Exercise 1: Straight Line 1

MARKS

1. Find the gradient of the line perpendicular to the line $2y - 5x + 6 = 0$. **(2)**

2. State the points where the line with equation $3x - 2y + 18 = 0$ intersects with the x- and y-axes. **(2)**

3. Find the angle made with the positive direction of the x-axis and the line $2y = 3x - 4$. Answer accurate to 1 d.p. **(2)**

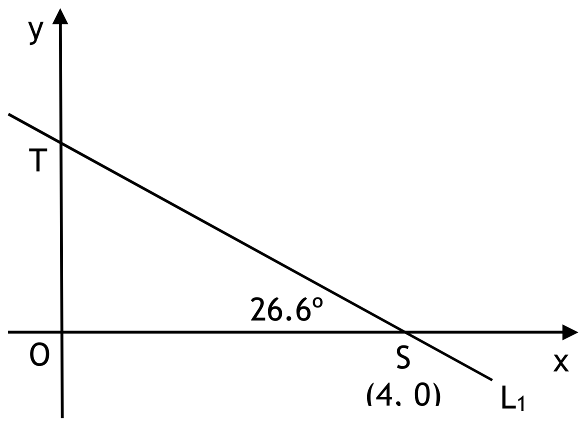
4. Points B and C have coordinates $(2, -3)$ and $(4, 3)$ respectively.

   Line L_1 passes through B and C.

   Find the equation of the line perpendicular to L_1 which passes through the point $(-1, 2)$. **(3)**

5. In the diagram below, line L_1 passes through the points S and T.

   S has coordinates $(4, 0)$ and angle TSO = 26.6°.



   a) Show that L_1 has the equation $x + 2y - 4 = 0$. **(3)**

   b) A second line L_2 passes through point T and is perpendicular to L_1.

   Find the coordinates of U, the point of intersection of line L_2 and the x-axis. **(4)**

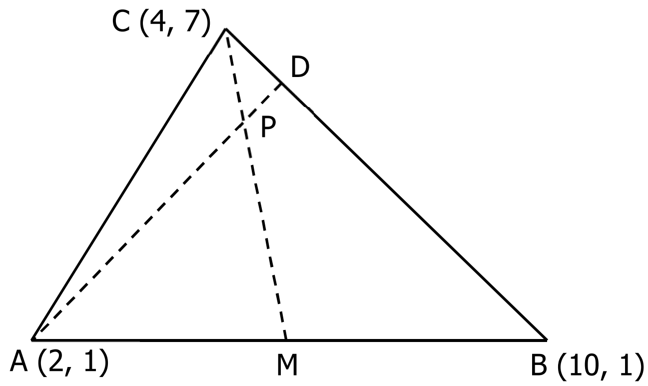
TOTAL = 16 MARKS

Exercise 2: Straight Line 2

MARKS

1. Find the equation of the perpendicular bisector of the line joining the points P (4, -1) and Q (-2, 7). **(4)**

2. Triangle ABC is shown below.



a) Find the equation of median CM. **(3)**

b) Find the equation of altitude AD. **(3)**

c) Find the coordinates of P, the point of intersection of CM and AD. **(2)**

3. Triangle KLM has vertices K (3, 7), L (-4, 5) and M (-2, -2).

Prove that triangle KLM is isosceles. **(4)**

TOTAL = 16 MARKS

Exercise 3: Sets & Functions

MARKS

1. State suitable domains for:

   a) $f(x) = \dfrac{3}{1-x}$ b) $g(x) = \sqrt{2x+5}$ c) $h(x) = \dfrac{1}{x^2 - 3x + 2}$ **(4)**

2. $p(x) = x^2 + 1$, $q(x) = 3x - 1$.

   Evaluate $p(q(-1))$. **(2)**

3. $f(x) = 2x$, $g(x) = x^3 - 1$.

   a) Find an expression for $h(x)$, where $h(x) = g(f(x))$. **(2)**

   b) Find an expression for $g^{-1}(x)$ **(3)**

4. Two functions are defined on the set of real numbers by

   $$f(x) = x^2 - 2, \qquad g(x) = 2x + 1$$

   a) Show that $f(g(x)) = 4x^2 + 4x - 1$. **(2)**

   b) Show that the equation

   $$f(g(x)) = g(f(x))$$

   has real, equal roots. **(5)**

TOTAL = 18 MARKS

Exercise 4: Graphs of Functions

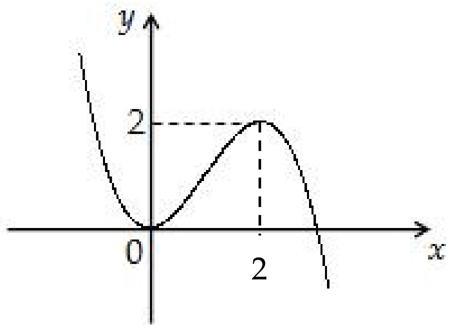
MARKS

1. On separate diagrams, sketch and annotate the graphs of:

   a) $y = 2x^2 + x - 6$ (annotate roots and y-intercept) **(3)**

   b) $y = 5 - (x + 3)^2$ (annotate TP and y-intercept) **(3)**

2. The diagram opposite shows part of the graph of the function $y = f(x)$.



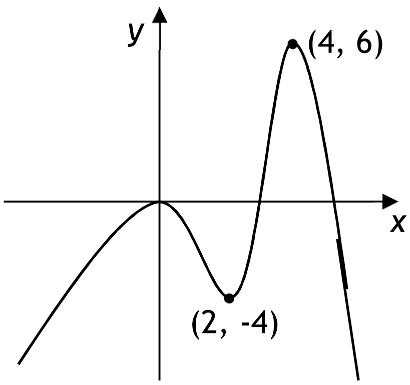
   On separate diagrams, show:

   a) $y = 2\,f(x)$ **(2)**

   b) $y = f(x) + 2$ **(2)**

   c) $y = f(2x)$ **(2)**

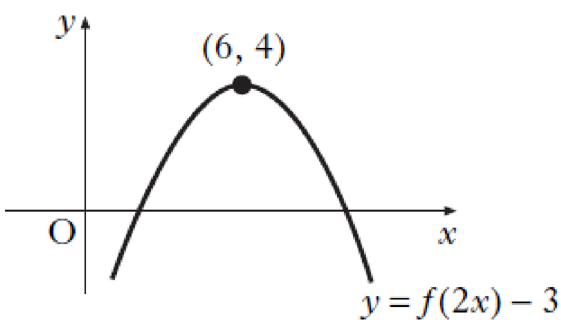
3.



The diagram opposite shows part of the graph of $y = g(x)$.

Sketch and annotate the graph of $y = 4 - g(\frac{1}{2}x)$ **(4)**

4. The diagram shows part of the graph of $y = f(2x) - 3$.



What are the coordinates of the turning point of $y = f(x)$? **(2)**

TOTAL = 18 MARKS

Exercise 5: Radians and Trig Graphs

MARKS

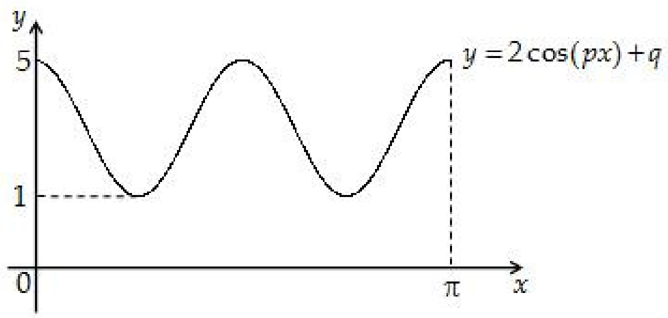
1. a) Convert to radians: (i) 120° (ii) 315° (iii) 75°

   b) Convert to degrees: (i) $\frac{5\pi}{6}$ (ii) $\frac{5\pi}{4}$ (iii) 4.38 **(6)**

2. State without a calculator the exact value of:

   a) $\sin 210^\circ$ b) $\cos \frac{11\pi}{6}$ **(4)**

3. The graph shown below has equation of the form $y = 2\cos(px) + q$.



   State the values of p and q. **(2)**

4. Sketch the graph of $y = 4 \sin 2x + 1$, where $0 \leq x \leq 2\pi$.

   Show clearly the heights of the maximum & minimum turning points and the point of intersection with the y-axis. **(3)**

TOTAL = 15 MARKS

Exercise 6: Recurrence Relations 1

MARKS

1. A sequence is defined by the recurrence relation

$$U_{n+1} = 1.2U_n + 4, \qquad U_0 = 20$$

What is the value of U_4? **(2)**

2. A sequence is defined by the recurrence relation

$$U_n = 3U_{n-1} + 12, \qquad U_0 = 6$$

Find the smallest value of n such that $U_n > 2500$. **(2)**

3. A sequence is defined by the recurrence relation

$$U_{n+1} = aU_n + b, \qquad U_1 = 4$$

Find U_3 in terms of a and b. **(2)**

4. Paul borrows £4500 on his credit card, which has a monthly interest rate of 1.2%.

He cuts up the card so he can't borrow on it again and pays £300 per month towards the balance.

a) Find, to the nearest penny, his balance after six months. **(4)**

b) Paul guesses that it should take him 15 months to pay off the card (since 15 x £300 = £4500).

Is he correct? Justify your answer. **(3)**

5. A sequence is defined by the recurrence relation

$$U_{n+1} = 14 - 2U_n$$

Find the value of U_0, given that $U_2 = 10$. **(3)**

TOTAL = 16 MARKS

Exercise 7: Recurrence Relations 2

MARKS

1. A sequence is defined by the recurrence relation

$$U_{n+1} = 0.8U_n + 4, \qquad U_0 = 10$$

a) Explain why the sequence tends to a limit as $n \to \infty$. **(1)**

b) Determine the value of the limit. **(2)**

2. On the first day of every month, 20% of the stock of a fish farm is sold to the local market. The farm is then restocked with 400 new fish.

The owners of the farm want to keep stock levels above 1000 fish at all times. Is this possible? Justify your answer. **(4)**

3. In the sequence generated by the recurrence relation $U_n = a\,U_{n-1} - b$,

$$U_1 = 5,\ U_2 = 10 \text{ and } U_3 = 30.$$

Find the values of a and b. **(4)**

4. A doctor prescribes a course of antibiotics to a hospital patient.

After an initial dose of 150mg, the patient receives an additional 100mg of the drug every hour.

The concentration of the drug in the bloodstream is known to drop by 35% every hour.

To be effective, the concentration of drug in the blood must reach a minimum of 250mg within 4 hours. However, the drug becomes toxic if its concentration ever reaches more than 300mg.

Is this an effective course of treatment? Give **two** reasons for your answer. **(5)**

TOTAL = 16 MARKS

Exercise 8: Differentiation 1

MARKS

1. Find f'(x) where:

   a) $f(x) = 4x^3 - 12x^{-2}$ b) $f(x) = 8x^2 - 2\sqrt{x}$

   c) $f(x) = \dfrac{4}{x^2} + \dfrac{x^3}{3}$ d) $f(x) = (2x - 3)(x + 4)$ **(7)**

2. A function is defined on the set of real numbers by

   $g(x) = 2x^3 - 4x^2 + x$

   Find the rate of change of the function when $x = -3$. **(3)**

3. A function is defined on the set of real numbers by

   $f(x) = x^3 - 2x^2 + 5x - 9$

   Find the values of x such that $f'(x) = 7$. **(4)**

4. Find the equation of the tangent to the curve

   $$y = 3x^3 + 4x^2 + 2x - 6$$

   at the point where $x = -1$. **(4)**

TOTAL = 18 MARKS

Exercise 9: Differentiation 2

MARKS

1. Find f'(x) where $f(x) = \dfrac{x^2 - x + 2}{\sqrt{x}}$, giving your answer with positive indices. **(4)**

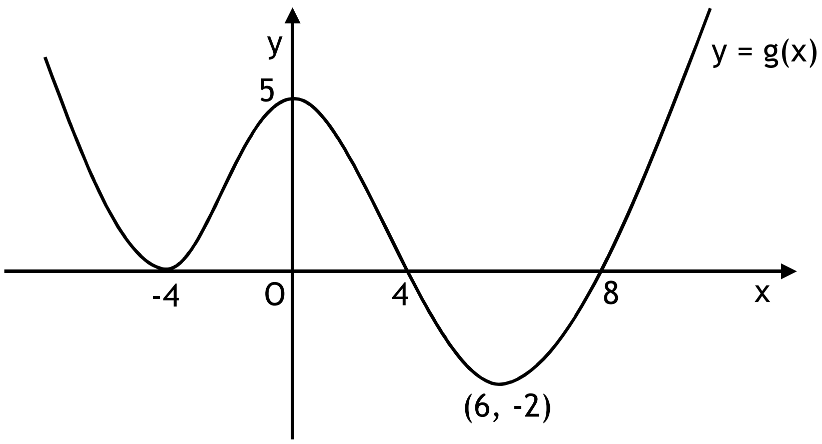
2. Determine the position and nature of the stationary points of the function $f(x) = 2x^3 - 24x$. **(7)**

3. Show that the function

   $f(x) = 3x^3 - 6x^2 + 4x$

   is never decreasing for all real values of x. **(4)**

4. Part of the graph of y = g(x) is shown below.



Sketch and annotate the graphs of:

a) $y = g'(x)$ **(2)**

b) $y = -g'(2x)$ **(3)**

TOTAL = 20 MARKS

Exercise 10: Integration

MARKS

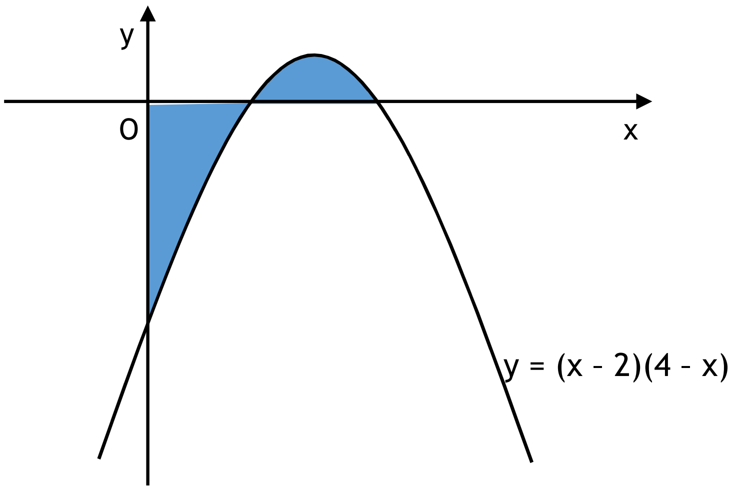
1. Find:

a) $\int x^8 dx$ b) $\int \frac{dg}{g^{1/5}}$ c) $\int \left(\frac{3p^4 - 2p^3 + 5}{p^2} \right) dp$ **(6)**

2. Evaluate:

a) $\int_{-2}^{1} 16v^2 \, dv$ b) $\int_{-1}^{3} (3x - 1)^2 dx$ **(7)**

3. The diagram below shows part of the graph of y = (x – 2)(4 – x).



Calculate the shaded area. **(8)**

TOTAL = 21 MARKS

Exercise 11: Quadratic Functions 1

MARKS

1. a) Express $x^2 - 6x + 10$ in the form $(x - p)^2 + q$. **(2)**

   b) Express $3x^2 - 6x + 10$ in the form $a(x - b)^2 + c$. **(3)**

2. State the nature of the roots of the equation $5 + 3x - 2x^2 = 0$. **(3)**

3. A parabola with equation $y = kx(x - 4)$ passes through the point $(-1, 10)$.

   What is the value of k? **(2)**

4. a) Express $x^2 - 6x + 15$ in the form $(x + p)^2 + q$. **(2)**

   b) Hence state the maximum value of the function $f(x) = \dfrac{3}{x^2 - 6x + 15}$,
   and state the value of x at which it occurs. **(4)**

TOTAL = 16 MARKS

Exercise 12: Quadratic Functions 2

MARKS

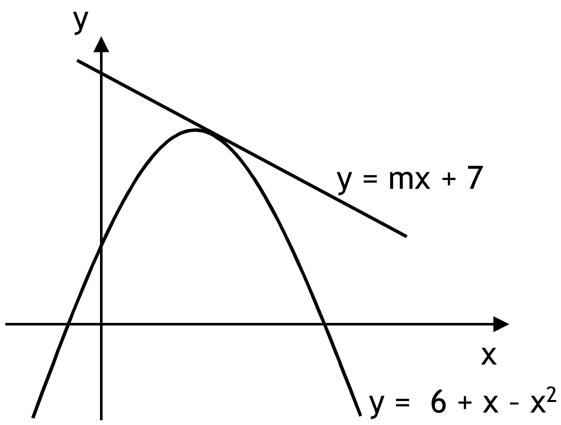
1. Solve $x^2 - 8x + 15 > 0$ **(3)**

2. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

   What is the value of k? **(3)**

3. Show that the line $y = 6x - 15$ is a tangent to the curve $y = x^2 + 2x - 11$. **(4)**

4. The line $y = mx + 7$ is a tangent to the parabola with equation $y = 6 + x - x^2$ as shown below.



   Find the value of m. **(5)**

5. Show that the roots of the equation

$$4x^2 + 2tx + t - 1 = 0$$

   are always real, for all values of t. **(5)**

TOTAL = 18 MARKS

Exercise 13: Polynomials

MARKS

1. $f(x) = 2x^3 + x^2 - 5x + 1$. State the value of $f(-2)$. **(2)**

2. State both the quotient and the remainder when $3x^3 + 2x^2 - 7x + 5$ is divided by $(x - 2)$. **(3)**

3. a) Show that $(x - 1)$ is a factor of $6x^3 + 5x^2 - 21x + 10$. **(3)**

   b) Hence factorise $6x^3 + 5x^2 - 21x + 10$ fully. **(2)**

4. a) (i) Show that $(x + 3)$ is a factor of $x^3 + 4x^2 - 3x - 18$ **(3)**

   (ii) Hence factorise $x^3 + 4x^2 - 3x - 18$ fully **(2)**

   b) Find the position and nature of the stationary points of the curve $y = x^3 + 4x^2 - 3x - 18$. **(7)**

   c) Sketch and annotate the curve $y = x^3 + 4x^2 - 3x - 18$. **(2)**

TOTAL = 24 MARKS

Exercise 14: The Circle 1

MARKS

1. State the centre and radius of each circle described below:

   a) $(x - 3)^2 + (y + 2)^2 = 64$ **(2)**

   b) $x^2 + y^2 - 6x + 10y - 8 = 0$ **(2)**

2. Explain why each equation below does NOT describe a circle:

   a) $x^2 - y^2 = 32$ **(1)**

   b) $x^2 + y^2 + 4x - 10y + 35 = 0$ **(2)**

3. A is the point (4, 8) and B is the point (-2, 14)

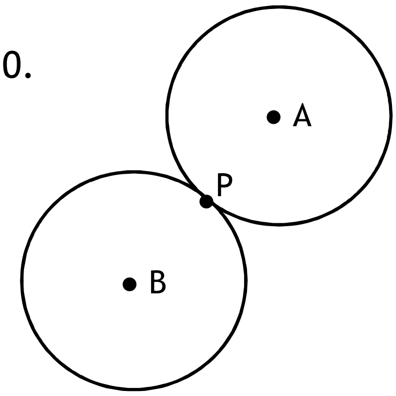
   State the equation of the circle which has line AB as its diameter. **(3)**

4. Circle A has equation $x^2 + y^2 - 6x - 12y + 20 = 0$.

   Circle B is congruent to Circle A.

   The circles touch at point P (0, 2).

   Determine the equation of Circle B. **(5)**



5. Find the coordinates of the points of intersection of the line $y = 4 - x$ and the circle $x^2 + y^2 + 8x + 4y - 38 = 0$ **(5)**

TOTAL = 20 MARKS

Exercise 15: The Circle 2

MARKS

1. Determine the equation of the tangent to the circle

   $(x - 1)^2 + (y - 2)^2 = 13$

   at the point (3, -1). **(4)**

2. A, B and C are the points (3, 5), (-6, 2) and (8, -10) respectively.

   a) Show that triangle ABC is right-angled at A. **(3)**

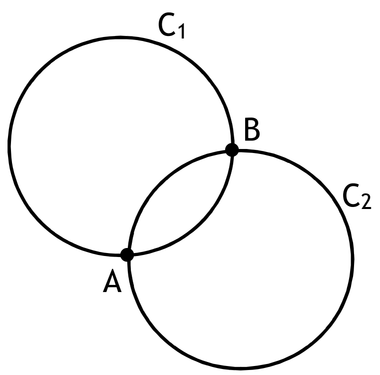
   b) Hence find the equation of the circle which passes through points A, B and C. **(3)**

3. Prove that the line $y = 7 - x$ is a tangent to the circle with equation

   $x^2 + y^2 + 2x - 4y - 13 = 0$.

   and state the coordinates of the point of contact. **(6)**

4. Two congruent circles C_1 and C_2 intersect at the points A and B as shown.



A is the point (2, -2) and B is the point (6, 6).

Find the equation of C_1 given that C_2 has the equation

$x^2 + y^2 - 20x + 2y + 36 = 0$. **(6)**

TOTAL = 22 MARKS

Exercise 16: Trig Formulae

MARKS

1. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{2}{\sqrt{5}}$.

   Find the exact values of:

   a) $\sin(A + B)$ **(2)**

   b) $\cos(B - A)$ **(2)**

2. Show that the exact value of $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$.

   (HINT: $15^\circ = 60^\circ - 45^\circ$) **(3)**

3. $\sin P = \frac{4}{5}$ and $\sin Q = \frac{2}{\sqrt{5}}$, where $0 \le P \le \frac{\pi}{2}$ and $0 \le Q \le \frac{\pi}{2}$.

   State the exact values of:

   a) $\sin(P - Q)$ **(3)**

   b) $\tan(P - Q)$ **(4)**

4. A is an acute angle such that $\tan A = \frac{2}{\sqrt{5}}$.

   Find the exact values of:

   a) $\sin 2A$ **(2)**

   b) $\cos 2A$ **(2)**

TOTAL = 18 MARKS

Exercise 17: Trig Equations

MARKS

1. Solve WITHOUT A CALCULATOR:

a) $\sin 2x = \frac{\sqrt{3}}{2}$ $\quad 0 \le x \le 2\pi$ **(3)**

b) $\sqrt{3}\tan 3x + 1 = 0$ $\quad 0 \le x \le 180^\circ$ **(4)**

2. Solve accurate to one d.p.:

a) $9\sin^2 x = 1$ $\quad 0 \le x \le 270^\circ$ **(3)**

b) $3\cos^2 x + 5\cos x - 2 = 0$ $\quad 0 \le x \le 360^\circ$ **(4)**

TOTAL = 14 MARKS

Exercise 18: Double Angle Equations and Trig Identities

MARKS

1. Solve without a calculator:

   a) $\sin 2x - \cos x = 0$ $\quad 0 < x < 360°$ **(4)**

   b) $\cos 2x - \cos x = 2$ $\quad 0 < x < 2\pi$ **(5)**

2. Find the x-coordinates of the points of intersection of the curves

   $y = 2\sin 2\theta$ and $y = 3\cos\theta$ where $0 < \theta < 180°$.

   Give your answers accurate to 2 s.f. where appropriate. **(4)**

3. Show that $\tan X - \tan Y = \dfrac{\sin(X - Y)}{\cos X \cos Y}$ **(3)**

4. Prove that $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$ **(4)**

TOTAL = 20 MARKS

Exercise 19: Vectors 1

MARKS

1. Given $\mathbf{p} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{q} = 3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$, calculate $|3\mathbf{p} - 2\mathbf{q}|$. **(2)**

2. P, Q and R are the points with coordinates (4, -4, 3), (5, -6, 5) and (8, -12, 11) respectively.

   Show that P, Q and R are collinear and find the ratio in which Q divides PR. **(3)**

3. K, L and M are the points (3, 2, 1) (-4, 8, -5) and (-1, 5, 0) respectively.

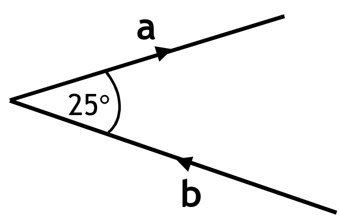
   Find the coordinates of N such that KLMN is a parallelogram. **(2)**

4. A is (4, -3, 2) and B is (-6, 2, 7).

   Find the coordinates of P given that it divides AB in the ratio 3:2. **(3)**

5. Find **a.b** in each case:

   a) $\mathbf{a} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$

   b) 

   $|\mathbf{a}| = 4$, $|\mathbf{b}| = 6$ **(5)**

6. K is the point (3, 5, 1), L is the point (4, 8, 2) & M is the point (7, 7, g).

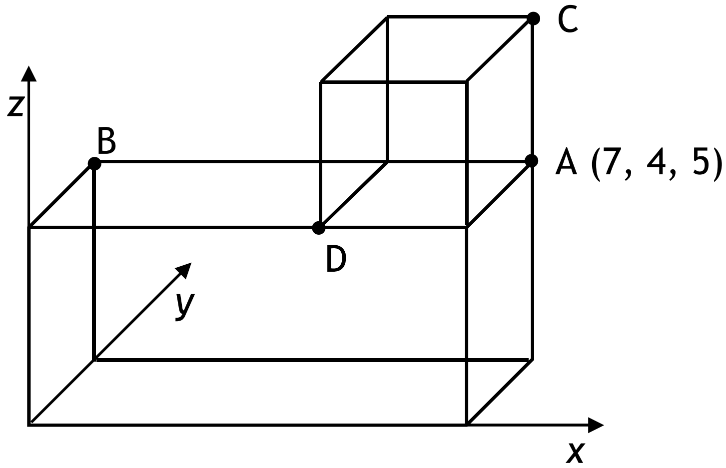
   Find the value of g such that triangle KLM is right-angled at L. **(3)**

TOTAL = 18 MARKS

Exercise 20: Vectors 2

MARKS

1. The diagram shows a cube placed on top of a cuboid, relative to suitable axes.

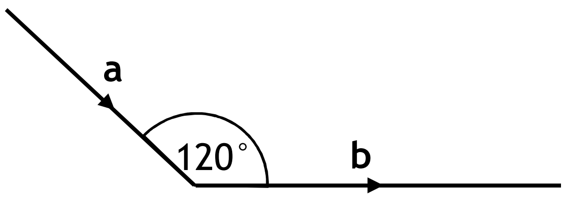


A is the point (7, 4, 5) as shown.

a) Write down the coordinates of: (i) B (ii) C (iii) D **(3)**

b) Calculate the size of angle BCD. **(5)**

2.



$|\mathbf{a}| = 6$.

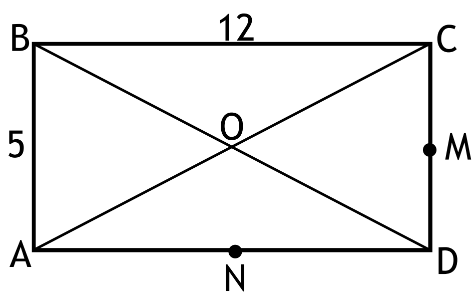
$\mathbf{a}.(\mathbf{a} + \mathbf{b}) = 60$.

Find $|\mathbf{b}|$ **(6)**

3. ABCD is a rectangle whose diagonals bisect at O.

M is the midpoint of BC, N is the midpoint of AB.

AD = 5 units, DC = 12 units.



a) Determine the exact value of cos DAC. **(2)**

b) Evaluate $\overrightarrow{AD}.\overrightarrow{NM}$ **(4)**

TOTAL = 20 MARKS

Exercise 21: Wave Function

MARKS

1. $8 \sin x^\circ + 15 \cos x^\circ$ is written in the form $k \cos (x - \alpha)^\circ$

   State the values of k and α, where $0 < \alpha < 360^\circ$. **(4)**

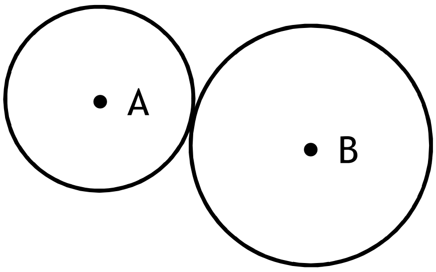
2. a) Express $3 \sin x^\circ - 2 \cos x^\circ$ in the form $k \cos (x - \alpha)^\circ$, where $0 < \alpha < 360^\circ$. **(4)**

   b) Hence state the coordinates of the maximum and minumum turning points of $y = 3 \sin x^\circ - 2 \cos x^\circ$ where $0 < x < 360^\circ$. **(3)**

3. Two circular cogs on a piece of machinery revolve in opposite directions.

   The height of a point on wheel A is given by $y = 4 \sin x + 6$, where x is the number of seconds since the cog started its rotation.

   The height of a point on wheel B is given by $y = 5 \cos x + 4$.



   a) Show that, when the two points are at the same height,

$$4 \sin x - 5 \cos x = -2$$ **(1)**

   b) (i) Write $4 \sin x - 5 \cos x$ in the form $k \sin (\theta + \alpha)$, where $0 < \alpha < 2\pi$ **(4)**

   (ii) Hence find the times when the points are at the same height during the interval $0 < x < 2\pi$ **(4)**

TOTAL = 20 MARKS

Exercise 22: Further Calculus 1

MARKS

1. Find the derivatives of:

   a) $f(x) = 3 \sin x$ b) $g(x) = -4 \cos x$ c) $y = 5 \cos x - \dfrac{3}{x^2}$ **(5)**

2. Find the rate of change of the function $f(x) = \dfrac{1}{2}\cos x$ when $x = \dfrac{2\pi}{3}$ **(4)**

3. Find the area bounded by the curve $y = 4 \sin x$, the x-axis and the lines $x = \dfrac{\pi}{2}$ and $x = \dfrac{3\pi}{2}$

   HINT: make a sketch first! **(5)**

4. Differentiate:

   a) $y = (3x - 5)^8$ b) $f(x) = \dfrac{1}{\sqrt{15-4x}}$ **(6)**

5. Differentiate:

   a) $f(x) = 3\cos 2x$ b) $g(x) = 2\sin\left(\dfrac{\pi}{4} - 3x\right)$. **(4)**

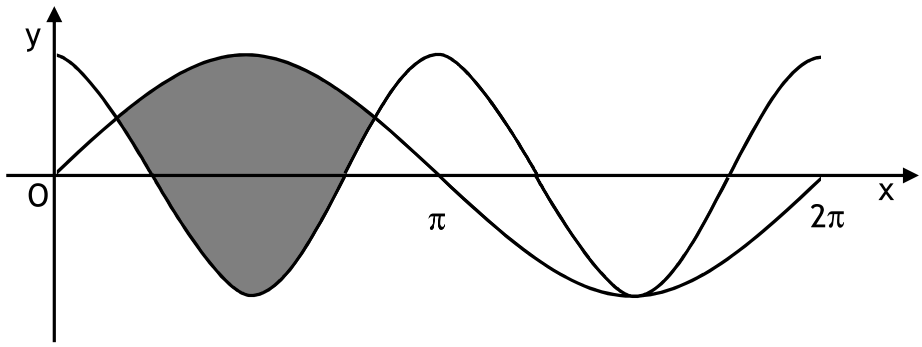
TOTAL: 24 MARKS

Exercise 23: Further Calculus 2

MARKS

1. Evaluate: $\int_0^1 \sqrt{1+3x}\,dx$ **(5)**

2. Part of the curves y = sin x and y = cos 2x are shown below.



Find the shaded area, given that the curves intersect where $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$ and $x = \frac{3\pi}{2}$. **(6)**

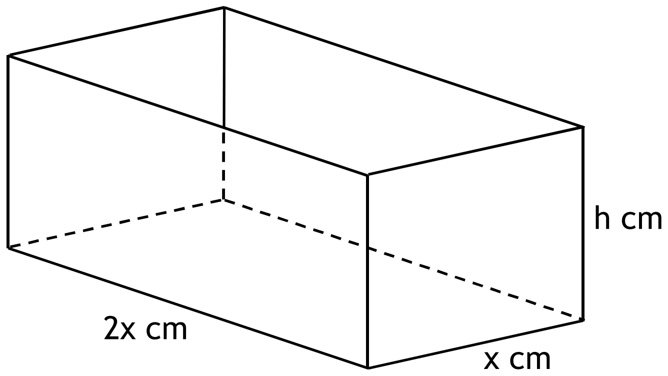
3. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = -6(5 - 2x)^2$.

Find the equation of the curve in terms of x and y, given that it passes through the point (2, -3). **(5)**

(SEE QUESTION 4 ON PAGE 26)

4. An open-topped glass fish tank has a volume of 288 litres.

The base is a rectangle with sides in the ratio 2:1 as shown.



a) Show that the surface area of the tank can be expressed as

$$A(x) = 2x^2 + \frac{864000}{x}$$ **(3)**

b) Find the value of x for which the surface area is a minimum. **(6)**

TOTAL = 25 MARKS

Exercise 24: Logarithmic and Exponential Functions 1

MARKS

1. Given that $\log_2 y = 3 \log_2 x + 3$, express y in terms of x. **(3)**

2. Find the value of *p* such that $\log_3 \left(\frac{1}{27}\right) = p$ **(2)**

3. Evaluate $5 \log_8 2 + \log_8 4 - \log_8 16$ **(4)**

4. Solve:

   a) $4^{(x + 1)} = 7$ (answer to 3 s.f) **(4)**

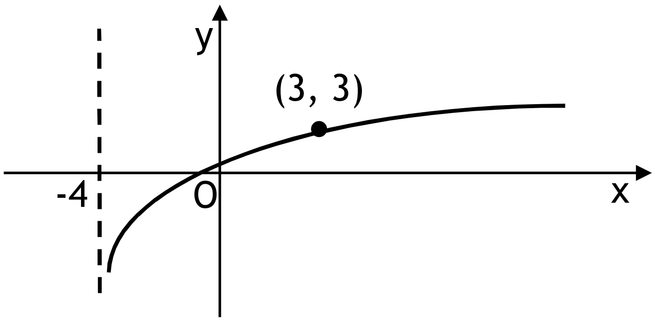
   b) $\log_2 (x - 4) + \log_2 (x + 2) = 4$ where $x > 4$. **(5)**

TOTAL = 18 MARKS

Exercise 25: Logarithmic and Exponential Functions 2

MARKS

1. Part of the graph of $y = \log_a (x + b)$ is shown.



State the values of a and b. **(2)**

2. The mass of a radioactive substance decays according to the formula

$$A_t = A_0 e^{-kt}$$

where *t* is the time in years.

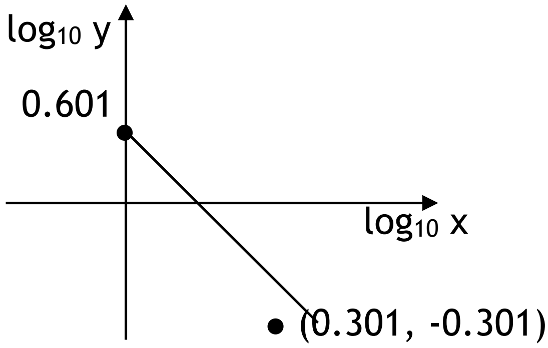
a) A 50g sample of the substance decays to 40g in 12 years.

Find the value of *k* accurate to 3 s.f. **(4)**

b) The half-life of a radioactive substance is defined as the time taken for half of the initial sample to decay.

Calculate the half-life of this substance. **(3)**

3. The results of an experiment are plotted on logarithmic axes and the graphs shown below is obtained.



The data conforms to the rule $y = kx^n$

a) Find the equation of the line in terms of $\log_{10} x$ and $\log_{10} y$. **(2)**

b) Hence determine the values of k and n. **(4)**

TOTAL = 15 MARKS