Compound Angle Formulae: sin(A + B) and sin(A – B)

Finding the value of a compound angle is not quite as simple as adding together the values of the component angles, e.g. $\sin 90° \neq \sin 60° + \sin 30°$. The following formulae must be used:

$\sin(A + B) = \sin A\cos B + \cos A\sin B$	$\sin(A - B) = \sin A\cos B - \cos A\sin B$

Example 1: Expand each of the following:

a) $\sin(X + Y)$

b) $\sin(Q + 3P)$

Example 2: Find the exact value of $\sin 75°$.

Example 3: A and B are acute angles where $\tan A = \frac{12}{5}$ and $\tan B = \frac{3}{4}$. Find the value of sin(A + B).

Example 4: Expand each of the following:

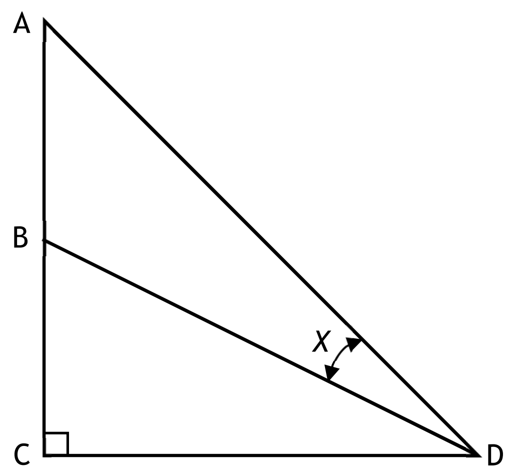
a) $\sin(\alpha - \beta)$

b) $\sin\left(2B - \frac{2\pi}{3}\right)$

Example 5: In the diagram opposite:

AC = CD = 2 units, and AB = BC = 1 unit.

Show that $\sin X$ is exactly $\frac{1}{\sqrt{10}}$.



cos(A + B) and cos (A – B)

cos(A + B) = cosAcosB - sinAsinB

cos(A - B) = cosAcosB + sinAsinB

Example 6: Expand the following:

a) cos(X – Y)

b) cos(X + 315)°

Example 7:

a) Show that $\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

b) Hence find the exact value of $\cos\frac{\pi}{12}$

To summarise:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

Double Angle Formulae

sin2A = sin(A + A)

=

cos2A = cos(A + A)

=

Since $\cos^2x° = 1 – \sin^2x°$ and $\sin^2x° = 1 – \cos^2x°$, we can further expand the formula for cos2A:

cos2A = $\cos^2A – \sin^2A$

=

cos2A = $\cos^2A – \sin^2A$

=

To summarise:

sin2A = 2sinAcosA

cos2A **= $\cos^2A$ - $\sin^2A$**

= $2\cos^2A$ - 1

= 1 – $2\sin^2A$

Example 8: Express the following using double angle formulae:

a) sin2X

b) sin6Y

c) cos2X (sine version)

d) cos8H (cosine version)

e) sin5Q

f) cosθ (cos and sin version)

Example 9: sinθ = $\frac{2}{\sqrt{13}}$, where θ is an acute angle. Find the exact values of:

a) sin(2θ)

b) cos(2θ)

Solving Trig Equations

Example 10 (Nat 5 Revision): Solve:

a) $\cos x^\circ = 0.588$ $\{ 0 \le x \le 360^\circ \}$

b) $3\sin x^\circ + 2 = 0$ $\{ 0 \le x \le 360^\circ \}$

Trig equations can also often involve multiple angles, compound angles or powers of sin, cos or tan. In all cases, you must pay close attention to the range of allowed values for *x*.

Example 11: Solve $\cos 2x^\circ = \frac{1}{2}$ for $0 \le x \le 360$

Example 12: Solve $3 \tan (3x - 105)° = 5$ for $0 \leq x \leq 240°$

Powers of sin, cos and tan

If a trig equation involves $\sin^2 x$ etc, rearrange as normal and take the square root of both sides. **Remember that this gives both a positive AND negative answer!**

Example 13: Solve $4\cos^2 x - 3 = 0$ for $0 \leq x \leq 2\pi$

Trig equations can also be written in forms which resemble quadratic equations: to solve these, treat them as such, and solve by factorisation.

Example 14: Solve $6\sin^2 x° - \sin x° - 2 = 0$ for $0 \leq x \leq 360°$

Solving Mixed Trig Equations

To solve trig equations with combinations of double- and single-angle angle terms:

- **Rewrite the double angle term using the formulae (use the formulae list!)**
- **Factorise**
- **Solve each factor for *x***

Example 15: Solve $\sin 2x° - 2\sin x° = 0$, $0 \le x \le 360°$

When the term is $\cos 2X$, the version of the double angle formula we use depends on the other trig term in the equation: use $2\cos^2 x - 1$ if the other term is $\cos x$; $1 - 2\sin^2 x$ if the other term is $\sin x$.

Example 16: Solve $2\cos 2x - 7\cos x = 0$, $0 \le x \le 2\pi$

Trigonometric Identities

NOTE: these are important formulae which are **not provided** in the exam paper formula sheets!

$$\frac{\sin x°}{\cos x°} = \tan x°$$

$$\sin^2 x° + \cos^2 x° = 1$$

Note that due to the second formula, we can also say that:

$$\cos^2 x° = 1 - \sin^2 x°$$

AND

$$\sin^2 x° = 1 - \cos^2 x°$$

To prove that an identity is true, we need to show that the expression on the left hand side of the equals sign can be changed into the expression on the right hand side.

Example 12: Prove that:

a) $\cos^4\alpha - \sin^4\alpha = \cos^2\alpha - \sin^2\alpha$

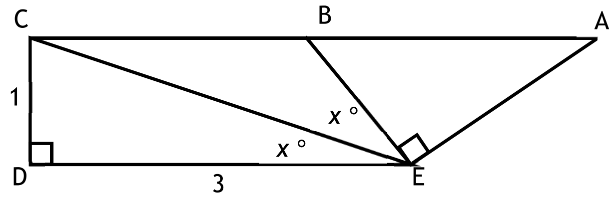
b) $\dfrac{1}{\tan x} - \tan x = \dfrac{2}{\tan 2x}$

Past Paper Example 1: In the diagram,

$\angle DEC = \angle CEB = x°$, and $\angle CDE = \angle BEA = 90°$.

CD = 1 unit; DE = 3 units.

By writing $\angle DEA$ in terms of x, find the exact value of $\cos(D\hat{E}A)$.



Past Paper Example 2: Find the points of intersection of the graphs of $y = 3\cos 2x° + 2$ and $y = 1 - \cos x°$ in the interval $0 \leq x \leq 360°$.

Past Paper Example 3: Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \leq x \leq 2\pi$$