Sketching Graphs (Revision)

In the exam, diagrams are provided whenever the question involves a graph. However, this is often not the case when working from the textbook: it is therefore important that we are able to sketch basic graphs where necessary, as the question usually becomes simpler when you can picture it.

Example 1: in the spaces provided, make a **basic** sketch of the graph(s) of the function(s) stated.

a) $y = 2x + 1$

b) $3x + 4y - 12 = 0$

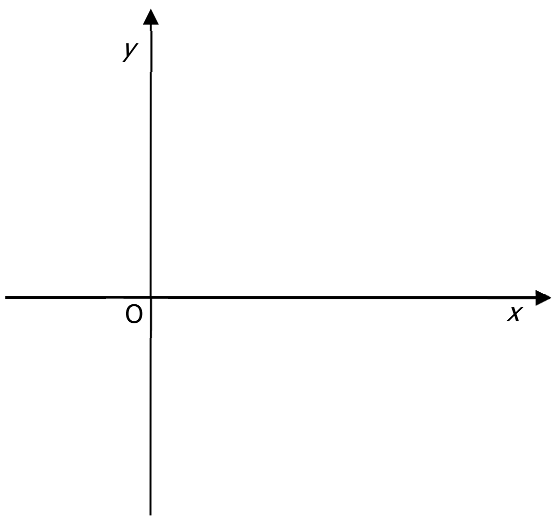
c) $y = -1$ **and** $x = 5$

d) $y = x^2$ **and** $y = 4$

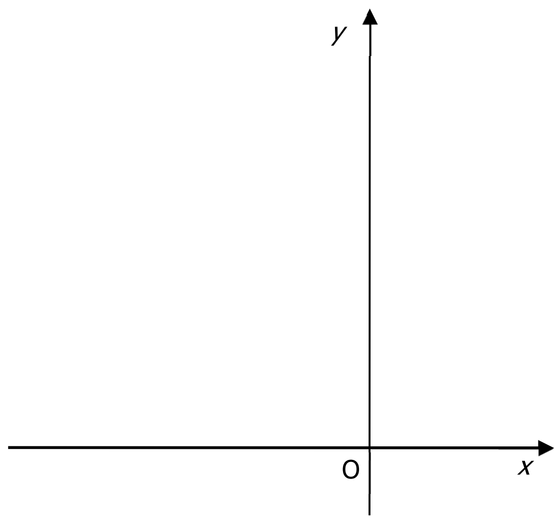
e) $y = x^2 - 4$

f) $y = (x - 2)^2$ **and** $y = 2x - x^2$

Example 2: Sketch and annotate the graph of $y = x^2 - 2x - 8$



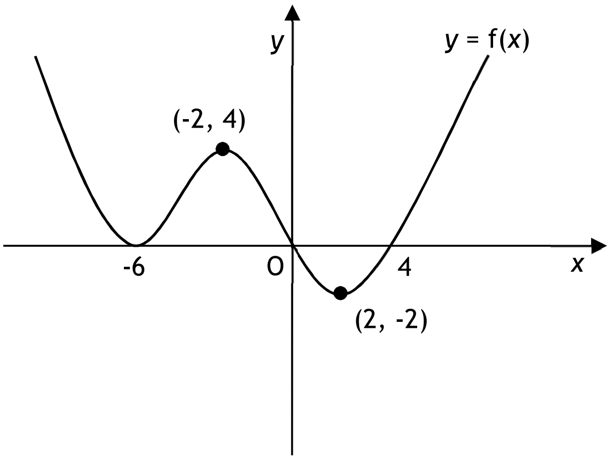
Example 3: Sketch and annotate the graph of $y = 11 - (x + 3)^2$



Transformation of Graphs

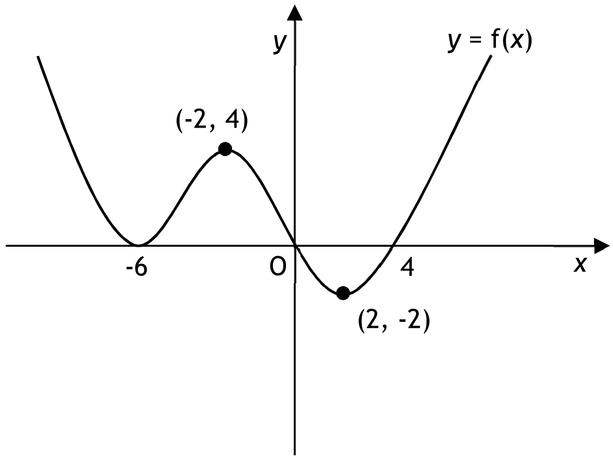
We have seen how the graph of y = sin(x) is different to that of y = sin(2x), and how $y = x^2$ differs from $y = (x - 1)^2$. The six operations below are used to transform the graph of a function:

y = f(*x*) + a



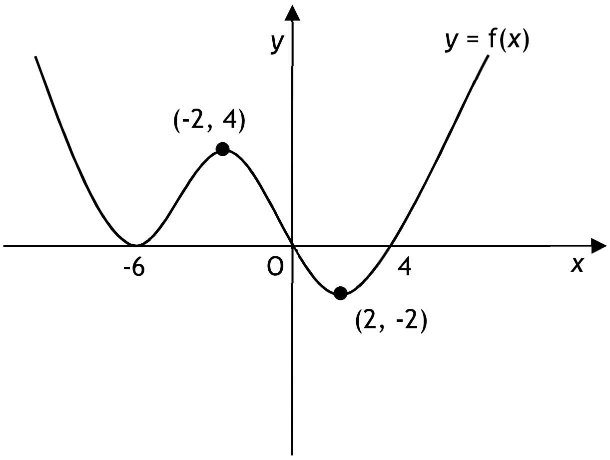
y = f(*x*) + a is obtained by **sliding** y = f(*x*):
Vertically upwards if a > 0
Vertically downwards if a < 0

y = f(*x* + a)



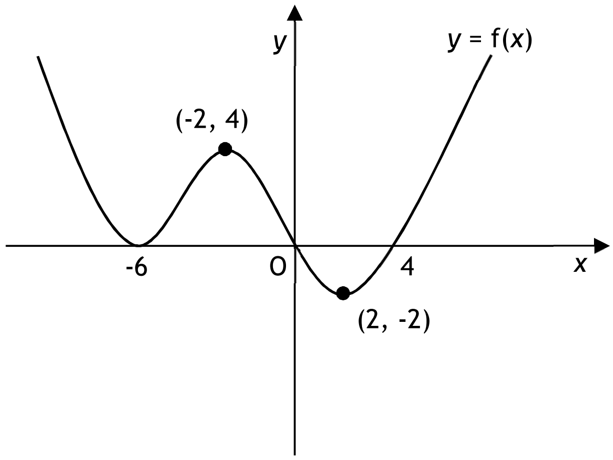
y = f(*x* + a) is obtained by **sliding** y = f(*x*):
Horizontally left if a > 0
Horizontally right if a < 0

y = k f(*x*)



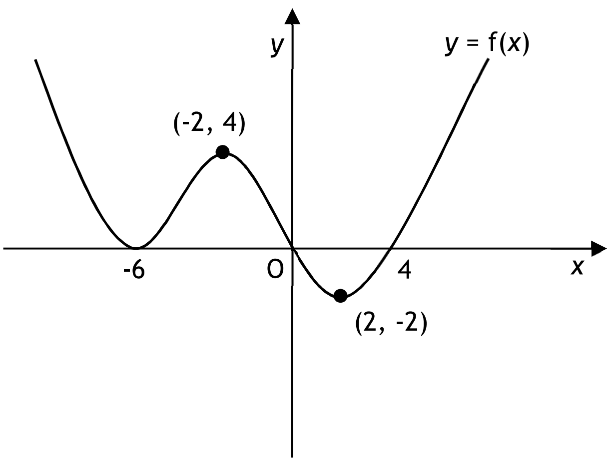
y = k f(*x*) is obtained by **vertically:**
stretching *y* = f(*x*) if k > 1
compressing *y* = f(*x*) if 0 < k < 1

y = f(k*x*)



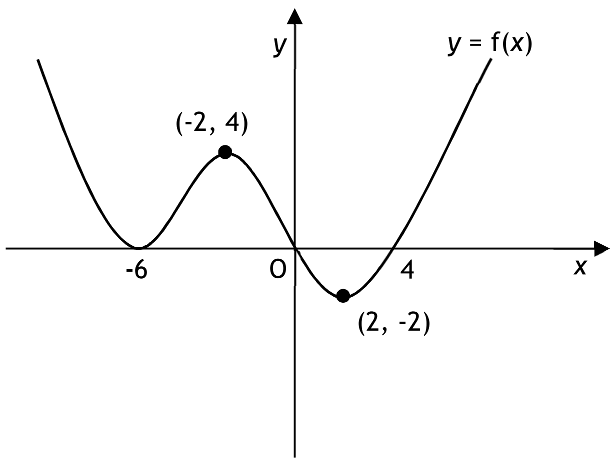
y = f(k*x*) is obtained by **horizontally:**
compressing *y* = f(*x*) if k > 1
stretching *y* = f(*x*) if 0 < k < 1

y = - f(*x*)



y = - f(*x*) is obtained by **reflecting** y = f(*x*):
in the *x* - axis

y = f(-*x*)

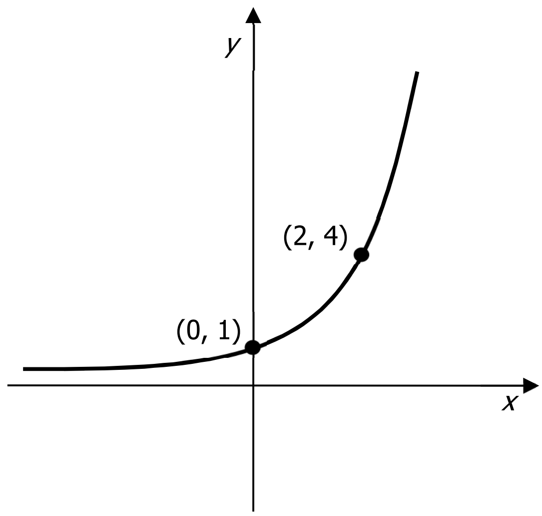


y = f(-*x*) is obtained by **reflecting** y = f(*x*):
in the *y* - axis

Example 4: To the graph of $y = 2^x$, add:

a) $y = 2^x - 3$

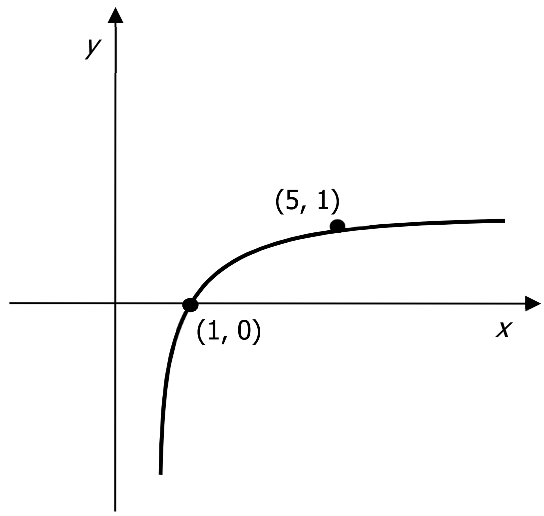
b) $y = 2^{(x - 2)}$



Example 5: To the graph of $y = \log_5 x$, add:

a) $y = 2\log_5 x$

b) $y = \log_5 (x + 1)$



Multiple Transformations

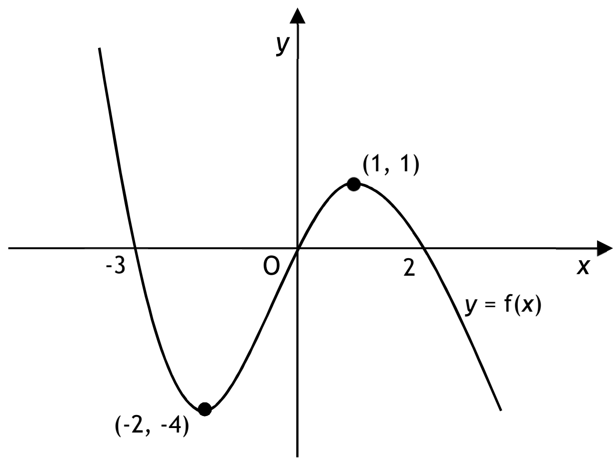
Often, are asked to perform more than one transformation on a graph. Where appropriate, always leave sliding vertically until last.

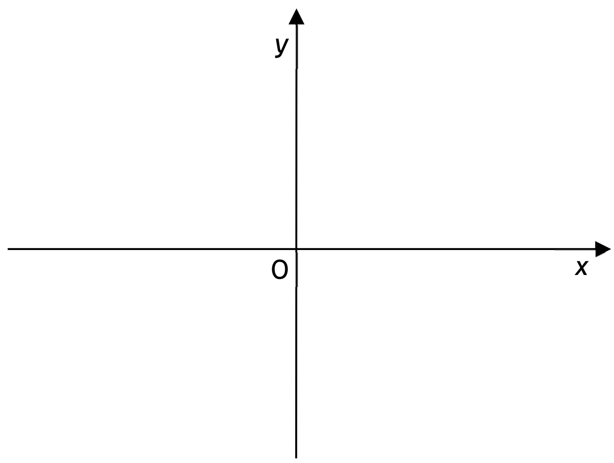
Example 6: Part of the graph of $y = f(x)$ is shown.

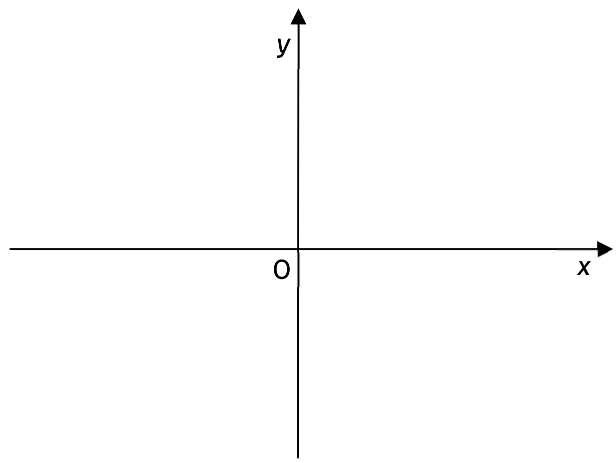
On separate diagrams, sketch:

a) $y = f(-x) + 2$

b) $y = -\frac{1}{2} f(x + 1)$





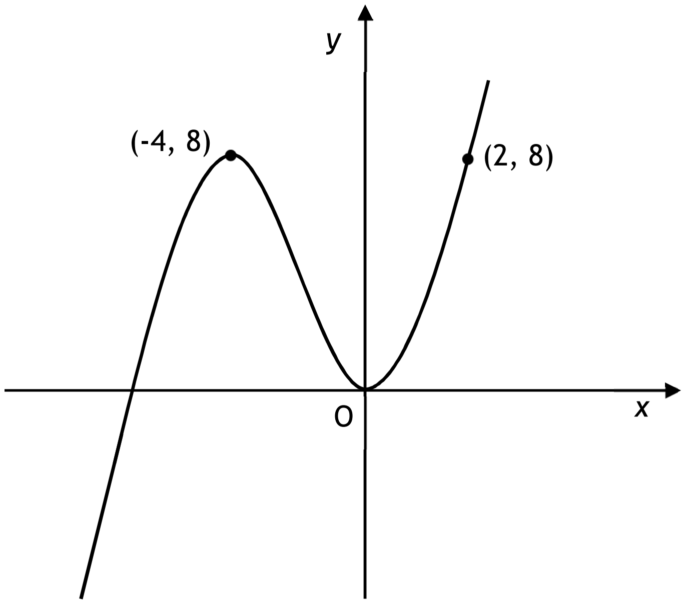


Past Paper Example: The diagram shows a sketch of the function $y = f(x)$.

To the diagram, add the graphs of:

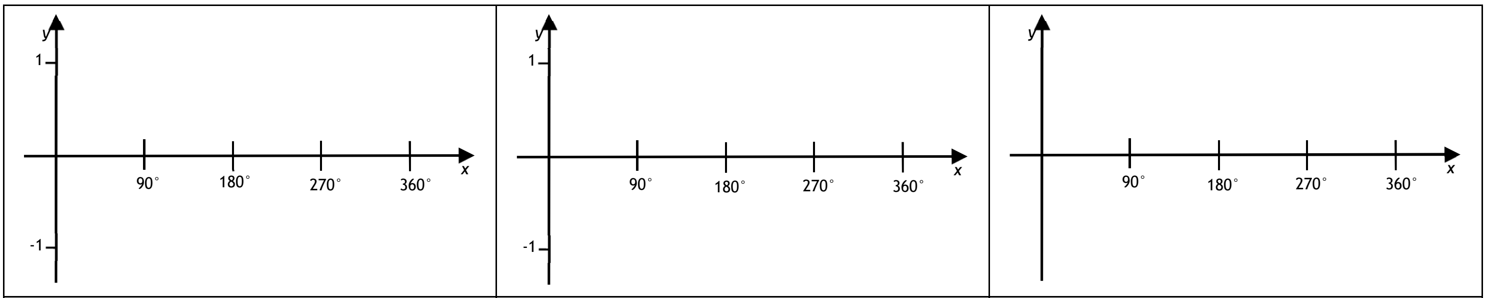
a) $y = f(2x)$

b) $y = 1 - f(2x)$.



Trigonometric Graphs

Example 1: Sketch the graphs of $y = \sin x°$, $y = \cos x°$ and $y = \tan x°$ below.



$y = \sin x°$	$y = \cos x°$	$y = \tan x°$

For trig graphs, how soon the graph repeats itself horizontally is known as the **period**, and half of the vertical height is known as the **amplitude.**

Function	Period	Amplitude
$y = \sin x°$		
$y = \cos x°$		
$y = \tan x°$		

$y = a \sin bx° + c$ and $y = a \cos bx° + c$:

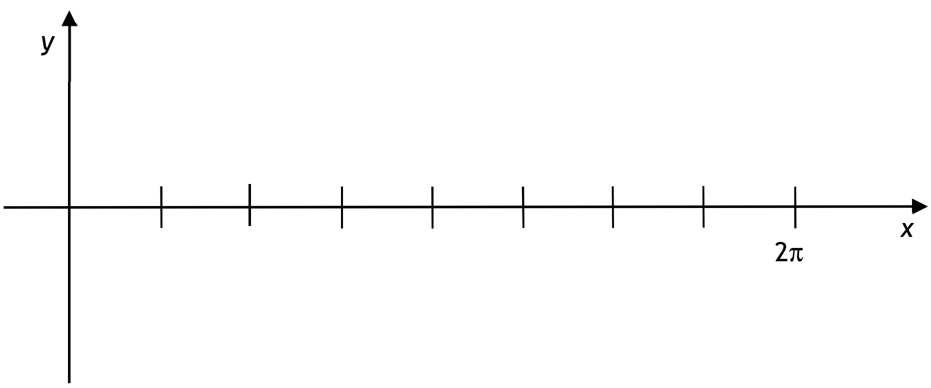
a = amplitude
b = waves in 360°
c = vertical shift

$y = a \tan bx° + c$:

b = "waves" in 180°
c = vertical shift

Compound Angles

A compound angle is one containing two parts, e.g. $(x - 60)°$. The graphs of trig functions including compound angles are shifted left or right along the x – axis (60° to the right in the example above).



Example 2: On the axes opposite, sketch:

a) $y = \sin x$

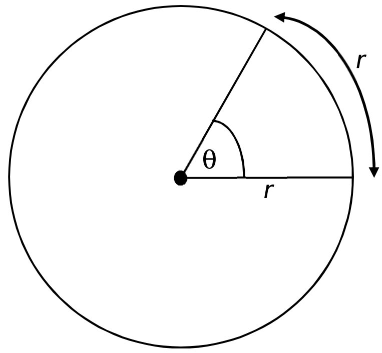
b) $y = a \sin\left(x - \frac{\pi}{4}\right)$

(where a > 0)

Radians

If we draw a circle and make a sector with an arc of exactly one radius long, then the angle at the centre of the sector is called a **radian.**

Remember that Circumference = $\pi D = 2\pi r$. This means that there are 2π radians in a full circle.



$360° = 2\pi$ radians

$180° = \pi$ radians

To convert $x°$ into radians:

Simplify the fraction $\frac{x\pi}{180}$

To convert $\frac{a\pi}{b}$ radians into degrees:

Find the fraction $\frac{a}{b}$ of 180°

Example 3: Convert:

a) 90° to radians

b) 60° to radians

c) 225° to radians

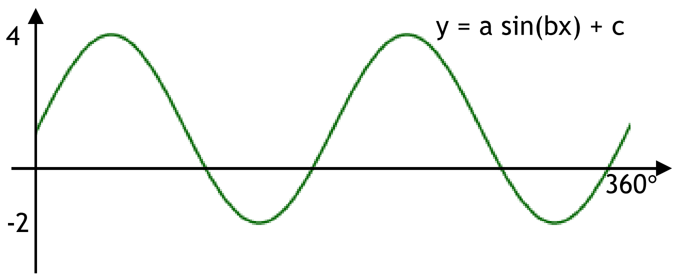
d) $\frac{\pi}{4}$ radians to degrees

e) $\frac{4\pi}{3}$ radians to degrees

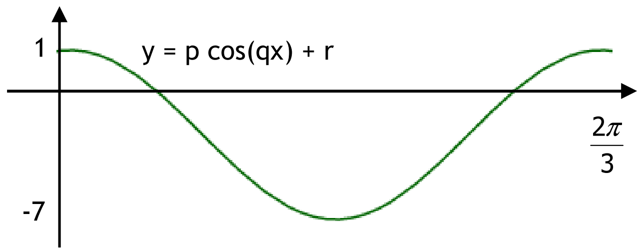
f) $\frac{11\pi}{6}$ radians to degrees

Example 4: y = a sin(bx) + c and y = p cos(qx) + r are shown below. State the values of:

a) a, b and c

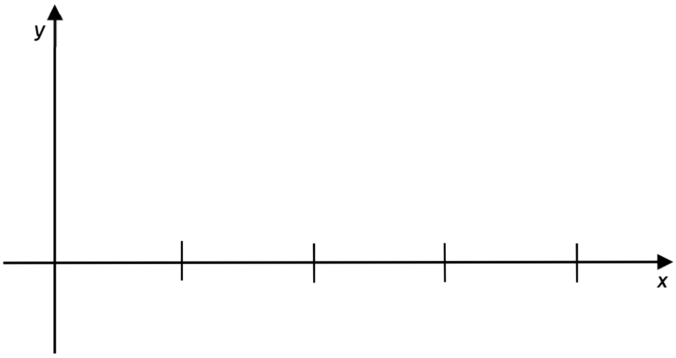


b) p, q and r

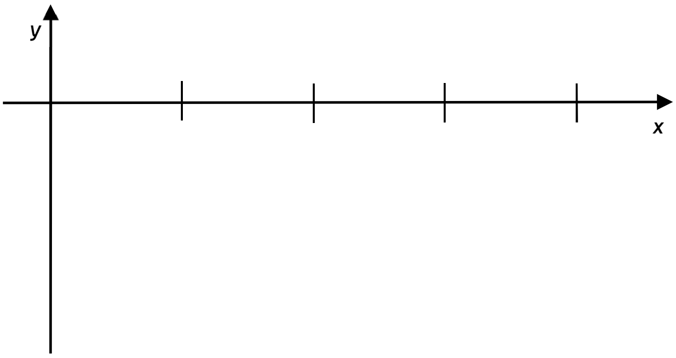


Example 5: Sketch the following graphs, showing the coordinates of the maximum and minimum points.

a) $y = 5\cos 2x° + 3 \quad \{0 \leq x \leq 360°\}$

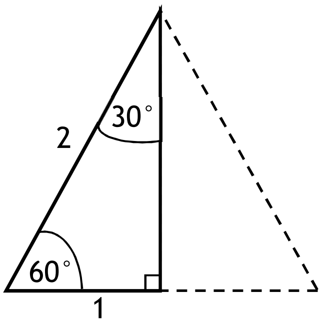


b) $y = -3\sin 3x - 2 \quad \{0 \leq x \leq 2\pi\}$

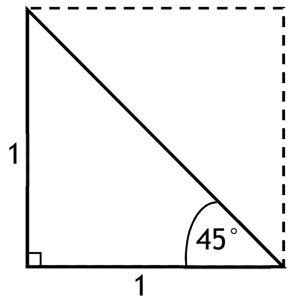


Exact Values

Consider the following triangles:



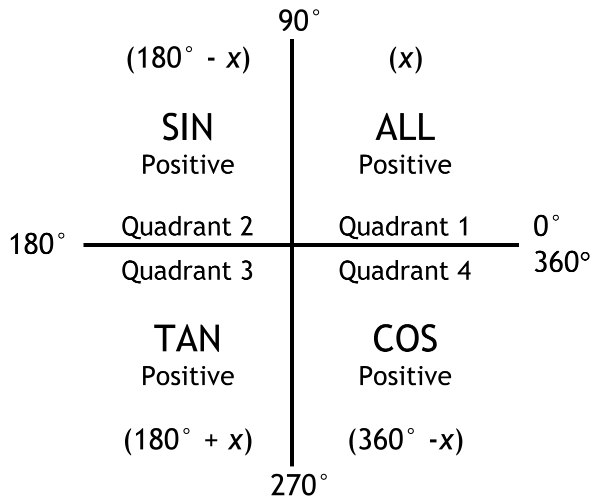
A right-angled triangle made by halving an equilateral triangle of side 2 units



A right-angled triangle made by halving an square of side 1 unit

Once we have found the lengths of the missing sides (by Pythagoras' Theorem), the following table of values can be constructed:

	0°	30°	45°	60°	90°
	0	$\left(\frac{\pi}{6}\right)$	$\left(\frac{\pi}{4}\right)$	$\left(\frac{\pi}{3}\right)$	$\left(\frac{\pi}{2}\right)$
Sin					
Cos					
Tan					



Example 6: State the exact values of:

a) sin 150°

b) tan 315°

c) $\cos\frac{7\pi}{6}$