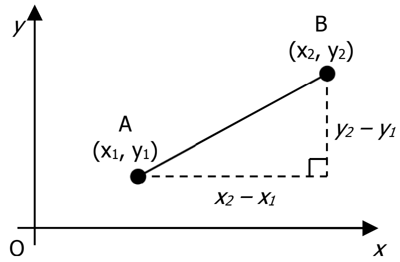
The Straight Line

Revision from National 5

The graph of $y = mx + c$ is a **straight line**, where m is the gradient and $(0, c)$ is the y-intercept.

Gradient is a measure of the steepness of a line. The gradient of the line joining points A (x_1, y_1) and B (x_2, y_2) is given by:



$$\mathbf{m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}}$$

Example 1: Find:

a) the gradient and y-intercept of the line $y = 2x + 5$

b) the equation of the line with gradient – 4 and y-intercept (0, -2)

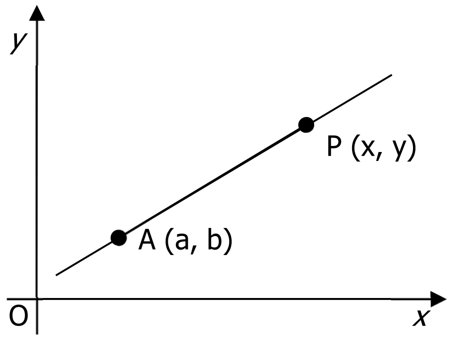
c) the gradient of the line joining P (-2, 4) and Q (3, -1)

d) the gradient of the line $3y + 4x - 11 = 0$

Equation of a Straight Line: $y - b = m(x - a)$

Points A (a, b) and P (x, y) both lie on a straight line.

The gradient of the line $m = \frac{y - b}{x - a}$. Rearranging this gives:



$$\mathbf{y - b = m(x - a)}$$

NOTE: when you are asked to find the equation of a straight line, it must only have **one** number term, e.g. $y - 5 = 2(x + 3)$ should be expanded and simplified to $y = 2x + 11$.

Example 2: Find the equations of the lines:

a) through (4, 5) with m = 2

b) joining (-1, -2) and (3, 10)

c) parallel to the line $x - 2y + 4 = 0$ and passing through the point (2, -3)

The General Equation of a Straight Line: Ax + By + C = 0

Example 3: Find the equation of the line through (-5, -1) with m = $-\frac{2}{3}$, giving your answer in the form Ax + By + C = 0.

Example 4: Sketch the line 5x – 2y – 24 = 0 by finding the points where it crosses the x – and y – axes.

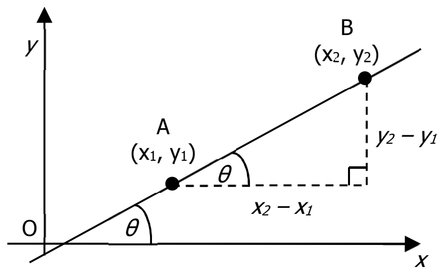
The Angle with the x-axis

The gradient of a line can also be described as the angle it makes with the ***positive*** direction of the x -axis.

As the y-difference is OPPOSITE the angle and the x-difference is ADJACENT to it, we get:

$$\mathbf{m_{AB} = \tan\theta}$$

(where θ is measured ANTI-CLOCKWISE from the x-axis)



Example 5: Find the angle made with the positive direction of the x -axis and the lines:

a) $y = x - 1$

b) $y = 5 - \sqrt{3}x$

c) joining the points (3, -2) and (7, 4)

Gradients of straight lines can be summarised as follows:

a) lines sloping **up** from left to right have **positive** gradients and make **acute** angles with the positive direction of the x-axis
b) lines sloping **down** from left to right have **negative** gradients and make **obtuse** angles with the positive direction of the x-axis
c) lines with **equal** gradients are parallel
d) **horizontal** lines (parallel to the ***x* -axis**) have gradient **zero** and equation y = a
e) **vertical** lines (parallel to the ***y* -axis**) have gradient **undefined** and equation x = b

Collinearity

If three (or more) points lie on the same line, they are said to be **collinear.**

Example 6: Prove that the points D (-1, 5), E (0, 2) and F (4, -10) are collinear.

Perpendicular Lines

If two lines are perpendicular to each other (i.e. they meet at 90°), then:

$m_1m_2 = -1$

Example 7: State whether these pairs of lines are perpendicular:

a) y = 2x - 5
6y = 10 – 3*x*

b) 2x – 3y = 5
3x = 2y + 9

When asked to find the gradient of a line perpendicular to another, follow these steps:

1. **Find the gradient of the given line**
2. **Flip it upside down**
3. **Change the sign (e.g. negative to positive)**

Example 8: Find the gradients of the lines perpendicular to:

a) the line *y* = 3*x* - 12

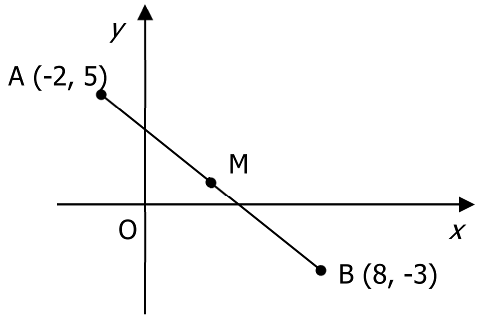
b) a line with gradient = -1.5

c) the line 2*y* + 5*x* = 0

Example 9: Line L has equation x + 4y + 2 = 0. Find the equation of the line perpendicular to L which passes through the point (-2, 5).

Midpoints and Perpendicular Bisectors

The **midpoint** of a line lies exactly halfway along it. To find the coordinates of a midpoint, find halfway between the *x* – and *y* – coordinates of the points at each end of the line (see diagram).



The *x* – coordinate of M is halfway between -2 and 8, and its *y* – coordinate is halfway between 5 and -3.

In general, if M is the midpoint of A (x_1, y_1) and B (x_2, y_2):

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The **perpendicular bisector** of a line passes through its midpoint at 90°.

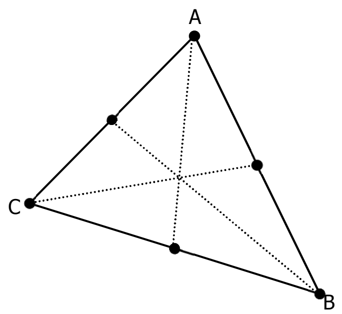
Example 10: Find the perpendicular bisector of the line joining F (-4, 2) and G (6, 8).

To find the equation of a perpendicular bisector:

- Find the gradient of the line joining the given points
- Find the perpendicular gradient (flip and make negative)
- Find the coordinates of the midpoint
- Substitute into $y - b = m(x - a)$

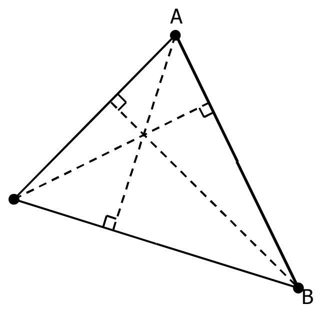
Lines Inside Triangles: Medians, Altitudes & Perpendicular Bisectors

In a triangle, a line joining a corner to the **midpoint** of the opposite side is called a **median.**



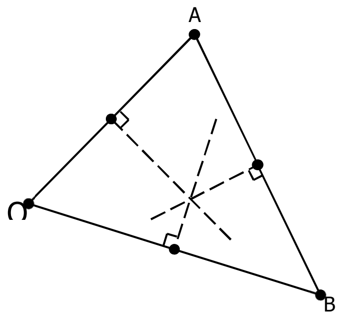
The medians are **concurrent** (i.e. meet at the same point) at the **centroid**, which divides each median 2:1

A line through a corner which is **perpendicular** to the opposite side is called an **altitude.**



The altitudes are concurrent at the **orthocentre**

A line at 90° to the midpoint is called a **perpendicular bisector.**

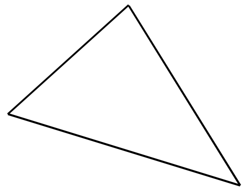


The perpendicular bisectors are concurrent at the **circumcentre**

For all triangles, the centroid, orthocentre and circumcentre are **collinear.**

Example 11: A triangle has vertices P (0, 2), Q (4, 4) and R (8, -6).

a) Find the equation of the median through P.



To find the equation of a median:

- Find the midpoint of the side opposite the given point
- Find the gradient of the line joining the given point and the midpoint
- Substitute into $y - b = m(x - a)$

b) Find the equation of the altitude through R.

To find the equation of an altitude:

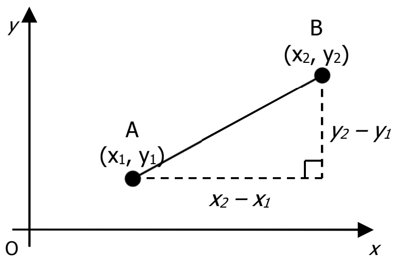
- Find the gradient of the side opposite the given point
- Find the perpendicular gradient (flip and make negative)
- Substitute into $y - b = m(x - a)$

Distance between Two Points

The distance between any two points A (x_1, y_1) and B (x_2, y_2) can be found easily by Pythagoras' Theorem.

If d is the distance between A and B, then:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



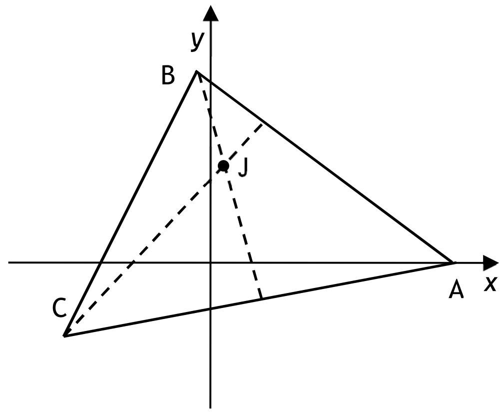
Example 12: Calculate the distance between:

a) A (-4, 4) and B (2, -4)

b) X (11, 2) and Y (-2, -5)

Example 13: A is the point (2, -1), B is (5, -2) and C is (7, 4). Show that BC = 2AB.

Past Paper Example 1: A triangle has vertices A (7, 0), B (-1, 8) and C (-3, -2). Find:



a) The equation of the altitude through C.

b) The equation of the median through B.

c) The coordinates of J, the point of intersection between the altitude and median.

Past Paper Example 2:

a) Find the equation of l_1, the perpendicular bisector of the line joining P (3, -3) to Q (-1, 9).

b) Find the equation of l_2 which is parallel to PQ and passes through R (1, -2).

c) Find the point of intersection of l_1 and l_2 and hence find the shortest distance between PQ and l_2.