CNHS Higher HW Solutions

Week 6 [15/03/19]

Qs 76 - 90

76. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 8u_n + 12$ with $u_0 = 4$.

(a) Explain why this sequence has a limit.
(b) Find the value of the limit.

(a) limit exists as -1<0.8<1 (b) L= $\dfrac{12}{1-0.8} = \dfrac{12}{0.2} = \dfrac{120}{2} = 60$

77. Calculate the limit as $n \to \infty$ of the sequence defined by $u_{n+1} = 0 \cdot 9u_n + 10$ with $u_0 = 1$.

L= $\dfrac{10}{1-0.9} = \dfrac{10}{0.1} = \dfrac{100}{1} = 100$

78. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

(a) Write down the condition on *k* for the sequence to have a limit.

(b) The sequence tends to a limit of 5 as $n \to \infty$.
Find the value of *k*.

(a) -1<k<1 (b) $\dfrac{3}{1-k} = 5....3 = 5 - 5k....5k = 2....k = \dfrac{2}{5}$

79. A slow puncture causes a tyre to lose 11% of its pressure each day.
To compensate for this, every morning the tyre is inflated by adding 3 units of pressure.

(a) If u_n is the pressure of the tyre after *n* days, write down a recurrence relation for u_{n+1} in terms of u_n.

(b) If this pattern continues indefinitely, explain why the pressure in the tyre will approach a limit and find the value of this limit correct to 1 decimal place.

(a) u_{n+1}=0.89u_n+3 (b) -1<0.89<1 so limit exists…L= $\dfrac{3}{1-0.89} = \dfrac{3}{0.11} = \dfrac{300}{11} = 27.3$

80. A sequence u_1, u_2, u_3, … is generated by the recurrence relation $u_{n+1} = au_n + b$.

(a) The first two terms are $u_1 = 4$ and $u_2 = 5$. Write down an equation in *a* and *b*.

(b) The third term is $u_3 = 7$. Write down a second equation in *a* and *b*.

(c) Find the values of *a* and *b*.

(a) 4a+b=5 (b) 5a+b=7 (c) a = 2 and b = - 3

81. At the same time every day a doctor gives a patient a 250 mg dose of an antibiotic.
It is known that over a 24 hour period, the amount of antibiotic in the bloodstream is reduced by 80%.

(a) Write down a recurrence relation for the amount of antibiotic in the patient's bloodstream immediately after a dose.

(b) It is known that more than 350 mg of the antibiotic in the bloodstream will result in unpleasant side effects.

Is it safe to administer this antibiotic over an extended period of time?
Explain your answer.

(a) $U_{n+1} = 0.2U_n + 250$ (b) Limit = 312.5mg < 350mg so it is safe.

82. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 6$ with $u_2 = 14$.

Find the value of u_0.

$U_0 = -1$

83. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 4u_n + 90$.

(a) Explain why the sequence will approach a limit as $n \to \infty$.

(b) Find the limit of this sequence.

(a) $-1 < 0.4 < 1$ (b) 150

84. A toad falls to the bottom of a well.

Each day, the toad climbs 13 feet and then rests overnight. During the night, it slides down $\frac{1}{4}$ of its height above the floor of the well.

(a) Write down a recurrence relation for t_{n+1} in terms of t_n, where t_n is the height reached by the toad at the end of n days.

(a) Given that the well is 50 feet deep, determine whether or not the toad will eventually escape from the well.

t_{n+1} = 0.75 t_n + 13 L = 52…52>50 so yes it will escape.

85. Calculate the discriminant and state the nature of the roots of each quadratic equation.

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 + 2x + 7 = 0$ (c) $4x^2 - 12x + 9 = 0$

(a) b^2-4ac=12 real and distinct (b) b^2-4ac= - 24 no real roots (c) b^2-4ac=0 real and equal

86. The equation $2x^2 + kx + k = 0$ has equal roots, where $k \neq 0$.
Find the value of *k*.

k^2-4.2.k=0…k^2-8k=0…k(k-8)=0….**k=8**

87. A function *f* is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

Find the remainder when $f(x)$ is divided by:

(a) $(x-1)$ (b) $(x+2)$

(a) **r = 4** (b) **r = - 11**

88. Show that $(x-3)$ is a factor of $f(x) = x^3 + 4x^2 - 11x - 30$ and hence factorise $f(x)$ fully.

r=0 therefore (x-3) is a factor…….**(x-3)(x+2)(x+5)**

89. Given that $(x-3)$ is a factor of $2x^3-9x^2+kx-3$, find the value of k.

3k-30=0….3k=30….**k=10**

90. A function f is defined by $f(x)=2x^3-7x^2+9$ where x is a real number.

(a) Show that $(x-3)$ is a factor of $f(x)$ and hence factorise $f(x)$ fully.

(b) Find the coordinates of all the points where the curve with equation $y=f(x)$ crosses the coordinate axes.

(a) (x-3) (x-$\frac{3}{2}$) (2x+2) (b) x = 3 , x = $\frac{3}{2}$, x = -1