CNHS Higher HW Solutions

Week 5 [08/03/19]

Qs 61 - 75

61. Find $f'(x)$ when: (a) $f(x)=x(x-1)^2$ (b) $f(x)=\dfrac{2x^6-5}{x^2}$

(a) $f(x)=x(x^2-2x+1)=x^3-2x^2+x \ldots f'(x)=\mathbf{3x^2-4x+1}$
(b) $f(x)=2x^4-5x^{-2} \ldots f'(x)=\mathbf{8x^3+10x^{-3}}$

62. A parabola has equation $y=\frac{1}{2}x^2-8x+34$.

The gradient of the tangent to the parabola at the point P is 4.
Find the coordinates of P.

$m=y'=x-8=4 \ldots x=12$
$y=\frac{1}{2}12^2-8.12+34=72-96+34=10$ **P(12,10)**

63. Find the coordinates of the stationary points of the curve $y=x^3-3x^2+4$ and justify their nature.

$y'=3x^2-6x=0$ @$x=0$, $y=0^3-3 \times 0^2+4=4$ nature table/2nd derivative
$3x(x-2)=0$ @$x=2$, $y=2^3-3(2)^2+4=0$
$x=0, x=2$

(0,4) is a max t.p.
(2,0) is a min t.p.

64. The amount of a drug, in milligrams, remaining in a person's bloodstream *t* minutes after being administered is given by $A(t)=300\sqrt{t}-15t$. Find the rate of change of *A* with respect to *t* when $t=36$.

$A'(t)=\dfrac{150}{t^{\frac{1}{2}}}-15 \ldots A'(36)=\dfrac{150}{36^{\frac{1}{2}}}-15=\dfrac{150}{6}-15=25-15=\mathbf{10}$

65. Given that $f(x)=4x^{\frac{3}{2}}-2\sqrt{x}$, find $f'(x)$.

$f'(x)=12x^{\frac{1}{2}}-x^{\frac{-1}{2}}$

66. Find the equation of the tangent to the curve with equation $y=\frac{8}{x}$ at the point P where $x=4$.

$m=y'=\dfrac{-8}{4^2}=\dfrac{-1}{2} \ldots y=\dfrac{8}{4}=2$ giving pt (4,2)…. **x+2y-8=0**

67. Show that the line $y = 8x - 11$ is a tangent to the parabola $y = 3x^2 - 4x + 1$ and find the coordinates of the point of contact.

(a) b^2-4ac=0 or repeated root…point of contact **(2 , 5)**

68. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

(a) Find the value of x for which the gradient of the tangent at P is 12.

(b) Hence find the equation of the tangent at P.

(a) x = 2 (b) y = 12x - 8

69. A ball is thrown vertically upwards.
The height, h metres, of the ball after t seconds is given by the formula $h = 30t - 5t^2$.

(a) The velocity, v metres per second, of the ball after t seconds is given by $v = \frac{dh}{dt}$.

Find a formula for v in terms of t and hence find the velocity of the ball after 3 seconds.

(b) Explain your answer to part (a) in terms of the ball's movement.

(a) $v = \frac{dh}{dt}$ **= 30 – 10t…@ 3 secs = 0** **(b) Ball has reached maximum height**

70. A curve has equation $y = x^4 - 2x^3 + 5$.
Find the equation of the tangent to the curve at the point where $x = 2$.

y = 5

71. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds.

A mathematical model suggests that P and x are related by the formula $P = 12x^3 - x^4$ for $0 < x < 12$.

Find the value of x in the interval $0 < x < 12$ which gives the maximum profit, justifying your answer.

x = 9 (£9000)

72. Find the stationary points on the curve with equation $y = x^3 - 3x^2 - 24x - 28$ and justify their nature.

(-2, 0) is a max t.p. and (4, - 108) is a min t.p.

73. The area, A cm^2, of tin required to make a box with a square base is given by the formula

$$A = x^2 + \frac{250}{x}, \qquad x > 0,$$

where x cm is the side length of the base.

Find the value of x for which this area is a minimum, justifying your answer.

x = 5cm

74. The equation of a curve is $y = 2x^3 - 3x^2$.

(a) Find the coordinates of the two points where the curve crosses the x-axis.

(b) Find the coordinates of the stationary points on the curve and determine their nature.

(a) (0, 0) and $(\frac{3}{2}, 0)$ **(b) (0, 0) is a max t.p and (1, 0) is a min t.p**

75. (a) A sequence is defined by the recurrence relation $u_{n+1} = 2u_n - 5$ with $u_0 = 6$.

Find the value of u_3.

(b) A second sequence us defined by the recurrence relation $v_{n+1} = \frac{1}{3}v_n + 4$.

If $v_2 = 10$, find the value of v_1.

(a) U_1=7, U_2=9, U_3=2x9-5=**13** **(b)** $\frac{1}{3}v_1 + 4 = 10 \ldots \frac{1}{3}v_1 = 6 \ldots v_1 = 18$