CNHS Higher HW Solutions

Week 4 [01/03/19]

Qs 46 - 60

46. Write each quadratic expression in the form $p(x+q)^2+r$:

(a) $2x^2+4x+7$ (b) $15+2x-x^2$

(a) $2(x+1)^2+5$ (b) $-(x-1)^2+16$

47. What is the value of $\sin\frac{5\pi}{6}-\tan\frac{3\pi}{4}$?

Do not use a calculator!

1½

48. Find $f'(x)$ when:

(a) $f(x)=2x^3+4x^2-3x+1$ (b) $f(x)=\frac{1}{2}x^4+\frac{1}{3}x^3+x$

$f'(x)=6x^2+8x-3$ $f'(x)=2x^3+x^2+1$

49. Given $f(x)=3x^2(2x-1)$, find the value of $f'(-1)$.

$f(x)=3x^2(2x-1)=6x^3-3x^2\ldots f'(x)=18x^2-6x\ldots f'(-1)$ =18x(-1)^2-6x(-1)=18+6=**24**

50. Find the gradient of the tangent to the curve with equation $y=x^3-4x+1$ at the point where $x=-2$.

@ $x=-2$, y=(-2)^3-4.(-2)+1=-8+8+1=1…pt (-2,1)
m=y'=3x^2-4…@ $x=-2$, m=3x(-2)^2-4=12-4=8

y-1=8(x+2)
y-1=8x+16
y =8x+17

51. Find $f'(x)$ when: (a) $f(x)=\frac{2}{x^5}$ (b) $f(x)=8\sqrt{x}$

(a) f(x)=2x^{-5}…f '(x)= **$-10x^{-6}$** (b)f(x)=8x$^{1/2}$…f '(x)=**$4x^{-\frac{1}{2}}$**

52. Find $f'(x)$ when: (a) $f(x) = 2x^{\frac{3}{2}} + \dfrac{3}{x^2}$ (b) $f(x) = \dfrac{1}{4x^3}$

(a) $f(x) = 2x^{\frac{3}{2}} + 3x^{-2}$ …f ‘(x)=3x $^{\frac{1}{2}}$ -6x^{-3} (b) $f(x) = \dfrac{1}{4}x^{-3}$ …f ‘(x)= $\dfrac{-3}{4}x^{-4}$

53. Find the equation of the tangent to the curve $y = x^2 - 3x + 6$ at the point (4, 10).

m=y’=2x-3…@x=4, m=2(4)-10=-2...y-10=-2(x-4)…y-10=-2x+8…**y= -2x+18**

54. Recall that $f'(x)$ measures the rate of change of a function $f(x)$

(a) If $s(t) = t^2 - 5t + 8$, find the rate of change of *s* with respect to *t* when $t = 3$.

s‘(t)=2t-5…s’(3)=2(3)-5 = 6 – 5 = 1

(b) The volume of a sphere is given by the formula $V = \dfrac{4}{3}\pi r^3$.

Find the rate of change of *V* with respect to *r* when $r = 2$.

V’(r)= $4\pi r^2$ …V’(2)= $4\pi 2^2 = 16\pi$

55. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature.

y’=3x^2-6x-9=0	@x= -1, y= (-1)^3-3 x (-1)^2-9(-1) +12=17	nature table/2nd derivative
x^2-2x-3=0	@x=3, y=3^3-3(3)^2-9(3) +12= -15	
(x-3)(x+1)=0		**(-1,17) is a max t.p.**
x= -1, x=3		**(3,-15) is a min t.p.**

56. Given that $f(x) = \sqrt{x} + \dfrac{2}{x^2}$, find the value of $f'(4)$.

$f(x) = x^{\frac{1}{2}} + 2x^{-2} \ldots f'(x) = \dfrac{1}{2}x^{\frac{-1}{2}} - 4x^{-3} = \dfrac{1}{2\sqrt{x}} - \dfrac{4}{x^3} \ldots f'(4) = \dfrac{1}{2\sqrt{4}} - \dfrac{4}{4^3} = \dfrac{1}{4} - \dfrac{4}{64} = \dfrac{3}{16}$

57. Find the equation of the tangent to the curve $y = x^3 - 9x$ at the point where $x = -2$.

@x=-2, y=$(-2)^3$-9(-2)=-8+18=10..pt (-2,10)
m=y'=$3x^2$-9…@x=-2, m=$3x(-2)^2$-9=12-9=3

y-10=3(x+2)
y-10=3x+6
y=3x+16

58. Find the stationary points on the curve with equation $y = x^3 - 9x^2 + 24x - 2$ and determine their nature.

y'=$3x^2$-18x+24=0
x^2-6x+8=0
(x-4)(x-2)=0
x= 2, x=4

@x= 2, y= 2^3-9 x 2^2 +24(2)-2=18
@x=4, y=4^3-9$(4)^2$+24(4) -2 = 14

nature table/2nd derivative

(2,18) is a max t.p.
(4,14) is a min t.p.

59. If $p = \frac{4}{x^3}$, find the rate of change of p with respect to x when $x = 2$.

$p = \frac{4}{x^3}$ =$4x^{-3}$…p'=$-12x^{-4}$=$\frac{-12}{x^4} \ldots \frac{-12}{2^4} = \frac{-12}{16} = \frac{-3}{4}$

60. Given $f(x) = 2\sqrt{x} + \frac{3}{x^2}$, find the exact value of $f'(4)$.

f(x)=$2x^{½}$ +$3x^{-2}$ …f '(x)=$x^{-½}$ -$6x^{-3}$=$\frac{1}{\sqrt{x}} - \frac{6}{x^3}$ …f '(4)= $\frac{1}{\sqrt{4}} - \frac{6}{4^3} = \frac{1}{2} - \frac{6}{64} = \frac{26}{64} = \frac{13}{32}$