CNHS Higher HW Solutions

Week 3 [22/02/19]

Qs 29 - 45

29. Solve the equation $2\cos x = \sqrt{3}$, where $0 \le x < 2\pi$.

$\cos x = \frac{\sqrt{3}}{2}$ …x= $\frac{\pi}{6}, \frac{11\pi}{6}$

30. (a) Express $2x^2 + 8x + 7$ in the form $a(x+b)^2 + c$.

(b) Hence **write down** the coordinates of the turning point on the parabola with equation $y = 2x^2 + 8x + 7$.

(a) $2(x+2)^2 - 1$ (b) (-2,-1)

31. The function f is defined by $f(x) = \frac{5}{x^2 - 3x + 2}$.

Which two values of x must be excluded from the domain of the function f?

$x^2 - 3x + 2 \neq 0 \ldots (x+3)(x-1) \neq 0 \ldots\ldots\ldots x \neq -3, x \neq 1$

32. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ in the interval $\frac{\pi}{2} \le x \le \pi$?

$\sin x = \frac{\sqrt{3}}{2}$ …acute angle = $\frac{\pi}{3}$ …angle in interval = $\frac{2\pi}{3}$

33. (a) Express $3x^2 - 12x + 10$ in the form $a(x+b)^2 + c$.

(b) Hence **write down** the coordinates of the turning point on the parabola with equation $y = 3x^2 - 12x + 10$.

(a) $3(x-2)^2-2$ (b) (2,-2)

34. Find the solution of the equation $\sqrt{2}\cos x + 1 = 0$ in the interval $\pi \le x \le \frac{3\pi}{2}$.

cos x = $\frac{-1}{\sqrt{2}}$ ….R.A.A.= $\frac{\pi}{4}$ …x= $\frac{3\pi}{4}$ $\frac{5\pi}{4}$

35. Functions *f* and *g* are defined on the set of all real numbers by $f(x) = 2x + 1$ and $g(x) = 4x - 2$. Find $f(f(x)) + g(g(x))$ in the form $ax + b$.

f(f(x)) = 2(2x+1)+1=4x+3
g(g(x)) = 4(4x-2)-2=16x-10 4x+3+16x-10=**20x-7**

36. The function *f* is defined on the set of real numbers by $f(x) = 3x - 8$.
Find an expression for the inverse function $f^{-1}(x)$ and find the value of $f^{-1}(7)$.

$f^{-1}(x) = \frac{x+8}{3}$ … $f^{-1}(7) = \frac{7+8}{3} = 5$

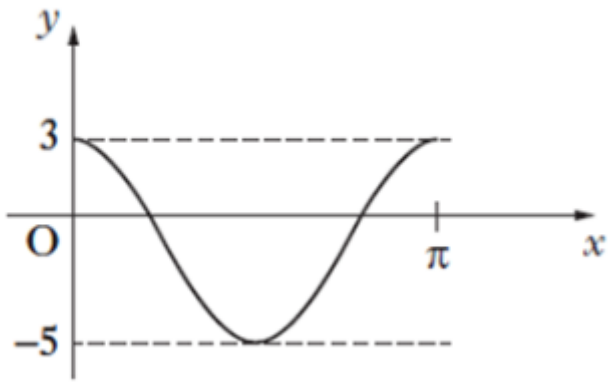
37. Express $f(x) = (2x - 1)(2x + 5)$ in the form $a(x + b)^2 + c$.

$4(x+1)^2 - 9$

38. Functions *f* and *g* are given by $f(x) = 3x^2 - 1$ and $g(x) = x^2 + 2$.
Express $f(g(x))$ in the form $ax^4 + bx^2 + c$.

$3x^4 + 12x^2 + 11$

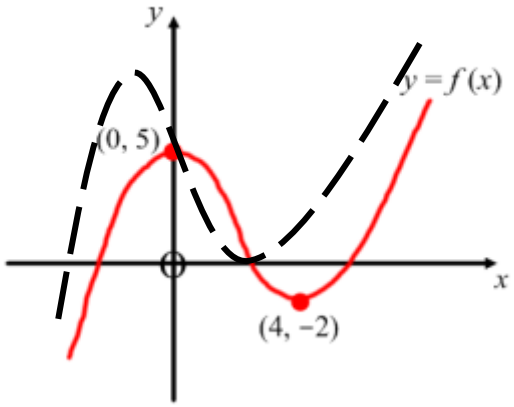
39. The diagram below shows the graph of $y = a\cos(bx) + c$



Write down the values of *a*, *b* and *c*.

a = 4 b = 2 c = - 1

40. The diagram shows the graph of $y = f(x)$.
The graph has a maximum turning point at (0, 5) and a minimum turning point at (4, –2).



Sketch the graph of $y = f(x-1)+2$.

Sketch as above ANNOTATED with max tp (1,7) and min tp (5,0)

41. **Do not use a calculator in this question!**

(a) State the value of $\sin\frac{\pi}{4}$ (b) Work out the value of $\cos\frac{2\pi}{3}$.

(c) Hence evaluate $4\sqrt{2}\sin\frac{\pi}{4}\cos\frac{2\pi}{3}$.

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{-1}{2}$ **(c) 2**

42. Functions *f* and *g* are defined on the set of all real numbers by $f(x) = x^2 + 3$ and $g(x) = x + 4$.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Show that the equation $f(g(x)) + g(f(x)) = 0$ has no real roots.

(a) $(x + 4)^2 + 3$ and $x^2 + 7$ (b) $b^2-4ac=4^2-4x1x13= -36\ldots-36<0$ so no real roots

43. The functions f and g are defined on suitable domains by

$$f(x) = 4x - 3 \quad \text{and} \quad g(x) = \sqrt{x+1}.$$

(a) A third function, h, is defined by $h(x) = g(f(x))$.
Find an expression for $h(x)$.

(b) State the largest possible domain for h.

(a) by $h(x) = g(f(x)) = \sqrt{4x-3+1} = \sqrt{4x-2}$ **(b)** $\mathbf{x \geq \frac{1}{2}}$

44. The function f is defined on the set of all real numbers by $f(x) = 3x - 10$.
Find a formula for the inverse function $f^{-1}(x)$.

$\mathbf{\frac{x+10}{3}}$

45. Functions f and g are defined by $f(x) = 3x + 5$ and $g(x) = 2 - x$.
Find an expression for $f(g(x))$ and find the value of x for which $f(g(x)) = 32$.

$f(g(x)) = 11 - 3x$ $\quad x = -7$