CNHS Higher HW Solutions

Week 10 [12/04/19]

Qs 136 - 150

136. If $y = \frac{1}{x^3} - \cos 2x$, find $\frac{dy}{dx}$.

$\frac{dy}{dx}$ = - $3x^{-4}$ + 2sin2x

137. (a) Given that $f(x) = \sin^3 x$, find $f'(x)$.

[*Hint*: write $f(x) = (\sin x)^3$ and use the chain rule]

(a) If $y = (1 + \cos 2x)^4$, find $\frac{dy}{dx}$.

(a) $3\sin^2 x\cos x$ (b) – $8\sin 2x(1+\cos 2x)^3$

138. Use the chain rule to find $f'(x)$ when:

(a) $f(x) = (5x+2)^4$ (b) $f(x) = \sqrt{8x+1}$

(a) $f'(x) = 20(5x+2)^3$ (b) $\frac{4}{\sqrt{8x+1}}$

139. Find: (a) $\int 6\cos 2x\,dx$ (b) $\int 2\sin 4x\,dx$

(a) 3 sin 2x + c (b) – ½ cos 4x + c

140. A curve has equation $y = \sqrt{x^2 + 5}$.

(a) Use the chain rule to find $\frac{dy}{dx}$.

(b) Hence find the equation of the tangent to the curve at the point where $x = 2$.

(a) $\frac{1}{2}(x^2 + 5)^{-\frac{1}{2}} . 2x = \frac{x}{\sqrt{x^2+5}}$ (b) 3y = 2x + 5

141. (a) A function f is defined by $f(x) = (1 - x^3)^{\frac{1}{3}}$.
Use the chain rule to find $f'(x)$.

(b) Find $\int 6\cos 2x dx$.

(a) $-x^2(1-x^3)^{-2/3}$ (b) 3sin2x+c

142. (a) Use the chain rule to differentiate $f(x) = (1 + \sin x)^4$.

(b) Find $\int \left(\frac{2}{x^4} + \cos 5x \right) dx$.

(a) $4\cos x(1+\sin x)^3$ (b) $\frac{-2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$

143. (a) For what value of k does the equation $kx^2 - 6x + 1 = 0$ have equal roots?

(b) Find $\int \sqrt{6x+1}dx$. [*Hint*: use $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$]

(a) k=9 (b) $\frac{(6x+1)^{3/2}}{9} + c$

144. An oil production platform is to be connected by a pipeline to a refinery on shore. The length of the underwater part of the pipeline is x kilometres.

The total cost of building a pipeline is $C(x)$ million pounds, where

$$C(x) = 2x + 100 - \sqrt{x^2 - 243}.$$

Show that $x = 18$ gives the minimum cost and find this minimum cost.

Justify with nature table. £127 million

145. (a) Given that $f(x)=\sqrt{3x^2+2}$, use the chain rule to find $f'(x)$.

(b) Evaluate $\int_0^{\frac{\pi}{6}} \cos 2x\,dx$.

(a) $3x(3x^2+2)^{-1/2}$ (b) $\frac{\sqrt{3}}{4}$

146. (a) Find $\int \frac{1}{\sqrt{3x+4}}dx$. [*Hint*: use $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}+C$]

(b) Given that $\int_4^a \frac{1}{\sqrt{3x+4}}dx = 2$, find the value of *a*.

(a) $\frac{2}{3}(3x+4)^{\frac{1}{2}}+c$ (b) a = 15

147. A circle with centre C(–2, 1) passes through the point P(5, –2).

(a) Use the distance formula (or otherwise) to find the radius of the circle.

(b) Hence write down the equation of the circle.

(a) radius = $\sqrt{58}$ (b) $(x+2)^2+(y-1)^2 = 58$

148. A circle has equation $x^2+y^2+8x+6y-75=0$.
Find the centre and radius of this circle.

centre (- 4 , - 3) radius = 10

149. A circle has equation $(x-3)^2+(y+2)^2=25$.

(b) Write down the coordinates of C, the centre of the circle.

(b) Find the equation of the tangent at the point P(6, 2) on the circle.

(a) (3,-2) (b) 4y = - 3x + 26 or equivalent

150. Find the coordinates of the two points of intersection of the line with equation $y = 2x + 5$ and the circle with equation $x^2 + y^2 - 6x - 2y - 30 = 0$.

(-3,-1) and (1,7)